Solving Vendor Selection Problem with Fuzzy Parameters Using Interval Programming with Fuzzy Numbers

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Abstract—Vendor Selection (VS) problem is considered as a critical decision that effects on the financial position for any firms. Hence the purchasing managers consider the selection process of the right vendors is a major function of the outsourcing process. Cost, quality, and services are three common criteria for VS problem. In the real cases the parameters of any problems are usually vague or not completely known, so that the Fuzzy Set Theory (FST) is one of the most powerful existing tools capable of addressing ambiguity and blurring information in parameters. A fuzzy parameters for VS problem are dealt with Linear Membership Function (LMF) through using $\alpha$—cut that convert a fuzzy numbers to Interval Multiple Objective Linear Programming (IMOLP). The formulation of IMOLP is determined based on the value of $\alpha$, then IMOLP model is divided to upper and lower sub-models. A compromise interval solution will obtain from solving the sub-models as conventionally linear programming. The proposed approach is clarified with an illustrative example.

Keywords—Vendor selection problem; Interval linear programming; Multi-objective programming; Fuzzy numbers; $\alpha$ cut; Compromise interval solution.

I. INTRODUCTION

Under intense competition in the industrial markets, firms are looking for a secure and stable supply process in which the needs of firms from raw materials and components necessary for the inventory and manufacturing process are outsourcing from a number of vendors capable of performing the supply process with the highest level of efficiency to achieve the company’s objectives that keep the company Always in competition. This VS process is the most important role for procurement management because about 80% of the final product cost depends on the purchasing cost of the raw materials and components needed for the manufacturing process. The VS process is the process of identifying a number of vendors from among the company's nominated vendors to ensure that the company obtains the quality, price, and delivery. The VS process includes several criteria that vary according to the company's activity. However, the most common standards in most companies are quality, price, and delivery. The process of selecting a vendor is a complex process because it is inherently a Multi-Attribute Decision Making process and because of the different performance of each vendor according to the criteria.

In real cases, the information and data entered into the company, which represent the parameters of the model in the process of formulation, are often vague and incomplete. Because of this, the FST is one of the most powerful existing tools capable of addressing ambiguity and blurring information in parameters.

In this paper, we apply the LMF corresponding to fuzzy parameters of VS problem through using $\alpha$—cut a fuzzy numbers. The proposed method adopts the $\alpha$—cut method to convert a fuzzy VS problem to IMOLP. Then the IMOLP divided to upper and lower sub-models. The proposed approach applied to obtain a compromise interval solution that can be effective to help a manager to select right vendors.

This paper is organized as follows: Section 2 introduces a brief literature review about VS problem and different methods that applied to deal with this problem. In section 3, various preliminaries are reviewed. Problem statement is introduced in section 4. In section 5, a methodology for solving VS problem is given. In section 6, an implement of methodology on illustrative numerical example is given. Finally discussion the conclusion is reviewed in section 7.

II. LITERATURE REVIEW

In multi-criteria VS problem, after first study of Dickson [1] on VS was recognized 23 criteria to select the suitable vendor, since then many articles presented significant criteria for VS problem [2-7]. A numerous researches and articles
applied a several methodologies to evaluate and select the right vendors. A mathematical programming model is the most common approaches have been used for VS problem. They contain many approaches, for example Linear Programming (LP), Mixed Integer Programming (MIP), Goal Programming (GP), and Multiple Objective Programming (MOP). According to [8] a linear weighting method is applied to evaluate vendors based on criteria, a LP model developed by [9] to achieve the criteria of VS at minimum aggregate price. The author[10] presented a GP approach to achieve minimum cost and maximum quality and delivery respectively for VS problem. The author[11] presented the data envelopment analysis for select proper vendors. A mixed-integer non-linear programming approach is discussed by [12]. A MOP has been introduced by [13]. In realistic cases, due to the imprecision of the data related to coefficient of VS problem formulation such as capacity of vendors, prices of the units and (or) available budget, the (FST) introduced by [14] is a powerful tool for handling uncertainty and obtaining a best result than other ordinary models. A fuzzy MIP model that includes fuzzy parameters for objectives and constraints has been treated by [15, 16] proposed a fuzzy method for VS Problem that helps a Decision maker (DM) to assign distinct weights to various criteria. In [17], interactive methodology using S-curve membership functions is presented to treat VS problem with fuzzy coefficients. The author [18] presented a fuzzy multi-criteria method to for VS problem with vague data. a new concept of solution in [19] is introduced, negative and positive solution by applied integrated fuzzy theory with TOPSIS to rank vendors based on ideal performance. In [20], a VS problem is formulated with uncertain demand to overcome imprecision data and multi-units.

III. PRELIMINARIES

In this section some basic and important definitions in which needed in the next section.

Definition 3.1. A FST was initiated by [14]. He defined a fuzzy set as \( \{ (x, \mu_A(x)) : x \in X \} \) where \( \mu_A(x) \) measures the degree to which element \( x \) belongs to a fuzzy set \( \tilde{A} \), i.e., \( \mu_A(x) : \rightarrow [0,1] \).

Definition 3.2. An \( \alpha \)- cut of a fuzzy set \( \tilde{A} \) is an ordinary set \( L_\alpha(\tilde{A}) \) where \( L_\alpha(\tilde{A}) = \{ x \in R : \mu_A(x) \geq \alpha \} , \alpha \rightarrow [0,1] \).

Definition 3.3. A Triangular Fuzzy Numbers (T.F.Ns) can be expressed as \( \tilde{A} = (a, b, c) \). T.F.Ns can be illustrated as interval \( [\bar{A}_\alpha, \bar{A}_\alpha] \), where

\[
\bar{A}_\alpha = \alpha(b-a) + a \\
\bar{A}_\alpha = -\alpha(c-b) + c
\]

IV. VENDOR SELECTION METHODOLOGY WITH BOTH FUZZY AND INTERVAL PROGRAMMING

The same multi-objective VS problem model introduced by [21] is considered as follows:

4.1 Notation

(i.) \( D \): Demand for the product item through stable period.

(ii.) \( P_i \): Price of a per unit product item of the ordered quantity \( x_i \) to the vendor \( i \).

(iii.) \( F_i \): Accepted units percentage provided by vendor \( i \).

(iv.) \( S_i \): Percentage of on-time deliveries by vendor \( i \).

(v.) \( C_i \): Upper capacity for vendor \( i \).

\[ \min z_1(x) = \sum_{i=1}^{n} P_i(x_i) \]  \hspace{1cm} (1)

\[ \max z_2(x) = \sum_{i=1}^{n} F_i(x_i) \]  \hspace{1cm} (2)

\[ \max z_3(x) = \sum_{i=1}^{n} S_i(x_i) \]  \hspace{1cm} (3)

\[ \text{s.t} \]

\[ \sum_{i=1}^{n} x_i = D \]  \hspace{1cm} (4)
\[ x_i \leq C_i \quad \forall i = 1, n \]  \hspace{1cm} (5) \[ x_i \geq 0 \]  \hspace{1cm} (6)

### 4.2 Fuzzy Vendor selection Problem:

In real cases situations, the parameters of VS problem are imprecise. Due to vagueness of information, a VS problem is considered with fuzzy parameters. The model of VS problem is transformed to fuzzy model with fuzzy parameters as:

\[
\min z_i(x) = \sum_{i=1}^{n} \tilde{P}_i(x_i)
\]

\[
\max z_2(x) = \sum_{i=1}^{n} \tilde{F}_i(x_i)
\]

\[
\max z_3(x) = \sum_{i=1}^{n} \tilde{S}_i(x_i)
\]

s.t.
\[
\sum_{i=1}^{n} x_i = \tilde{D}
\]
\[ x_i \leq C_i \quad \forall i = 1, n \]  \hspace{1cm} (11) \[ x_i \geq 0 \]  \hspace{1cm} (12)

Where \( \tilde{P}_i, \tilde{F}_i, \tilde{S}_i \) and \( \tilde{D}_i \) \( \forall i = 1, 2, ..., n \) are fuzzy parameters. These fuzzy parameters are characterized by Tr.F.Ns. The LMF is considered for both fuzzy goals and constraints respectively. We apply the \( \alpha \) – cut of the fuzzy numbers \( \tilde{P}_i, \tilde{F}_i, \tilde{S}_i \) and \( \tilde{D}_i \) defined as the crisp set \( (\tilde{P}_i, \tilde{F}_i, \tilde{S}_i, \tilde{D}_i)_{\alpha} \) for which the degree of the membership functions for these parameters exceeds the level of \( \alpha \).

\[
(\tilde{P}_i, \tilde{F}_i, \tilde{S}_i, \tilde{D}_i)_{\alpha} = \{P_i, F_i, S_i, D_i : \forall i = 1, 2, ..., n \ \mu_{P_i}(P_i) \geq \alpha, \mu_{F_i}(F_i) \geq \alpha, \mu_{S_i}(S_i) \geq \alpha, \mu_{D_i}(D_i) \geq \alpha\}
\]

The \( \alpha \) – cut method is used to convert the (Tr.F.Ns) to (IMOLP). The fuzzy VS problem transformed into IMOLP as:

\[
\min z_i(x) = \sum_{i=1}^{n} P_i^\alpha(x_i)
\]

\[
\max z_2(x) = \sum_{i=1}^{n} F_i^\alpha(x_i)
\]

\[
\max z_3(x) = \sum_{i=1}^{n} S_i^\alpha(x_i)
\]

s.t.
\[
\sum_{i=1}^{n} x_i = \tilde{D}^\alpha
\]
\[ x_i \leq C_i \quad \forall i = 1, n \]  \hspace{1cm} (17) \[ x_i \geq 0 \]  \hspace{1cm} (18)

According to [22] the model (13-18) divided into two sub-models as in (19-25) and (26-32), the optimum values of objectives for lower bound obtaining from solving (19-25), similarly the optimum values of objectives for upper bound obtaining from solving (26-32) as:

\[
\min z_i(x) = \sum_{i=1}^{n} P_i^-(x_i)
\]

\[
\max z_2(x) = \sum_{i=1}^{n} F_i^-(x_i)
\]
\[
\text{max } z_3(x) = \sum_{i=1}^{n} S_i^-(x_i) \quad (21)
\]

s.t.
\[
\sum_{i=1}^{n} x_i \geq \tilde{D}^- \quad (22)
\]
\[
\sum_{i=1}^{n} x_i \leq \tilde{D}^+ \quad (23)
\]
\[
x_i \leq C_i \quad \forall i = 1, n \quad (24)
\]
\[
x_i \geq 0 \quad (25)
\]

\[
\text{min } z_3(x) = \sum_{i=1}^{n} P_i^+(x_i) \quad (26)
\]
\[
\text{max } z_2(x) = \sum_{i=1}^{n} F_i^+(x_i) \quad (27)
\]
\[
\text{max } z_3(x) = \sum_{i=1}^{n} S_i^+(x_i) \quad (28)
\]

s.t.
\[
\sum_{i=1}^{n} x_i \geq \tilde{D}^- \quad (29)
\]
\[
\sum_{i=1}^{n} x_i \leq \tilde{D}^+ \quad (30)
\]
\[
x_i \leq C_i \quad \forall i = 1, n \quad (31)
\]
\[
x_i \geq 0 \quad (32)
\]

[23] developed [24] approach for fuzzy decision making. [23] converted the fuzzy MOP model to equivalent crisp linear model and solved it conventionally. We depend on this approach to handle the fuzzy parameters of VS problem with conventional programming model. [23] proposed fuzzy linear program by solving the multiple objective individually to obtain the upper bound \( Z^i_j = \max Z_j \) and lower bound \( Z^0_j = \min Z_j \).

Based on defining the values of upper and lower bound for the parameters, assuming the LMF for maximization and minimization fuzzy goals as:

\[
\mu_{Z_j^i}(x) = \begin{cases} 
1, & \text{for } Z_i \leq Z_j(x) \\
\frac{Z_j(x) - Z^0_i}{Z^1_i - Z^0_i}, & \text{for } Z^0_i < Z_j(x) < Z^1_i \\
0, & \text{for } Z^0_i \geq Z_j(x)
\end{cases} \quad (33)
\]

\[
\mu_{Z_j^0}(x) = \begin{cases} 
1, & \text{for } Z_j(x) \leq Z^0_i \\
\frac{Z^1_i - Z_j(x)}{Z^1_i - Z^0_i}, & \text{for } Z^0_i < Z_j(x) < Z^1_i \\
0, & \text{for } Z_j(x) \geq Z^1_i
\end{cases} \quad (33)
\]

Now, based on the LMF defined in (33) and (34), we adopt the approach suggested by [23]. By introduce the auxiliary variable \( \lambda \) corresponding to the grades of overall achievement for the objectives. the equivalent crisp model is obtained as:

\[
\text{max } \lambda \quad (35)
\]

subjectto
\[
\mu_j(z_j(x)) \geq \lambda, \quad \text{for fuzzy goals} \]
\[
x_i \in X, \quad \forall i = 1, n; \quad \text{(deterministic constraints)}
\]
\[
0 \leq u \leq 1,
\]
\[
x_i \geq 0, \quad \forall i = 1, n.
\]
4.3 Steps of the proposed method:

(1) Formulate the fuzzy VS problem model.

(2) Ask the DM to select initial value of $\alpha \in [0,1]$.

(3) Based on value of $\alpha$ cut, construct the IMOLP model.

(4) The IMOLP model divided into upper and lower bounds sub-models respectively.

(5) Solve the objectives of a multi-objective VS problem individually with other constraints to determine the both upper and lower bound respectively.

(6) Educe the membership function (33), (34) for each fuzzy goal.

(7) Formulate (35) based on step (6).

(8) By any linear software package solver, solve (35) and obtain the compromise interval solution.

The steps (5) - (8) is repeated for both bounds sub-model.

V. ILLUSTRATIVE EXAMPLE

The same case study used in [21] and [25] is considered.

Table 1: fuzzy data for the VS problem

<table>
<thead>
<tr>
<th>Vendor $i$</th>
<th>$P_i$%</th>
<th>$F_i$%</th>
<th>$S_i$%</th>
<th>$C_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>$\tilde{5}$</td>
<td>$0.80$</td>
<td>$0.90$</td>
<td>400</td>
</tr>
<tr>
<td>$V_2$</td>
<td>$\tilde{7}$</td>
<td>$0.90$</td>
<td>$0.80$</td>
<td>450</td>
</tr>
<tr>
<td>$V_3$</td>
<td>$\tilde{4}$</td>
<td>$0.85$</td>
<td>$0.85$</td>
<td>450</td>
</tr>
</tbody>
</table>

Step 1: construct fuzzy VS problem. The parameters are characterized by T.F.Ns:

$Min \ z_1 = (4.5,6)x_1 + (5.7,8)x_2 + (2,4.6)x_3$

$Max \ z_2 = (0.70,0.80,0.85)x_1 + (0.75,0.90,0.95)x_2 + (0.80,0.85,0.90)x_3$

$Max \ z_3 = (0.80,0.90,0.95)x_1 + (0.75,0.80,0.90)x_2 + (0.70,0.85,0.90)x_3$

s.t.

$x_1 + x_2 + x_3 = (700,800,850)$

$x_1 \leq 400$

$x_2 \leq 450$

$x_3 \leq 450$

Step 2: To transform the fuzzy VS problem into IMOLP, the $\alpha$ - cut is applied. By assuming $\alpha = 0$, we have the following problem:

$Min \ z_1 = [4,6]x_1 + [5,8]x_2 + [2,6]x_3$

$Max \ z_2 = [0.70,0.85]x_1 + [0.75,0.95]x_2 + [0.80,0.90]x_3$

$Max \ z_3 = [0.80,0.95]x_1 + [0.75,0.90]x_2 + [0.70,0.90]x_3$

s.t.

$x_1 + x_2 + x_3 = [700,850]$

$x_1 \leq 400$

$x_2 \leq 450$

$x_3 \leq 450$

Step 4: Divide the IMOLP model into lower and upper bounds sub-models:

According to (4) and (5) the two sub-models are considered as:

The lower bound model:

$Min \ z_1 = 4x_1 + 5x_2 + 2x_3$

$Max \ z_2 = 0.70x_1 + 0.75x_2 + 0.80x_3$
\textbf{Max } z_3 = 0.80x_1 + 0.75x_2 + 0.70x_3 \\
\textbf{s.t.} \\
x_1 + x_2 + x_3 \leq 850 \\
x_1 + x_2 + x_3 \geq 700 \\
x_1 \leq 400 \\
x_2 \leq 450 \\
x_3 \leq 450 \\
x_1, x_2, x_3 \geq 0 \\

The Upper bound model: \\
\textbf{Min } z_1 = 6x_1 + 8x_2 + 6x_3 \\
\textbf{Max } z_2 = 0.85x_1 + 0.95x_2 + 0.90x_3 \\
\textbf{Max } z_3 = 0.95x_1 + 0.90x_2 + 0.90x_3 \\
\textbf{s.t.} \\
x_1 + x_2 + x_3 \leq 850 \\
x_1 + x_2 + x_3 \geq 700 \\
x_1 \leq 400 \\
x_2 \leq 450 \\
x_3 \leq 450 \\
x_1, x_2, x_3 \geq 0 \\

For Solving sub-model (1):
Step 5: Solving each objective individually subject to given constraints to find the lower and upper bound.
\[ 1900 \leq Z_1^* \leq 3850, \quad 515 \leq Z_2^* \leq 660, \quad 535 \leq Z_3^* \leq 657. \]
Step 6: Use (6)-(7) to construct the membership functions.
Step 7: Using (8) to obtain the crisp model that equivalent to fuzzy model which is as:
\[ \text{Max } \lambda \text{ subject to :} \]
\[ \lambda \leq \frac{3850 - (4x_1 + 5x_2 + 2x_3)}{1950}, \]
\[ \lambda \leq \frac{0.70x_1 + 0.75x_2 + 0.70x_3 - 505}{155}, \]
\[ \lambda \leq \frac{0.80x_1 + 0.75x_2 + 0.70x_3 - 502.5}{155}, \]
\[ x_1 + x_2 + x_3 \leq 850. \]
\[ x_1 + x_2 + x_3 \geq 700. \]
\[ x_1 \leq 400, \]
\[ x_2 \leq 450, \]
\[ x_3 \leq 450, \]
\[ x_1, x_2, x_3 \geq 0 \]
Step 8: After solving the last model in step 7 by using (Lingo software application), we get:

The level achievement of objectives is \( \lambda = 0.74 \) for the optimum value \( z_1 = 2409.862, \ z_2 = 624.2259, \) \( \) and \( z_3 = 616.9725 \) and optimal solution \( x_1 = 377.4656, \ x_2 = 0, \) and \( x_3 = 450. \)
Now, the steps (5)–(8) are repeated in order to solve the second sub-model (upper):

Step 5: Solving each objective individually subject to given constraints to find the lower and upper bound.

\[ 4200 \leq Z_1^+ \leq 5100, \quad 610 \leq Z_2^+ \leq 787, \quad 650 \leq Z_3^+ \leq 785. \]

Step 6: Use (6)-(7) to construct the membership functions.

Step 7: Using (8) to obtain the crisp model that equivalent to fuzzy model which is as:

\[
\begin{align*}
\text{Max } \lambda \\
\text{subject to:} \\
\lambda &\leq \frac{6000 - (6x_1 + 8x_2 + 6x_3)}{1800}, \\
\lambda &\leq \frac{0.85x_1 + 0.95x_2 + 0.90x_3 - 610}{177.5}, \\
\lambda &\leq \frac{0.95x_1 + 0.90x_2 + 0.90x_3 - 650}{135},
\end{align*}
\]

\[ x_1 + x_2 + x_3 \leq 850. \]

\[ x_1 + x_2 + x_3 \geq 700. \]

\[ x_1 \leq 400, \]

\[ x_2 \leq 450, \]

\[ x_3 \leq 450, \]

\[ x_1, x_2, x_3 \geq 0 \]

Step 8: After solving the last model in step 7, by using (Lingo software application), we get:

The level achievement of objectives is:

\[ \lambda = 0.61 \text{ for the optimum value } z_1 = 2371.676, z_2 = 617.5434, \text{ and } z_3 = 609.3353. \]

and optimal solution \( x_1 = 367.919, \quad x_2 = 0, \quad \text{and } x_3 = 450. \)

The compromise interval solution is obtained as:

<table>
<thead>
<tr>
<th>( \lambda^2 )</th>
<th>( z_1^+ )</th>
<th>( z_2^+ )</th>
<th>( z_3^+ )</th>
<th>( x_1^+ )</th>
<th>( x_2^+ )</th>
<th>( x_3^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.61,0.74]</td>
<td>[2372,2410]</td>
<td>[618,624]</td>
<td>[609,617]</td>
<td>[368,377]</td>
<td>0</td>
<td>450</td>
</tr>
</tbody>
</table>

The overall achievement level for objectives 0.74 is greater than from that of [25] = 0.687 and [21] =0.611.

VI. CONCLUSIONS

A process of selection the right vendor represents a safety valve for companies to achieve all firm goals including minimum cost, rejected items, and late delivered items. Hence maintain his Competitive position of the company in the market.

In this paper, an effective fuzzy approach has been introduced to treat the imprecision and ambiguity of the parameters of VS problem. The proposed method is consider as a powerful and applicable tool that help managers to select the proper vendor to accomplish all fuzzy criteria according to preferences of decision maker.

REFERENCES


