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Discrete Model of a Prey-Predator Eco-System with Mortality Rates and Limited-Unlimited Resources

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***Abstract*- The present paper is devoted to an investigation on a discrete model of two interacting species a prey-predator eco-system with limited - unlimited resources and mortality rates. The system comprises of a prey (S_1), a predator (S_2) that survives upon S_1. The basic equations for this model constitute as two first order non-linear coupled ordinary difference equations. All possible equilibrium points are identified based on the model equations and criteria for their stability are discussed. The model would be stable if absolute value of each of the eigen values of the characteristic equation is less than one.**

***Keywords*— Absolute value, Eigen value, Equilibrium state, Prey, Predator, Stable, Unstable.**

I. INTRODUCTION

Ecology, a branch evolutionary biology, deals with living species that coexist in a physical environment sustain themselves on common resources. It is a common observation that the species of the same nature can not flourish in isolation without any interaction with species of different kinds. The Ecological interactions can be broadly classified as Ammensalism, Competition, Commensalism, Neutralism, Mutualism, Predation, Parasitism and so on. Svirezhev et al [1] and Volterra [2] pioneered theoretical ecology significantly and opened new eras in the field of life and biological sciences. The authors Rogers et al [3], Varma [4] and Veilleux [5] discussed prey-predator, competing ecological models. Colinvaux [6] and Smith [7] studied basic concepts of population models in ecology.

Mathematical modeling has been playing an important role for the last half a century in explaining several phenomena concerned with individuals and groups of populations in nature. The general concepts of modeling have been discussed by several authors Kapur [8], Kushing [9], Meyer [10] and Pieiou [11]. Srinivas [12] studied the competitive ecosystem of two species and three species with limited and unlimited resources. Later, Narayan et al [13] studied prey-predator ecological models with partial cover for the prey and alternate food for the predator. Further, Acharyulu et al [14] derived mathematical model of ecological Ammensalism with limited resources. The present author Prasad [15-25] investigated continuous and discrete models on three and four species syn-ecosystems.

II. NOTATION ADOPTED

$N_1(t)$: The population strength of prey species (S_1).
$N_2(t)$: The population strength of predator species (S_2).
t : Time instant.
a_i : Natural growth rates of S_i, i = 1, 2.
a_{ii} : Self inhibition coefficient of S_i, i = 1, 2.
a_{12}, a_{21} : Interaction coefficients of S_1 due to S_2 and S_2 due to S_1.

Further the variables N_1, N_2 are non-negative and the model parameters a_1, a_2, a_{11}, a_{12}, a_{22}, a_{21} are assumed to be non-negative constants.

III. BOTH PREY AND PREDATOR SPECIES WITH LIMITED RESOURCES

Consider the non-linear autonomous system of discrete difference equations

$$N_1(t)-N_1(t-1)=-a_1N_1(t-1)-a_{11}N_1^2(t-1)-a_{12}N_1(t-1)N_2(t-1) \quad (1)$$

$$N_2(t)-N_2(t-1)=-a_2N_2(t-1)-a_{22}N_2^2(t-1)+a_{21}N_1(t-1)N_2(t-1) \quad (2)$$

The equations (1) and (2) can be written in terms of recurrence relations as

$$N_1(t)=g_1(N_1,N_2)=\alpha_1N_1(t-1)-a_{11}N_1^2(t-1)-a_{12}N_1(t-1)N_2(t-1) \quad (3)$$

$$N_2(t)=g_2(N_1,N_2)=\alpha_2N_2(t-1)-a_{22}N_2^2(t-1)+a_{21}N_1(t-1)N_2(t-1) \quad (4)$$

where $\alpha_i = (1 - a_i),\ i = 1,2$ (5)

The equilibrium states for the given system are obtained by solving the equations at

$$N_i(t+1) = N_i(t)\ ,\ i = 1,2 \tag{6}$$

(i) *Fully washed out state*

$$E_1 : \bar{N}_1 = 0, \bar{N}_2 = 0$$

ii) *The state in which only the prey (S_1) is washed out.*

$$E_2 : \bar{N}_1 = 0, \bar{N}_2 = \frac{\alpha_2 - 1}{a_{22}}$$

(iii) *The state in which only the predator (S_2) is washed out.*

$$E_3 : \bar{N}_1 = \frac{\alpha_1 - 1}{a_{11}}, \bar{N}_2 = 0$$

(iv) *The Co-existent state (or) normal steady state.*

$$E_4 : \bar{N}_1 = \frac{(\alpha_1 - 1)a_{22} - (\alpha_2 - 1)a_{12}}{a_{11}a_{22} + a_{12}a_{21}}, \bar{N}_2 = \frac{(\alpha_1 - 1)a_{21} + (\alpha_2 - 1)a_{11}}{a_{11}a_{22} + a_{12}a_{21}}$$

3.1 Stability of the Equilibrium States

The basic equations can be linearized about the equilibrium point $E(\bar{N}_1, \bar{N}_2)$

We get

$$g_1(N_1, N_2) = g_1(\bar{N}_1, \bar{N}_2) + \frac{\partial\,[g_1(\bar{N}_1, \bar{N}_2)]}{\partial N_1}(N_1 - \bar{N}_1) + \frac{\partial\,[g_1(\bar{N}_1, \bar{N}_2)]}{\partial N_2}(N_2 - \bar{N}_2)$$

$$N_1(t) = \left(\alpha_1 - 2a_{11}\bar{N}_1 - a_{12}\bar{N}_2\right) N_1 - a_{12}\bar{N}_1 N_2 \tag{7}$$

$$g_2(N_1, N_2) = g_2(\bar{N}_1, \bar{N}_2) + \frac{\partial\,[g_2(\bar{N}_1, \bar{N}_2)]}{\partial N_1}(N_1 - \bar{N}_1) + \frac{\partial\,[g_2(\bar{N}_1, \bar{N}_2)]}{\partial N_2}(N_2 - \bar{N}_2)$$

$$N_2(t) = a_{21}\bar{N}_2 N_1 + \left(\alpha_2 - 2a_{22}\bar{N}_2 + a_{21}\bar{N}_1\right) N_2 \tag{8}$$

The above equations can be written in the matrix notation as $N(t) = A.N(t-1)$

where $A = \begin{bmatrix} \alpha_1 - 2a_{11}\bar{N}_1 - a_{12}\bar{N}_2 & -a_{12}\bar{N}_1 \\ a_{21}\bar{N}_2 & \alpha_2 - 2a_{22}\bar{N}_2 + a_{21}\bar{N}_1 \end{bmatrix}$ and $N = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$ (9)

The characteristic equation for the system is $|A - \lambda I|$ (10)

The equilibrium point $E(\bar{N}_1, \bar{N}_2)$ is stable, if absolute value of each of the eigen values of the characteristic equation A is less than one.

3.1.1 Stability of E_1

In this case, we have

$$A = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} = \begin{bmatrix} 1 - a_1 & 0 \\ 0 & 1 - a_2 \end{bmatrix} \tag{11}$$

The eigen values of the matrix A are $1 - a_1,\ 1 - a_2$. Since, the absolute value of both of these eigen values is less than one only when $0 < a_1, a_2 < 1$.

Hence, $E_1(0, 0)$ is **stable** when $0 < a_1, a_2 < 1$.

3.1.2 Stability of E_2

In this case, we have

$$A = \begin{bmatrix} \alpha_1 - a_{12}\left(\dfrac{\alpha_2 - 1}{a_{22}}\right) & 0 \\ a_{21}\left(\dfrac{\alpha_2 - 1}{a_{22}}\right) & 2 - \alpha_2 \end{bmatrix} = \begin{bmatrix} 1 - a_1 + \dfrac{a_{12}a_2}{a_{22}} & 0 \\ \dfrac{a_{21}a_2}{a_{22}} & 1 + a_2 \end{bmatrix} \tag{12}$$

The eigen values of the matrix A are $1-a_1+\dfrac{a_{12}a_2}{a_{22}}$ *and* $1+a_2$.

Since, the absolute value of one of the eigen value is not less than one. i.e, $|1+a_2|\geq 1$.

Hence, E_2 is **unstable**.

3.1.3 Stability of E_3

In this case, we have

$$A=\begin{bmatrix} 2-\alpha_1 & a_{12}\left(\dfrac{1-\alpha_1}{a_{11}}\right) \\ 0 & \alpha_2+a_{21}\left(\dfrac{\alpha_1-1}{a_{11}}\right) \end{bmatrix}=\begin{bmatrix} 1+a_1 & \dfrac{a_{12}a_1}{a_{11}} \\ 0 & 1-a_2+\dfrac{a_{21}a_1}{a_{11}} \end{bmatrix} \quad (13)$$

The eigen values of the matrix A are $1+a_1$ *and* $1-a_2+\dfrac{a_{21}a_1}{a_{11}}$

Since, the absolute value of one of the eigen value is not less than one. i.e, $|1+a_1|\geq 1$.

Hence, E_3 is **unstable**.

3.1.4 Stability of E_4

In this case, we have $A=\begin{bmatrix} \beta_1 & -a_{12}\bar{N}_1 \\ a_{21}\bar{N}_2 & \beta_2 \end{bmatrix}$ (14)

where

$$\beta_1=\frac{a_{11}a_{22}(1+a_1)+a_{12}a_{21}-a_{11}a_{12}a_2}{a_{11}a_{22}+a_{12}a_{21}};\ \beta_2=\frac{a_{11}a_{22}(1+a_2)+a_{12}a_{21}+a_{21}a_{22}a_1}{a_{11}a_{22}+a_{12}a_{21}} \quad (15)$$

The characteristic equation is $\lambda^2-\lambda\left(\beta_1+\beta_2\right)+\left(\beta_1\beta_2+a_{12}a_{21}\bar{N}_1\bar{N}_2\right)=0$ (16)

Let λ_1 *and* λ_2 be the eigen values of the characteristic equation (16)

Since, the absolute value of both of these eigen values is less than one only when $-1<\lambda_1,\lambda_2<1;\ \lambda_1,\lambda_2\neq 0$.

Therefore, E_4 is **stable**, only when $-1<\lambda_1,\lambda_2<1;\ \lambda_1,\lambda_2\neq 0$

VI. THE PREY SPECIES WITH UNLIMITED RESOURCES AND PREDATOR SPECIES WITH LIMITED RESOURCES

Consider the non-linear autonomous system of discrete difference equations

$$N_1(t)-N_1(t-1)=-a_1N_1(t-1)-a_{12}N_1(t-1)N_2(t-1) \quad (17)$$

$$N_2(t)-N_2(t-1)=-a_2N_2(t-1)-a_{22}N_2^2(t-1)+a_{21}N_1(t-1)N_2(t-1) \quad (18)$$

The equations (2.17) and (2.18) can be written in terms of recurrence relations as

$$N_1(t)=\alpha_1N_1(t-1)-a_{12}N_1(t-1)N_2(t-1) \quad (19)$$

$$N_2(t)=\alpha_2N_2(t-1)-a_{22}N_2^2(t-1)+a_{21}N_1(t-1)N_2(t-1) \quad (20)$$

The equilibrium points for the given system are obtained as,

(i) *Fully washed out state.*

$$E_1:\bar{N}_1=0,\bar{N}_2=0$$

(ii) *The state in which only the prey (S_1) is washed out.*

$$E_2:\bar{N}_1=0,\bar{N}_2=\frac{\alpha_2-1}{a_{22}}$$

(iii) *The Co-existent state (or) normal steady state.*

$$E_3:\bar{N}_1=\frac{\left(\alpha_1-1\right)a_{22}-\left(\alpha_2-1\right)a_{12}}{a_{12}a_{21}},\bar{N}_2=\frac{\alpha_1-1}{a_{12}}$$

4.1 Stability of the Equilibrium States

The basic equations can be linearized about the equilibrium point $E(\bar{N}_1, \bar{N}_2)$

We get $A = \begin{bmatrix} \alpha_1 - a_{12}\bar{N}_2 & -a_{12}\bar{N}_1 \\ a_{21}\bar{N}_2 & \alpha_2 - 2a_{22}\bar{N}_2 + a_{21}\bar{N}_1 \end{bmatrix}$ (21)

The equilibrium state E_1 is **stable** as established in **3.1.1**. Now we will discuss the stability of all other equilibrium states.

4.1.1 Stability of E_2

In this case, we have

$$A = \begin{bmatrix} \alpha_1 - a_{12}\left(\dfrac{\alpha_2 - 1}{a_{22}}\right) & 0 \\ a_{21}\left(\dfrac{\alpha_2 - 1}{a_{22}}\right) & 2 - \alpha_2 \end{bmatrix} = \begin{bmatrix} 1 - a_1 + \dfrac{a_2 a_{12}}{a_{22}} & 0 \\ -\dfrac{a_2 a_{21}}{a_{22}} & 1 + a_2 \end{bmatrix} \quad (22)$$

The eigen values of the matrix A are $1 - a_1 + \dfrac{a_2 a_{12}}{a_{22}}$ *and* $1 + a_2$. Since, the absolute value of one of the eigen value is not less than one. i.e, $|1 + a_2| \geq 1$.

Hence, E_2 is **unstable**.

4.1.2 Stability of E_3

In this case, we have

$$A = \begin{bmatrix} 1 & \dfrac{a_{12}(\alpha_2 - 1) - a_{22}(\alpha_1 - 1)}{a_{21}} \\ \dfrac{a_{21}(\alpha_1 - 1)}{a_{12}} & \dfrac{a_{12} - a_{22}(\alpha_1 - 1)}{a_{12}} \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{a_{22}a_1 - a_2 a_{12}}{a_{21}} \\ -\dfrac{a_1 a_{21}}{a_{12}} & \dfrac{a_{12} + a_{22}a_1}{a_{12}} \end{bmatrix} \quad (23)$$

The characteristic equation is $\lambda^2 + \delta_1\lambda + \delta_2 = 0$ (24)

where $\delta_1 = -\dfrac{a_1 a_{22} + 2a_{12}}{a_{12}}; \delta_2 = \dfrac{a_{12} + a_{22}a_1 + a_{22}a_1^2 - a_1 a_2 a_{12}}{a_{12}}$ (25)

Let λ_1 *and* λ_2 be the eigen values of the characteristic equation (24).

Since, the absolute value of both of these eigen values is less than one only when $-1 < \lambda_1, \lambda_2 < 1;\ \lambda_1, \lambda_2 \neq 0$

Therefore, E_3 is stable, only when $-1 < \lambda_1, \lambda_2 < 1;\ \lambda_1, \lambda_2 \neq 0$

V. THE PREY SPECIES WITH LIMITED RESOURCES AND PREDATOR SPECIES WITH UNLIMITED RESOURCES

Consider the non-linear autonomous system of discrete difference equations

$$N_1(t) - N_1(t-1) = -a_1 N_1(t-1) - a_{11}N_1^2(t-1) - a_{12}N_1(t-1)N_2(t-1) \quad (26)$$

$$N_2(t) - N_2(t-1) = -a_2 N_2(t-1) + a_{21}N_1(t-1)N_2(t-1) \quad (27)$$

The equations (2.26) and (2.27) can be written in terms of recurrence relations as

$$N_1(t) = \alpha_1 N_1(t-1) - a_{11}N_1^2(t-1) - a_{12}N_1(t-1)N_2(t-1) \quad (28)$$

$$N_2(t) = \alpha_2 N_2(t-1) + a_{21}N_1(t-1)N_2(t-1) \quad (29)$$

The equilibrium points for the given system are obtained as

(i) *Fully washed out state.*

$$E_1 : \bar{N}_1 = 0, \bar{N}_2 = 0$$

(ii) *The state in which only the predator (S_2) is washed out.*

$$E_2 : \bar{N}_1 = \frac{\alpha_1 - 1}{a_{11}}, \bar{N}_2 = 0$$

(iii) The Co-existent state (or) normal steady state.

$$E_3 : \bar{N}_1 = \frac{1-\alpha_2}{a_{21}}, \bar{N}_2 = \frac{(\alpha_1 - 1)a_{21} + (\alpha_2 - 1)a_{11}}{a_{12}a_{21}}$$

5.1 Stability of the Equilibrium States

The basic equations can be linearized about the equilibrium point $E(\bar{N}_1, \bar{N}_2)$

We get $A = \begin{bmatrix} \alpha_1 - 2a_{11}\bar{N}_1 - a_{12}\bar{N}_2 & -a_{12}\bar{N}_1 \\ a_{21}\bar{N}_2 & \alpha_2 + a_{21}\bar{N}_1 \end{bmatrix}$ (30)

The equilibrium state E_1 is **stable** as established in **3.1.1**. Now we will discuss the stability of all other equilibrium states.

5.1.1 Stability of E_2

In this case, we have

$$A = \begin{bmatrix} 2-\alpha_1 & a_{12}\left(\frac{1-\alpha_1}{a_{11}}\right) \\ 0 & \alpha_2 + a_{21}\left(\frac{\alpha_1 - 1}{a_{11}}\right) \end{bmatrix} = \begin{bmatrix} 1+a_1 & \frac{a_{12}a_1}{a_{11}} \\ 0 & 1-a_2-\frac{a_{21}a_1}{a_{11}} \end{bmatrix} \quad (31)$$

The eigen values of the matrix A are $1+a_1$ *and* $1-a_2-\frac{a_{21}a_1}{a_{11}}$

Since, the absolute value of one of the eigen value is not less than one. i.e, $|1+a_1| \geq 1$.

Hence, E_2 is **unstable**.

5.1.2 Stability of E_3

In this case, we have

$$A = \begin{bmatrix} \frac{a_{21} + a_{11}(\alpha_2 - 1)}{a_{21}} & \frac{a_{12}(\alpha_2 - 1)}{a_{21}} \\ \frac{a_{21}(\alpha_1 - 1) + a_{11}(\alpha_2 - 1)}{a_{12}} & 1 \end{bmatrix} = \begin{bmatrix} \frac{a_{21} - a_2 a_{11}}{a_{21}} & -\frac{a_2 a_{12}}{a_{21}} \\ -\frac{a_1 a_{21} + a_2 a_{11}}{a_{12}} & 1 \end{bmatrix} \quad (32)$$

The characteristic equation is $\lambda^2 + \mu_1\lambda + \mu_2 = 0$ (33)

where $\mu_1 = \frac{2a_{21} - a_2 a_{11}}{a_{21}}; \mu_2 = \frac{a_{21} - a_2 a_{11} - a_{11}a_1^2 - a_1 a_2 a_{21}}{a_{21}}$ (34)

Let λ_1 *and* λ_2 be the eigen values of the characteristic equation (33)

Since, the absolute value of both of these eigen values is less than one only when $-1 < \lambda_1, \lambda_2 < 1;\ \lambda_1, \lambda_2 \neq 0$

Therefore, E_3 is **stable**, only when $-1 < \lambda_1, \lambda_2 < 1;\ \lambda_1, \lambda_2 \neq 0$

VI. BOTH PREY AND PREDATOR SPECIES WITH UNLIMITED RESOURCES

Consider the non-linear autonomous system of discrete difference equations.

$$N_1(t) - N_1(t-1) = -a_1 N_1(t-1) - a_{12}N_1(t-1)N_2(t-1) \quad (35)$$

$$N_2(t) - N_2(t-1) = -a_2 N_2(t-1) + a_{21}N_1(t-1)N_2(t-1) \quad (36)$$

The equations (35) and (36) can be written in terms of recurrence relations as.

$$N_1(t) = \alpha_1 N_1(t-1) - a_{12}N_1(t-1)N_2(t-1) \quad (37)$$

$$N_2(t) = \alpha_2 N_2(t-1) + a_{21}N_1(t-1)N_2(t-1) \quad (38)$$

The equilibrium points for the given system are obtained as

$$E_1 : \bar{N}_1 = 0, \bar{N}_2 = 0; \quad E_2 : \bar{N}_1 = \frac{1-\alpha_2}{a_{21}}, \bar{N}_2 = \frac{\alpha_1 - 1}{a_{12}}$$

6.1 Stability of the Equilibrium States

The basic equations can be linearized about the equilibrium point $E(\bar{N}_1, \bar{N}_2)$

We get $A = \begin{bmatrix} \alpha_1 - a_{12}\bar{N}_2 & -a_{12}\bar{N}_1 \\ a_{21}\bar{N}_2 & \alpha_2 + a_{21}\bar{N}_1 \end{bmatrix}$ (39)

The equilibrium state E_1 is **stable** as established in **3.1.1**. Now we will discuss the stability of the equilibrium state E_2.

6.1.1 Stability of E₂

In this case, we have

$$A = \begin{bmatrix} 1 & a_{12}\left(\frac{\alpha_2 - 1}{a_{21}}\right) \\ a_{21}\left(\frac{\alpha_1 - 1}{a_{12}}\right) & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{a_2 a_{12}}{a_{21}} \\ -\frac{a_1 a_{21}}{a_{12}} & 1 \end{bmatrix} \quad (40)$$

The eigen values of the matrix A are $1 \pm \sqrt{a_1 a_2}$

Since, the absolute value of one of the eigen values is greater than or is equal to one.
Hence, E_2 is **unstable**.

VII. CONCLUSION

This paper deals with an investigation on discrete model of two species a prey-predator eco-system with limited and unlimited resources. The system comprises of a prey (S_1), a predator (S_2) that survives upon S_1. All possible equilibrium points of the model are identified based on the model equations at the following four situations and criteria for their stability are discussed. The model would be stable if absolute value of each of the eigen values of the characteristic equation is less than one.

i) Both S_1 and S_2 with limited resources.
ii) $S_1$with unlimited resources and $S_2$with limited resources.
iii) $S_1$with limited resources and S_2 with unlimited resources.
iv) Both S_1 and S_2 with unlimited resources.

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