

# A High-Order Model for Forecasting Rice Production Based on Combined Fuzzy Time Series and Particle Swarm Optimization

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**Abstract**— Crop production is considered as one of the real world complex problem due to its non-deterministic nature and uncertain behavior. Particularly, forecasting of rice production for a lead year is pre-eminent for crop planning, agro based resource utilization and overall management of rice production. As such, main challenge in rice production forecasting is to generate realistic method that must be capable for handling complex time series data and generating forecasting with almost tiny error. However, first-order fuzzy time series models have proven to be insufficient for solving these problems for the best forecasting accuracy. For this reason, this paper presents a novel high-order model based on fuzzy time series (FTS) and particle swarm optimization (PSO) which overcomes the drawback mentioned above. First, the global information of fuzzy logical relationships is combined with the local information of latest fuzzy fluctuation to find the forecasting value in the defuzzification stage. Second, the particle swarm optimization technique is developed to adjust the lengths of intervals in the universe of discourse for the fuzzification stage. To illustrate the forecasting process and the effectiveness of the proposed model, two numerical datasets of average rice production of Viet Nam and enrolment of students of Alabama University are examined. The examined results show that the proposed model gets lower forecasting errors than those of other existing models

**Keywords**— Forecasting, FTS, high - order fuzzy relationships, particle swarm optimization, enrolments, rice production.

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## I. INTRODUCTION

For more than one decade, many forecasting models has successfully been used to deal with various domain problems, such as academic enrollments [1]-[9], crop production [10][11], stock markets [12]- [14]and temperature prediction [14][15]. The tradition forecasting methods cannot deal with forecasting problems in which the historical data needs to be represented by linguistic values. Fuzzy set theory was firstly presented by Zadeh [16] to handle problems with linguistic values. The concepts of fuzzy sets have been successfully adopted to time series by Song and Chissom [1]. They introduced both the time-invariant fuzzy time series [1] and the time-variant time series [2] model which use the max–min operations to forecast the enrolments of the University of Alabama. Unfortunately, their method needs max–min composition operations to deal with fuzzy rules. It takes a lot of computation time when fuzzy rule matrix is big. Therefore, Chen [4] proposed the first-order fuzzy time

series model by using simple arithmetic calculations instead of max-min composition operations [3]for better forecasting accuracy. After that, fuzzy time series has been widely studied for improving accuracy of forecasting in many applications. Huarng [5]presented effective approaches which can properly adjust the lengths of intervals to get better forecasting accuracy. Chen [6]proposed a new forecast model based on the high-order fuzzy time series to forecast the enrollments of University of Alabama. Yu[7] presented a new model which can refine the lengths of intervals during the formulation of fuzzy relationships and hence capture the fuzzy relationships more appropriately. Both the stock index and enrollment are used as the targets in the empirical analysis. Chen & Chung [8], [9] presented the first-order and high-order fuzzy time series model to deal with forecasting problems based on genetic algorithms. Singh[10],[11]presented simplified and robust computational methods for the forecasting rules based on one and various parameters as fuzzy relationships, respectively. Lee et al. [14] presented a method for forecasting the temperature and the TAIFEX based on the high-order fuzzy logical relation groups and genetic algorithm. They also used genetic algorithm and simulated annealing in it. Recently, Particle swarm optimization technique has been successfully applied in many applications. Based on Chen's model [4], Kuo et al. [17]developed a new hybrid forecasting model which combined fuzzy time series with PSO algorithm to find the proper length of each interval. Then, to improve previous model [17]. They continued to

present a new forecast method to solve the TAIFEX forecasting problem based on fuzzy time series and PSO algorithm [17][18]. Some other authors, propose some methods for the temperature prediction and the TAIFEX forecasting, based on two-factor fuzzy logical relationships [19] and use them in which combine with PSO algorithm in fuzzy time series [20]. In Addition, other hybrid techniques such as: Pritpal and Bhogeswar [21] presented a new model based on hybridization of fuzzy time series theory with artificial neural network (ANN). Matarneh et al. [22] use feed forward artificial neural network and fuzzy logic for weather forecasting achieve better results.

The above mentioned researches showed that the lengths of intervals and creating forecasting rules are two important issues considered to be serious influencing the forecasting accuracy and applied to different problems. However, most of the models were implemented for forecasting of other historical data and not rice production. In this paper, a forecasting model based on the fuzzy logical relationship groups and PSO is presented to forecast rice production for each year on basis of historical time series of rice data in Viet Nam. Firstly, a new forecasting rule which combines global information of fuzzy logical relationships with local information of latest fuzzy fluctuation is developed to find forecasting values. Then, the root mean square error (RMSE) value is applied to estimate the forecasting accuracy. Finally, a new hybrid forecasting model based on combined FTS and particle swarm optimization (PSO) is developed to adjust the length of each interval in the universe of discourse by minimizing RMSE value. The case study with the data of rice production of Viet Nam and the enrolment data at the University of Alabama show that the performance of proposed model is better than those of any existing models based on the high – order FTS.

The rest of this paper is organized as follows. In Section 2, a brief review of the basic concepts of FTS and particle swarm optimization algorithms are introduced. Section 3, first gives the details of fuzzy time series model based on the proposed forecasted rules to forecast rice production and then combines with the particle swam optimization algorithm to find the effective lengths of intervals in the universe of discourse during training phase. Section 4 evaluates the forecasting performance of the proposed method with the existing methods based on the enrolments data of the University of Alabama. Finally, Section 5 provides some conclusions

## II. BASIC CONCEPTS OF FUZZY TIME SERIES AND ALGORITHMS

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### A. Basic concepts of fuzzy time series

Conventional time series refer to real values, but fuzzy time series are structured by fuzzy sets [16]. Let  $U = \{u_1, u_2, \dots, u_n\}$  be an universal set; a fuzzy set  $A_i$  of  $U$  is defined as  $A_i = \{f_A(u_1)/u_1, f_A(u_2)/u_2, \dots, f_A(u_n)/u_n\}$ , where  $f_A$  is a membership function of a given set  $A$ ,  $f_A : U \rightarrow [0,1]$ ,  $f_A(u_i)$  indicates the grade of membership of  $u_i$  in the fuzzy set  $A$ ,  $f_A(u_i) \in [0, 1]$ , and  $1 \leq i \leq n$ .

General definitions of FTS are given as follows:

**Definition 1:** Fuzzy time series [1], [2]

Let  $Y(t) (t = \dots, 0, 1, 2 \dots)$ , a subset of  $R$ , be the universe of discourse on which fuzzy sets  $f_i(t)$  ( $i = 1, 2 \dots$ ) are defined and if  $F(t)$  is a collection of  $f_1(t), f_2(t), \dots$ , then  $F(t)$  is called a fuzzy time series on  $Y(t) (t = \dots, 0, 1, 2 \dots)$ . Here,  $F(t)$  is viewed as a linguistic variable and  $f_i(t)$  represents possible linguistic values of  $F(t)$ .

**Definition 2:** Fuzzy logic relationship (FLR) [2],[3]

If  $F(t)$  is caused by  $F(t-1)$  only, the relationship between  $F(t)$  and  $F(t-1)$  can be expressed by  $F(t-1) \rightarrow F(t)$ . According to [2] suggested that when the maximum degree of membership of  $F(t)$  belongs to  $A_i$ ,  $F(t)$  is considered  $A_j$ . Hence, the relationship between  $F(t)$  and  $F(t-1)$  is denoted by fuzzy logical relationship  $A_i \rightarrow A_j$  where  $A_i$  and  $A_j$  refer to the current state or the left - hand side and the next state or the right-hand side of fuzzy time series.

**Definition 3:**  $\gamma$  - order fuzzy logical relationships [6]

Let  $F(t)$  be a fuzzy time series. If  $F(t)$  is caused by  $F(t-1), F(t-2), \dots, F(t-\gamma+1), F(t-m)$  then this fuzzy relationship is represented by  $F(t-\gamma), \dots, F(t-2), F(t-1) \rightarrow F(t)$  and is called an  $m$  - order fuzzy time series.

**Definition 4:** Fuzzy relationship group (FRG) [4]

Fuzzy logical relationships, which have the same left-hand sides, can be grouped together into fuzzy logical relationship groups. Suppose there are relationships such as follows:

$$A_i \rightarrow A_{k1}, A_i \rightarrow A_{k2}, \dots$$

In previous study was proposed by Chen [1], the repeated fuzzy relations were simply ignored when fuzzy relationships were established. So, these fuzzy logical relationship can be grouped into the same FRG as :  $A_i \rightarrow A_{k1}, A_{k2} \dots$

### B. Particle swarm optimization algorithm (PSO)

Kennedy and Eberhart **Error! Reference source not found.** proposed traditional particle swarm optimization (PSO) techniques for dealing with optimization problems, where a set of potential solutions is represented by a swarm of particles and each particle is move through the search space for search the optimal solution. When particles moving, all particles (i.e, N particles) have fitness values which are evaluated by fitness functions and the position of the best particle among all particles found so far is recorded and each particle keeps its personal best position which has passed previously. At each times of moving, each jth element in the velocity vector  $V_{id} = [v_{id,1}, v_{id,2}, \dots, v_{id,n}]$  and each element  $x_{id,j}$  in the position vector  $X_{id} = [x_{id,1}, x_{id,2}, \dots, x_{id,n}]$  of particle id are calculated as follows:

$$V_{id}^{k+1} = \omega^k * V_{id}^k + C_1 * \text{Rand}() * (P_{best\_id} - x_{id}^k) + C_2 * \text{Rand}() * (G_{best} - x_{id}^k) \quad (1)$$

$$x_{id}^{k+1} = x_{id}^k + V_{id}^{k+1} \quad (2)$$

$$\omega^k = \omega_{max} - \frac{k * (\omega_{max} - \omega_{min})}{iter\_max} \quad (3)$$

The  $P_{best\_id}$  for jth particle is presented as  $P_{best\_id} = [p_{id,1}, p_{id,2}, \dots, p_{id,n}]$  and calculated as:

$$P_{best\_id}^{k+1} f(x) = \begin{cases} P_{best\_id}^{k+1}, & \text{if } fitness(x_{id}^{k+1}) > P_{best\_id}^k \\ fitness(x_{id}^{k+1}), & \text{if } fitness(x_{id}^{k+1}) \leq P_{best\_id}^k \end{cases} \quad (4)$$

The  $G_{best}$  at kth iteration is computed as:

$$G_{best} = \min_i(P_{best\_id}^k) \quad (5)$$

where  $V_{id}^k$  is the velocity of the particle id in kth iteration, and is limited to  $[-V_{max}, V_{max}]$ ,  $V_{max}$  is a constant pre-defined by user.  $X_{id}^k$  is the current position of a particle id in kth iteration. The symbol  $\omega$  denotes the inertial weight coefficient. The symbols  $C_1$  and  $C_2$  denote the self-confidence coefficient and the social confidence coefficient, respectively. In a standard PSO, the value of  $\omega$  decreases linearly during the whole running procedure, and  $C_1 = C_2 = 2$ . The symbol  $\text{Rand}()$  denotes a function can generate a random real number between 0 and 1 under normal distribution. The symbols  $P_{best\_id}$  denotes the personal best position of the particle id, respectively. The symbol  $G_{best}$  denotes the best one of all personal best positions of all particles within the swarm. The whole running procedure of the standard PSO is described in Algorithm 1.

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#### Algorithm 1: Standard PSO algorithm

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1. Initialize the N particles' positions  $X_{id}$  and velocities  $V_{id}$

2. **While** the stop condition (the optimal solution is found or the maximum moving step is reached) is not satisfied

**do**

2.1. **For** particle id, ( $1 \leq id \leq N$ ) **do**

- ✓ Calculate the fitness value of particle i
- ✓ if fitness value better than previous  $P_{best\_id}$  then
  - Set fitness value is new  $P_{best\_id}$  according to (4)

end if

**end for**

2.2. Update the global best position of all particles  $G_{best}$  according to (5).

2.3. **For** particle i, ( $1 \leq id \leq N$ ) **do**

- ✓ Move particle id to another position according to (1) and (2)

**end for**

2.4. Update  $\omega$  according to Eq.(3)

**end while**

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## III. A FORECASTING MODEL BASED ON THE FUZZY TIME SERIES AND PSO

### A. Forecasted model based on the high-order FLRGs

In the section, to verify the effectiveness of the proposed model, the annual data to represent the average rice production (thousand ton/ year) of Viet Nam between 1990-2010 is listed in Table 1 in which it taken from the site [www.gso.gov.vn](http://www.gso.gov.vn), more precisely from <https://www.gso.gov.vn/default.aspx?tabid=717> is used to illustrate the first - order fuzzy time series forecasting process. The step-wise procedure of the proposed model is detailed as follows:

**Table 1: The annual data of the average rice production (thousand ton/year) of Viet Nam**

Year	Actual rice data	Year	Actual rice data	Year	Actual rice data
1990	19225.1	1997	27523.9	2004	36148.9
1991	19621.9	1998	29145.5	2005	35832.9

1992	21590.4	1999	31393.8	2006	35849.5
1993	22836.5	2000	32529.5	2007	35942.7
1994	23528.2	2001	32108.4	2008	38729.8
1995	24963.7	2002	34447.2	2009	38950.2
1996	26396.7	2003	34568.8	2010	39988.9

**Step 1:** Define the universe of discourse U

Assume  $Y(t)$  be the historical data of rice production at year  $t$  ( $1990 \leq t \leq 2010$ ). The universe of discourse is defined as  $U = [D_{\min}, D_{\max}]$ . In order to ensure the forecasting values bounded in the universe of discourse U, we set  $D_{\min} = I_{\min} - N_1$  and  $D_{\max} = I_{\max} + N_2$ ; where  $I_{\min}, I_{\max}$  are the minimum and maximum data of  $Y(t)$ ;  $N_1$  and  $N_2$  are two proper positive to tune the lower bound and upper bound of the U. From the historical rice production data are shown in Table 1, we obtain  $I_{\min} = 19225.1$  và  $I_{\max} = 39988.9$ . Thus, the universe of discourse is defined as  $U = [I_{\min} - N_1, I_{\max} + N_2] = [19000, 40000]$  with  $N_1 = 225.1$  and  $N_2 = 11.1$

**Step 2:** Partition U into equal length intervals

Divide U into equal length intervals. Compared to the previous models in [4], [17], we cut U into seven intervals,  $u_1, u_2, \dots, u_7$ , respectively. The length of each interval is  $L = \frac{D_{\max} - D_{\min}}{7} = \frac{40000 - 19000}{7} = 3000$ . Thus, the seven intervals are defined as follows:

$u_i = (D_{\min} + (i-1)*L, D_{\min} + i*L]$ , with  $(1 \leq i \leq 7)$  gets seven intervals as:

$u_1 = (19000, 22000], u_2 = (22000, 25000], \dots, u_6 = (34000, 37000], u_7 = (37000, 40000]$ .

**Step 3:** Define the fuzzy sets for observation of rice production

Each interval in Step 2 represents a linguistic variable of “rice production”. For seven intervals, there are seven linguistic values which are  $A_1 =$  “very poor rice production”,  $A_2 =$  “poor rice production”,  $A_3 =$  “above poor rice production”,  $A_4 =$  “average rice production”,  $A_5 =$  “above average rice production”,  $A_6 =$  “good rice production”, and  $A_7 =$  “very good rice production” to represent different regions in the universe of discourse on U, respectively. Each linguistic variable represents a fuzzy set  $A_i$  and its definitions is described in (6) as follows:

$$\begin{aligned}
 A_1 &= \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \dots + \frac{0}{u_7} \\
 A_2 &= \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \dots + \frac{0}{u_7} \\
 &\text{-----} \\
 A_7 &= \frac{0}{u_1} + \frac{0}{u_2} + \dots + \frac{0.5}{u_6} + \frac{1}{u_7}
 \end{aligned} \tag{6}$$

For simplicity, the membership values of fuzzy set  $A_i$  either are 0, 0.5 or 1. The value 0, 0.5 and 1 indicate the grade of membership of  $u_j$  ( $1 \leq j \leq 7$ ), in the fuzzy set  $A_i$  ( $1 \leq i \leq 7$ ).

Where, where the symbol ‘+’ denotes fuzzy set union, the symbol ‘/’ denotes the membership of  $u_j$  which belongs to  $A_i$ .

**Step 4:** Fuzzify all historical data of rice production

To fuzzify all historical data, it’s necessary to assign a corresponding linguistic value to each interval first. The simplest way is to assign the linguistic value with respect to the corresponding fuzzy set that each interval belongs to with the highest membership degree. For example, the historical rice data of year 1990 is 19225.1, and it belongs to interval  $u_1$  because 19225.1 is within (19000, 22000]. So, we then assign the linguistic value “very poor rice production” (eg. the fuzzy set  $A_1$ ) corresponding to interval  $u_1$  to it. Consider two time serials data  $Y(t)$  and  $F(t)$  at year  $t$ , where  $Y(t)$  is actual data and  $F(t)$  is the fuzzy set of  $Y(t)$ . According to formula (6), the fuzzy set  $A_1$  has the maximum membership value at the interval  $u_1$ . Therefore, the historical data time series on date  $Y(1990)$  is fuzzified to  $A_1$ . The completed fuzzified results of rice production are listed in Table 2.

**Step 5:** Define all  $\gamma$  – order fuzzy logical relationships.

Based on Definition 2. To establish a  $\gamma$  - order fuzzy relationship, we should find out any relationship which has the  $F(t - \gamma), F(t - \gamma + 1), \dots, F(t - 1) \rightarrow F(t)$ , where  $F(t - \gamma), F(t - \gamma + 1), \dots, F(t - 1)$  and  $F(t)$  are called the current state and the next state of fuzzy logical relationship, respectively. Then a  $\gamma$  - order fuzzy logic relationship in the training phase is got by replacing the corresponding linguistic values.

For example, supposed  $\gamma = 2$  from Table 3, a fuzzy relation  $A_1, A_1 \rightarrow A_1$  is got as  $F(1990), F(1991) \rightarrow F(1992)$ . So on, we get the 2rd-order fuzzy relationships are shown in Table 3, where there are 17 relations; the first 16 relations are called the trained patterns, and the last one, is called the untrained pattern (in the testing phase). For the untrained pattern, relation 17 has the fuzzy relation  $A_7, A_7 \rightarrow \#$  as it is created by the relation  $F(2009), F(2010) \rightarrow F(2011)$ , since the linguistic value of  $F(2011)$  is unknown within the historical data, and this unknown next state is denoted by the symbol ‘#’.

Table 2. The results of fuzzification for rice production data

Year	Actual data	Fuzzy set	Membership value	Year	Actual data	Fuzzy set	Membership value
1990	19225.1	A1	[1 0.5 0 0 0 0 0]	2001	32108.4	A5	[0 0 0 0.5 1 0.5 0]
1991	19621.9	A1	[1 0.5 0 0 0 0 0]	2002	34447.2	A6	[0 0 0 0 0.5 1 0.5]
1992	21590.4	A1	[1 0.5 0 0 0 0 0]	2003	34568.8	A6	[0 0 0 0 0.5 1 0.5]
1993	22836.5	A2	[0.5 1 0.5 0 0 0 0]	2004	36148.9	A6	[0 0 0 0 0.5 1 0.5]
1994	23528.2	A2	[0.5 1 0.5 0 0 0 0]	2005	35832.9	A6	[0 0 0 0 0.5 1 0.5]
1995	24963.7	A2	[0.5 1 0.5 0 0 0 0]	2006	35849.5	A6	[0 0 0 0 0.5 1 0.5]
1996	26396.7	A3	[0 0.5 1 0.5 0 0 0]	2007	35942.7	A6	[0 0 0 0 0.5 1 0.5]
1997	27523.9	A3	[0 0.5 1 0.5 0 0 0]	2008	38729.8	A7	[0 0 0 0 0.5 1]
1998	29145.5	A4	[0 0 0.5 1 0.5 0 0]	2009	38950.2	A7	[0 0 0 0 0.5 1]
1999	31393.8	A5	[0 0 0 0.5 1 0.5 0]	2010	39988.9	A7	[0 0 0 0 0.5 1]
2000	32529.5	A5	[0 0 0 0.5 1 0.5 0]	2011	N/A	N/A	N/A

Table 3. The 2<sup>rd</sup>- order fuzzy logical relationships

No	Fuzzy relations	No	Fuzzy relations	No	Fuzzy relations	No	Fuzzy relations
1	A1, A1 → A1	5	A2, A2 → A3	9	A4, A5 → A6	13	A6, A6 → A6
2	A1, A1 → A2	6	A2, A3 → A3	10	A5, A5 → A5	14	A6, A6 → A7
3	A1, A2 → A2	7	A3, A3 → A4	11	A5, A5 → A6	15	A6, A7 → A7
4	A2, A2 → A2	8	A3, A4 → A5	12	A5, A6 → A6	16	A7, A7 → A7
						17	A7, A7 → #

Step 6: Establish all  $\gamma$  – order fuzzy logical relationships groups

Based on [4] all the fuzzy relationships having the same fuzzy set on the left-hand side or the same current state can be put together into one fuzzy relationship group. The fuzzy logical relationship as the same are counted only once. Thus, from Table 3 and based on Definition 4, we can obtain seven 2<sup>rd</sup> – order fuzzy relationship groups shown in Table 4.

Table 4: The complete result of the 2<sup>nd</sup> - order fuzzy relationship groups

No	Fuzzy relation group	No	Fuzzy relation groups	No	Fuzzy relation groups	No	Fuzzy relation groups
1	A1, A1 → A1, A2	4	A2, A3 → A3	7	A4, A5 → A5	10	A6, A6 → A6, A7
2	A1, A2 → A2	5	A3, A3 → A4	8	A5, A5 → A5, A6	11	A6, A7 → A7
3	A2, A2 → A2, A3	6	A3, A4 → A5	9	A5, A6 → A6	12	A7, A7 → #

Step 7: Calculate and defuzzify the forecasted output values

In this step, to enhance the forecasting accuracy, we propose a novel forecasting rule which combines the global information of fuzzy relationships with the local information latest fuzzy set appear in current state, named (ILF) to calculate the forecasting output values for the trained patterns in the training phase. In addition to, we use<sup>13</sup> to calculate the forecasting output values for the untrained patterns in the testing phase

Rule 1: For the training phase, the forecasted value of rice production at year t is computed according to formula (7) as follows:

$$\text{Forecasted\_value} = w_1 * \text{Global\_inf} + w_2 * \text{Local\_inf} \quad (7)$$

Where,

- ✓ The *Global\_inf* is the global information which can be determined by the fuzzy groups created in Step 6. Suppose that there is a  $\gamma$ - order fuzzy relationship group is presented as follows:  $A_{t-\gamma}, A_{t-\gamma+1}, \dots, A_{t-1} \rightarrow A_{t1}, A_{t2}, \dots, A_{tp}$

Base on Chen [ 1], the value of *Global\_inf* is calculated as follows:

$$\text{Global\_inf} = \frac{m_{t1} + m_{t2} + \dots + m_{tp}}{p}$$

Where,  $m_{k1}, m_{k2}, \dots, m_{kp}$  are the midpoints value of intervals  $u_1, u_2, \dots, u_p$  with respect to p linguistic values existing in the next states of fuzzy logical relationships, respectively.

- ✓ The **Local\_inf** is the local information which is derived by the ILF scheme. The ILF scheme is an estimating scheme determined by the next state and the latest past in the current state. Suppose that there is a  $\gamma$ - order fuzzy relationship as follows:  $A_{t-\gamma}, A_{t-\gamma+1}, \dots, A_{t-1} \rightarrow A_{t1}, A_{t2}, \dots, A_{tp}$ . The **Local\_inf** value is formulated as follows:

$$\mathbf{Local\_inf} = Lv_{tk} + \frac{Uv_{tk} - Lv_{tk}}{2} * \frac{m_{tk} - m_{t-1}}{m_{tk} + m_{t-1}}$$

Where,  $A_{t-1}$  and  $A_{tk}(1 \leq k \leq p)$  denote the latest past in the current state and the next state, respectively.

Here,  $m_{t-1}$  and  $m_{tk}$  are midpoints of the fuzzy intervals  $u_{t-1}$  and  $u_{tk}$  with respective to  $A_{t-1}$  and  $A_{tk}$ ;  $Lv_{tk}$ ,  $Uv_{tk}$  denote the lower bound and upper bound of interval  $u_{tk}$ ,  $t$  is forecasting time.

**Rule 2:** For the testing phase, we calculate forecasted value for a group which contains the unknown linguistic value of the next state at year 2011 according to Eq.(8), where the symbol  $w_h$  means the highest votes predefined by user,  $m$  is the order of the fuzzy relationship, the symbols  $M_{t1}$  and  $M_{ti}$  denote the midpoints of the corresponding intervals of the latest past and other past linguistic values in the current state. From Table 4, it can be shown that group 12 has the fuzzy relationship  $A_7, A_7, \rightarrow \#$  as it is created by the fuzzy relationship  $F(2009), F(2010) \rightarrow F(2011)$ ; since the linguistic value of  $F(2011)$  is unknown within the historical data, and this unknown next state is denoted by the symbol ‘#’.

$$\text{Forecasted}_{\text{for \#}} = \frac{(M_{t1} * w_h) + M_{t2} + \dots + M_{ti} + \dots + M_{ty}}{w_h + (\gamma - 1)}; \text{ with } (1 \leq i \leq \gamma) \quad (8)$$

From forecasted rules above and based on Table 4, we complete forecasted results rice production of Viet Nam the period from 1990 to 2011 based on 2nd-order FTS model with seven intervals are listed in Table 5.

**Table 5: The complete forecasted outputs for rice production of Viet Nam based on the 2<sup>nd</sup>- order FTS model**

Year	Actual data	Fuzzy set	<i>detsacroF</i> <i>eulav</i>	Year	Actual data	Fuzzy set	<i>detsacroF</i> <i>eulav</i>
1990	19225.1	A1	-----	2001	32108.4	A5	32500
1991	19621.9	A1	-----	2002	34447.2	A6	34033
1992	21590.4	A1	20500	2003	34568.8	A6	34750
1993	22836.5	A2	22051	2004	36148.9	A6	35500
1994	23528.2	A2	22750	2005	35832.9	A6	35500
1995	24963.7	A2	23500	2006	35849.5	A6	35500
1996	26396.7	A3	25045	2007	35942.7	A6	35500
1997	27523.9	A3	25750	2008	38729.8	A7	37030.5
1998	29145.5	A4	28790	2009	38950.2	A7	37750
1999	31393.8	A5	31786.5	2010	39988.9	A7	37750
2000	32529.5	A5	31750	2011	N/A	N/A	37750

The performance of proposed model can be assessed by comparing the difference between the forecasted values and the actual values. The widely used indicators in time series models comparisons are the root mean square error (RMSE) according to (9) as follows:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{t=\gamma}^n (F_t - R_t)^2} \quad (9)$$

Where,  $R_t$  denotes actual value at year  $t$ ,  $F_t$  is forecasted value at year  $t$ ,  $n$  is number of the forecasted data,  $\gamma$  is order of the fuzzy logical relationships

### B. Forecasting model based on combined the high – order FTS and PSO algorithm

To improve forecasted accuracy of the proposed, the effective lengths of intervals and fuzzy logical relationship groups which are two main issues presented in this paper. A novel method for forecasting rice production is developed by PSO algorithm to adjust the length each of intervals in the universe of discourse without increasing the number of intervals

In our model, each particle exploits the intervals in the universe of discourse of historical data  $Y(t)$ . Let the number of the intervals be  $n$ , the lower bound and the upper bound of the universe of discourse on historical data  $Y(t)$  be  $p_0$  and  $p_n$ , respectively. Each particle id is a vector consisting of  $n-1$  elements  $p_k$  where  $1 \leq k \leq n-1$  and  $p_k \leq p_{k+1}$ . Based on these  $n-1$  elements, define the  $n$  intervals as  $u_1 = [p_0, p_1]$ ,  $u_2 = [p_1, p_2]$ , ...,  $u_i = [p_{i-1}, p_i]$ , ... and  $u_n = [p_{n-1}, p_n]$ , respectively. When a particle moves to a new position, the elements of the corresponding new vector need to be sorted to ensure that each element  $p_i$  ( $1 \leq k \leq n-1$ ) arranges in an ascending order. The complete steps of the proposed model are presented in Algorithm 2.

**Algorithm 2:** The FTS-PSO algorithm

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**1. Initialize** all particles' positions  $X_{id}$ , velocities  $V_{id}$  and parameters of the proposed method. These parameters are:

- ✓ Number of particles is **50**
- ✓ Maximum number of iterations is **100**
- ✓ The value of inertial weigh  $\omega$  be linearly decreased from  $\omega_{max} = 0.9$  to  $\omega_{min} = 0.4$
- ✓ The coefficient C1 equal C2 as **2**
- ✓ The position of particle id be limited by:  $x_{min} + Rand( ) * (x_{max} - x_{min})$ ; where  $x_{min}$  and  $x_{max}$  are lower and upper bounds of universal set, respectively.
- ✓ The velocity of particle id be exceeded by  $v_{min} + Rand( ) * (v_{max} - v_{min})$

**2. While** the stop condition (maximum iterations or minimum RMSE criteria) is not satisfied **do**

2.1. For particle id, ( $1 \leq id \leq N$ ) **do**

- ✓ Define linguistic values according to all
- ✓ intervals defined by the current position of particle id
- ✓ Fuzzify all historical data by Step 4 in Subsection 3.1
- ✓ Create all  $\gamma$  – order fuzzy relationships by Step 5 in Subsection 3.1
- ✓ Make all  $\gamma$  – order fuzzy relationship groups by Step 6 in Subsection 3.1
- ✓ Calculate forecasting values by Step 7 in Subsection 3.1
- ✓ Compute the RMSE values for particle id based on Eq. (9)
- ✓ Update the personal best position of particle id according to the RMSE values mentioned above.

**end for**

2.2. Update the global best position of all particles according to the RMSE values mentioned above.

**3. For** particle i, ( $1 \leq id \leq N$ ) **do**

- ✓ move particle id to another position according to (1) and (2)

**end for**

- ✓ update  $\omega$  according to Eq. (3)

**end while**

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**IV. EXPERIMENTAL RESULTS**

In this paper, we apply the proposed model to forecast the rice production of Viet Nam with the whole historical data the period from 1990 to 2010 is listed in Table 1 and we also the proposed model to handle other forecasting problems, such as the empirical data for the enrolments of University of Alabama [4] from 1971 to 1992 are used to perform comparative study in the training phase.

**A. Experimental results for forecasting rice production of Viet Nam**

In this section, we apply the proposed method for forecasting the rice production from 1990 to 2010 are listed in Table 1. Our proposed model is executed 20 runs for each order, and the best result of runs at each order is taken to be the final result. During simulation with parameters are expressed in algorithm 2, the number of intervals is kept fix for the proposed model. The forecasted accuracy of the proposed method is estimated using the RMSE value (9). The forecasted results of proposed model under number of interval as 14 and various orders are listed in Table 6.

**Table 6: The completed forecasting results for rice production data of Viet Nam under deferent orders of FTS**

Year	Actual data	Forecasted value for each order of fuzzy relation					
		2 <sup>nd</sup> -order	3 <sup>rd</sup> -order	4 <sup>th</sup> -order	5 <sup>th</sup> -order	6 <sup>th</sup> -order	7 <sup>th</sup> -order
1990	19225.1	-----	-----	-----	-----	-----	-----
1991	19621.9	-----	-----	-----	-----	-----	-----
1992	21590.4	21052.75	-----	-----	-----	-----	-----
1993	22836.5	22494	22926.5	-----	-----	-----	-----
1994	23528.2	23832.25	22912.5	23609.75	-----	-----	-----
1995	24963.7	25076.5	24524	25009.25	24928.25	-----	-----
1996	26396.7	26116.75	26447	26376.75	26547.5	26386.75	-----
1997	27523.9	27659.25	27525.5	27634.5	27500.5	27528	27519.75
1998	29145.5	29483.75	29090.75	29041	29383.5	29134.25	29156.25
1999	31393.8	31145.75	31372	31314	31196.5	31393.5	31292.75

2000	32529.5	32202.25	32298.25	32360.75	32404.5	32332	32351.5
2001	32108.4	32199.25	32295.25	32346.75	32387.5	32322	32347
2002	34447.2	34855	34334.25	34429	34522.25	34515.75	34495.25
2003	34568.8	34832	34330.25	34422.5	34518.75	34510.75	34489.25
2004	36148.9	35834	35720.75	36023.25	35999.25	35945.75	35981.75
2005	35832.9	35834	35692.25	35998.25	35975.25	35921.25	35955.25
2006	35849.5	35834	35692.25	35998.25	35975.25	35921.25	35955.25
2007	35942.7	35834	36283.6	35998.25	35975.25	35921.25	35955.25
2008	38729.8	38061	38161.6	38805.25	38805.25	38848.25	38833.75
2009	38950.2	39046	38745.5	38795.75	38793.25	38836.25	38826.75
2010	39988.9	39046	39617	39807.75	39959.5	39967	39955
2011	N/A	39046	39572.8	39680.9	39749.9	39687.85	39613.15
<b>RMSE</b>		<b>369.34</b>	<b>297.37</b>	<b>127.5</b>	<b>139.75</b>	<b>108.05</b>	<b>116.2</b>

From Table 6, it can be seen that the performance of the proposed model is improved a lot with increasing number of orders in the same number of interval. Particularly, the proposed model gets the lowest RMSE value of **108.05** with 6<sup>th</sup>-order fuzzy logical relationship.

### B. Experimental results for forecasting enrolments

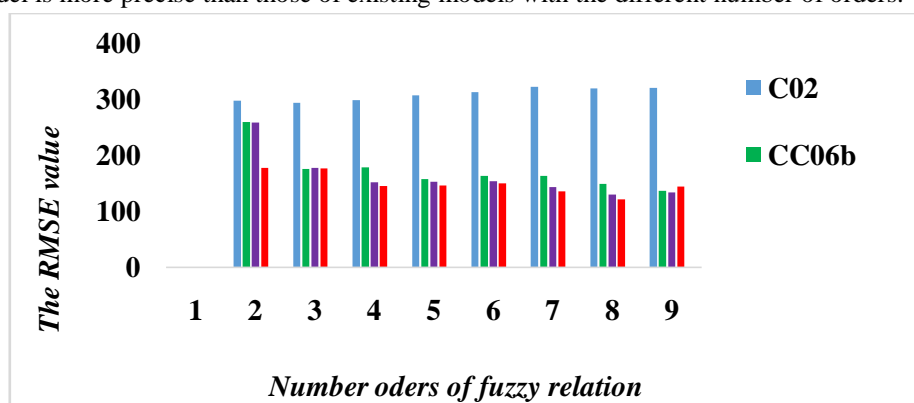
In order to verify the forecasting effectiveness of the proposed model under different number of orders and kept number of intervals of 7, three FTS models in, C02[6] and CC06[9]and HPSO[17], are examined and compared. The forecasted accuracy of the proposed method is estimated by using the RMSE function (9). From the parameters are expressed in Algorithm 2. Our proposed model is executed 20 runs, and the best result of runs is taken to be the final result. A comparison of the forecasting accuracy of all models mentioned above and the proposed model, are listed in [Table 7](#). where all models use the high-order FTS to forecast enrolments of university of Alabama in the training phase.

**Table 7: A comparison of the RMSE value between our model and C02 model, CC06b model, HPSO model under different number of orders and the number of interval is 7.**

Models	Number of order of fuzzy relations							
	2	3	4	5	6	7	8	9
02C	298.48	294.44	298.96	307.47	313.39	322.59	319.65	320.6
b06CC	260.45	176.42	178.91	158.06	164.26	164.22	149.63	136.87
OSPH	259.08	177.89	152.55	153.41	153.85	143.7	130.79	134.06
<b>Our model</b>	<b>178.24</b>	<b>177.09</b>	<b>145.63</b>	<b>146.85</b>	<b>149.56</b>	<b>136.09</b>	<b>121.61</b>	<b>144.52</b>

From Table 7, it can be seen that the accuracy of the proposed model is improved significantly. Particularly, our model gets the lower RMSE values than three models presented in C02 [6] and CC06 [9]and HPSO [17]. These finding suggest that the proposed model is able to provide effective forecasting capability for the high – order FTS model with different number of orders in the same number of interval is 7.

To be clearly visualized. From Fig.1, the graphical comparison clearly shows that the forecasting accuracy of the proposed model is more precise than those of existing models with the different number of orders.



**Fig. 1. A comparison of the MSE values for 7 intervals with different high-order fuzzy relationships.**



## V. CONCLUSIONS

In this paper, a new hybrid forecasting model based on combined FTS and PSO is presented to improve forecasting. First, the proposed model is implemented for forecasting of rice production of Viet Nam. Subsequently, we proved correctness and robustness of the proposed model by testing and verifying it on historical enrolments data and comparing with those of existing models in the training phase. The main contributions of this paper are illustrated in the following. First, the author shows that the forecasted accuracy of the fuzzy forecast model can be improved by considering more information of latest fuzzy fluctuation within all current states of all fuzzy relationships. Second, the PSO algorithm for the optimized lengths of intervals is developed to adjust the interval lengths by searching the space of the universe. Third, based on the performance comparison in Tables 7 and Fig 1 the author shows the proposed model outperforms previous forecasting models for the training phase with various orders in this domain. Although this study shows the superior forecasting capability compared with the existing forecasting models, but the proposed model is only tested by two problems: enrolments data and rice production dataset based on one factor FTS. To continue considering the effectiveness of the forecasting model in the future, the proposed model can be extended to deal with multidimensional time series data such as: Stock market prediction, weather forecast, traffic accident prediction and so on.

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