



# Performance Analysis of SVD and SPIHT Algorithm for Image Compression Application

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**Abstract-** This paper deals with performance evaluation of well known image compression algorithm i.e. wavelet based image compression Set Partition in hierarchical Tree (SPIHT) and decomposition algorithm known as Singular Value Decomposition (SVD). Due to multi resolution nature of wavelet transforms, SPIHT provides better image compression at higher compression ratio. The techniques are implemented in MATLAB and compared using the performance parameters PSNR, MSE, CR and compressed size. Images obtained with SPIHT technique yields higher compression ratio and better visual quality.

**Keywords-** Image compression, Singular Value Decomposition (SVD), Discrete Wavelet Transform (DWT), Set Partition in hierarchical Tree (SPIHT).

## I. INTRODUCTION

Nowadays the data are transmitted in the form of images, graphics, audio and video. These types of data require a lot of storage capacity and transmission bandwidth. With the growth of multimedia and internet, large amount of data are transmitted through network. Large amount of data can't be sent if there is low storage capacity and bandwidth. To solve this problem data has to be compressed by using one of the algorithms and then it can be sent easily. [1]

Image compression means minimizing the size in bytes of a graphics file without degrading the quality of the image to an unacceptable level. The basic principle behind image compression is that the neighbouring pixels are correlated and therefore contain redundant information. The foremost task then is to remove redundancies that aim at removing duplication and irrelevance data that omits parts of data that will not be noticed by human visual system.

The image compression techniques are categorized into two main classifications namely Lossy compression techniques and Lossless compression techniques [2]. Lossless compression works by compressing the overall image without removing any of the image's detail. Lossy compression works by removing image detail, but not in such a way that it is apparent to the viewer. Lossy techniques provide for greater compression ratios than lossless techniques. The approaches for lossy compression include lossy predictive coding and transform coding. Transform coding which applies a Fourier-

related transform such as DCT, DWT, are the most commonly used approach.

Most recently the Wavelet based image compression is introduced. In this type, the coders give a better improvement in the picture quality in the higher compression ratios. There are plenty of algorithms are found under this Wavelet based transforms such as Embedded Zero Tree Wavelet (EZW), Set Partition in hierarchical Tree Algorithm (SPIHT) etc.

Well known decomposition algorithm Singular Value Decomposition is based on dividing the image matrix  $A$  into product  $USV^T$ , which allows us to refactor a digital image in three matrices. The using of singular values of such refactoring allows us to represent the image with a smaller set of values, which can preserve useful features of the original image, but use less storage space in the memory, and achieve the image compression process.

In this paper, we will do comparison with widely used wavelet based image compression algorithm set partitioning in hierarchical tree and decomposition algorithm Singular Value Decomposition (SVD) on different performance measure such as Peak to Noise Ratio (PSNR), Mean Square Error (MSE) and CR.

The paper is organized as follows: Section II explains Wavelet Transformation; Section III explains Set Partition in hierarchical Tree Algorithm (SPIHT); Section IV explains Singular Value Decomposition; Section V introduces the experimental results and Section VI gives the conclusion.

## II. DISCRETE WAVELET TRANSFORM

Wavelet transform is the latest method of compression where its ability to describe any type of signals both in time and frequency domain. JPEG2000 which is the standards of international image coding is adopted the method of wavelet transform coding. An  $M \times N$  image is decomposed using wavelet transform. The image is decomposed into four sub-bands after passing a high-pass filter and low-pass filter. The four sub-bands are LL, HL, LH and HH respectively. LL sub band contains horizontal details of the image. LH sub band contains vertical details of the image and HH sub band contains the diagonal details of the image. The process is called the first level of wavelet decomposition. The low frequency sub-band can be continually decomposed into four sub-bands. [3]

The image of low frequency sub-band contains major information. The values of high frequency sub-band approximate zero, the more high frequency the more obvious this situation. For image, the part of the low frequency is primary part which can represent the image information. So researchers take full advantage of the characteristic after wavelet transform and employ proper method to process the image coefficients for achieving effective compression.

### III. SET PARTITIONING IN HIERARCHICAL TREE (SPIHT) ALGORITHM

The SPIHT algorithm was developed by Said and Pearlman in 1996. The SPIHT uses the fundamental idea of zero-tree coding from the EZW but is able to obtain a more efficient and better compression performance in most cases without having to use an arithmetic encoder. It uses wavelet sub band decomposition and imposes a quad tree structure across the sub bands in order to exploit the inter-band correlation.

SPIHT algorithm uses a special data structure – spatial orientation trees (SOT). This particular structure is not only made full use of different scales the correlation between the wavelet coefficients, but also give full consideration to the correlation of the same scale wavelet coefficients.

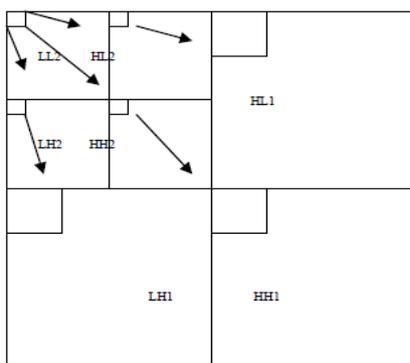


Fig 1: Tree structure used in the SPIHT algorithm.

The algorithm searches each tree, and partitions the tree into one of three lists: 1) the list of significant pixels (LSP) containing the coordinates of pixels found to be significant at the current threshold; 2) the list of insignificant pixels

(LIP), with pixels that are not significant at the current threshold; and 3) the list of insignificant sets (LIS), which contain information about trees that have all the constituent entries to be insignificant at the current threshold. [4-5]

The SPIHT algorithm consists of three stages: initialization, sorting pass and refinement pass. At the initialization stage the SPIHT first defines a start threshold according to the maximum value in the wavelet coefficients pyramid, then sets the LSP as an empty list and puts the coordinates of all coefficients in the coarsest level of the wavelet pyramid (LL band) in the LIP and those which have descendants to the LIS.

In the sorting pass, the elements in the LIP then in the LIS are sorted. For each pixel in the LIP it performs a significance test against the current threshold and outputs the test result (0 or 1) to the output bit stream. If a coefficient is significant, its sign is coded and then its coordinate is moved to the LSP. During the sorting pass of LIS, the SPIHT does the significance test for each set in the LIS and outputs the significance information (0 or 1). If a set is significant, it is partitioned into its offspring and leaves. The current threshold is divided by 2 and the sorting and refinement stages are continued until we achieve the target bit-rate. [6]

Steps of SPIHT Algorithm as follows:

1. Initialize the LIP, LIS, LSP table and determine the maximum threshold  
Threshold  $T=2^n$ ,  
Where,  $n = \lceil \log_2 \max | (i, j) | \rceil$
2. For a given threshold, searches LIP, LIS table to determine the importance of each wavelet coefficient in LIP table.
3. If coefficient (i,j) is significant, then the output "1" and the sign bit is sent out, the node removed from the LIP form, added to the LSP end of the table.
4. If coefficient (i,j) is not significant, then the output should be "0", do not remove this node, the corresponding coordinates are moved to the LIP or LIS respectively, for subsequent testing at a lower bit level.
5. For the same threshold value, scan each node fine in turn in LSP table fine: output not newly added node LSP table wavelet coefficients corresponding to the first binary representation of  $n+1$  bits, the scan end.
6. For the next scan: threshold  $T \leftarrow T/2$ ,  $n \leftarrow n-1$ , repeat step (3) and step (4) step (5), until the threshold values or bit rate compliance encoder requirements.

### IV. SINGULAR VALUE DECOMPOSITION (SVD)

Singular Value Decomposition (SVD) is said to be a significant topic in linear algebra by many renowned mathematicians. SVD method can transform matrix  $A$  into product  $USV^T$ , which allows us to refactoring a digital image in three matrices. The using of singular values of such refactoring allows us to represent the image with a smaller set of values, which can preserve useful features of the original image, but use less storage space in the memory, and achieve the image compression process. Let's say we have a matrix  $A$  with  $m$  rows and  $n$

columns, with rank  $r$  and  $r \leq n \leq m$ . Then the  $A$  can be factorized into three matrices:  $U$ ,  $S$ , and  $V$  such that [7]

$$A=U*S*V^T$$

Where,  $U$  is  $m$ -by- $k$  orthonormal columns  
 $S$  is  $k$ \* $k$  diagonal column  
 $V$  is  $k$ \* $n$  orthonormal columns  
 $U*S*V^T$  is the closest rank  $k$  approximation to  $A$

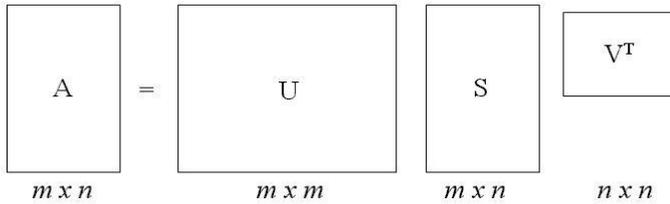


Fig 2: Decomposition of Matrix A

The purpose of transforming the matrix  $A$  into  $USV^T$ . We want to approximate the  $m \times n$  matrix  $A$  by using far fewer entries than in the original matrix. By using the rank of a matrix we remove the redundant information (the dependant entries). SVD can be used to provide the best lower-rank approximation to a matrix  $M$ , and thus be used for image compression. Its usefulness in applications concerning image processing has also been evaluated. Among these applications we can mention patron recognition, secret communication of digital images, movement estimation, classification and quantization. [8] To illustrate the SVD image compression process, we show detail procedures:

$$A = USV^T = \sum_{i=1}^r \sigma_i u_i v_i^T \tag{1}$$

That is  $A$  can be represented by the outer product expansion:

$$A = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \dots + \sigma_r u_r v_r^T \tag{2}$$

When compressing the image, the sum is not performed to the very last SVs; the SVs with small enough values are dropped. (Remember that the SVs are ordered on the diagonal.) The closet matrix of rank  $k$  is obtained by truncating those sums after the first  $k$  terms:

$$A_k = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \dots + \sigma_k u_k v_k^T \tag{3}$$

The integer  $k$  can be chosen confidently less then  $n$ , and the digital image corresponding to  $k$   $A$  still have very close the original image. However, they chose the different  $k$  will have a different corresponding image and storage for it. [9]

## V. EXPERIMENTAL RESULTS

The two schemes are discussed here and stimulated on the platform called MATLAB [10]. In this paper we select 8 bit gray scale image. The quality of the image is analyzed by measuring PSNR and MSE.

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [I(i,j) - K(i,j)]^2$$

$$PSNR = 20 \log_{10} \left( \frac{MAX_I}{\sqrt{MSE}} \right)$$

The set partition in hierarchical tree (SPIHT) image compression technique is applied to two images i.e. Pepper.tif and owl.gif image at different level of decomposition. SPIHT computation on pepper and owl images gives maximum PSNR of 36.37 and 30.01 at decomposition level 6 respectively. Table-I shows the performance evaluation of SPIHT in terms of PSNR, CR and compressed size at varying levels. Fig. 1 and Fig 3 shows the images at different level of decomposition of SPIHT algorithm

The Singular value decomposition (SVD) image compression technique is applied to pepeer.tif and owl.gif image with varying  $k$  values.SVD computation on Pepper.tif and owl.gif images gives maximum PSNR of 34.9638 and 30.559 at  $k=100$  respectively. Table-II shows the performance evaluation of SVD in terms of PSNR, CR and compressed size for varying  $k$  values. Fig. 2 and Fig 4 shows the images by varying rank of matrix  $k$ .

It has been observed that although SVD is giving near about same PSNR value as SPIHT but picture quality achieved maximum in case of SPIHT. Therefore SPIHT yields better result as compared to SVD.

## CONCLUSION

Two image compression techniques i.e. singular value decomposition (SVD) and set partition in hierarchical tree (SPIHT) are compared and contrast based on the subjective and objective performance measures.

Using SVD computation, an image matrix can be compressed to a significantly smaller sized matrix, and during decomposition, the image portray almost an identical to original image. This in turn saves a lot of memory space. Here the average value of  $k$  considered for Decomposition is 100 and gives optimal image quality as depicted.

The inherent multiresolution nature, wavelet-based coders facilitate progressive transmission of images. Therefore SPIHT algorithm is coupled with the power of multiresolution analysis, yields significant compression with little quality loss.

These techniques are successfully tested on owl.gif images. The results of the above techniques SVD and DWT-SPIHT are compared by using parameters such as Compressed Size, Compression Ratio, PSNR and MSE values from the reconstructed image. It has been observed that SPIHT algorithms provide a better performance in picture quality at higher compression ratio. Therefore SPIHT provides a better result as compared to SVD.

Experimental results:

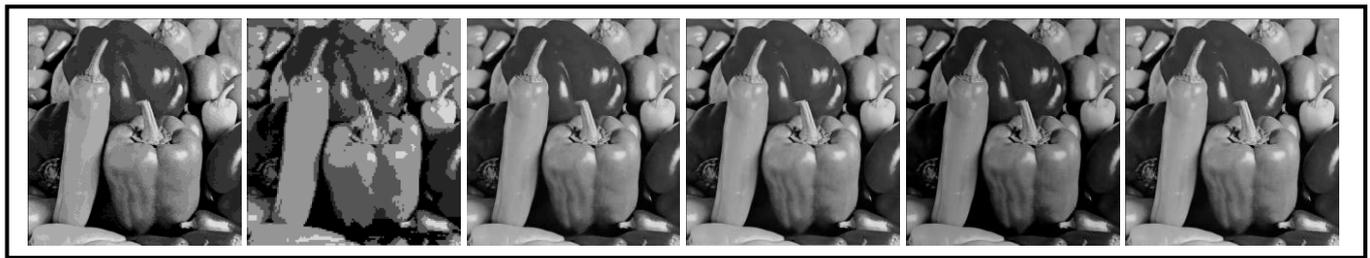


Figure 1: a) Original image b) level = 2 c) level =3 d) level=4 e) level =5 f) level =6

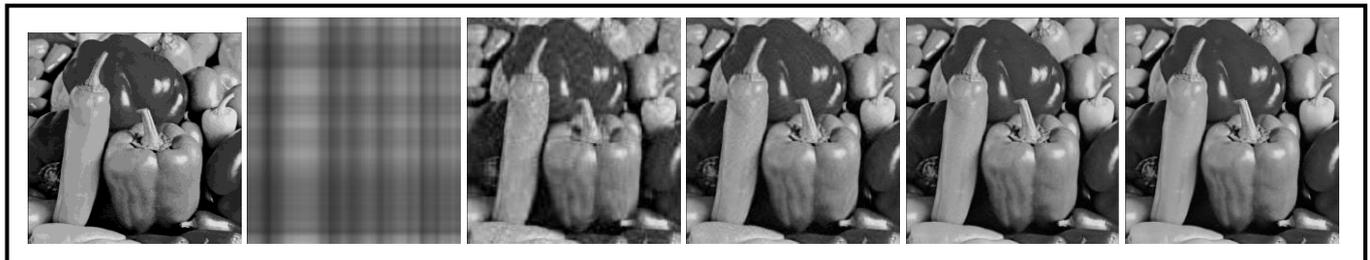


Figure 2 a): Original image b) k = 1 c) k =25 d) k=50 e) k =75 f) k =100



Figure 3: a) Original image b) level = 2 c) level =3 d) level =4 e) level =5 f) level =6

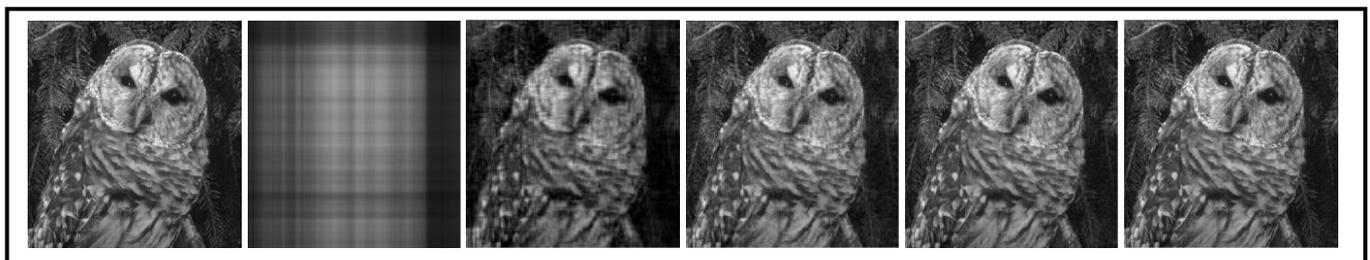


Figure 4 a): Original image b) level = 1 c) level =25 d) level =50 e) level =75 f) level =100

TABLE I  
PERFORMANCE EVALUATION OF DWT-SPIHT ALGORITHM

|                     | Pepper.tif<br>Original Size-268900 bytes |        |                 |                   | Own.tif<br>Original Size-298146 bytes |        |                 |                   |
|---------------------|--|--------|-----------------|-------------------|---------------------------------------|--------|-----------------|-------------------|
| Decomposition level | PSNR                                     | MSE    | Compressed size | Compression ratio | PSNR                                  | MSE    | Compressed Size | Compression Ratio |
| 2                   | 20.99                                    | 517.97 | 5142            | 52.29             | 19.88                                 | 669.12 | 7861            | 37.93             |
| 3                   | 32.80                                    | 34.10  | 42043           | 6.40              | 27.71                                 | 110.25 | 54998           | 5.42              |
| 4                   | 32.80                                    | 16.98  | 73274           | 3.67              | 29.51                                 | 72.77  | 83118           | 3.59              |
| 5                   | 36.29                                    | 15.28  | 80044           | 3.36              | 29.91                                 | 66.32  | 88968           | 3.35              |
| 6                   | 36.37                                    | 14.98  | 81233           | 3.31              | 30.01                                 | 64.91  | 90247           | 3.30              |

TABLE II  
PERFORMANCE EVALUATION OF SVD ALGORITHM

|                     | Pepper.tif<br>Original Size-268900 bytes |         |                 |                   | Own.tif<br>Original Size-298146 bytes |         |                 |                   |
|---------------------|--|---------|-----------------|-------------------|---------------------------------------|---------|-----------------|-------------------|
| Decomposition level | PSNR                                     | MSE     | Compressed size | Compression ratio | PSNR                                  | MSE     | Compressed Size | Compression Ratio |
| 1                   | 13.6318                                  | 2817.72 | 82038           | 3.2775            | 16.7112                               | 1386.64 | 82520           | 3.61302           |
| 25                  | 26.2068                                  | 155.74  | 201900          | 1.33185           | 23.4284                               | 295.248 | 213650          | 1.39549           |
| 50                  | 30.472                                   | 58.329  | 212723          | 1.26409           | 26.2831                               | 153.026 | 225953          | 1.3195            |
| 75                  | 33.0628                                  | 32.1222 | 216310          | 1.24312           | 28.5779                               | 90.2173 | 229913          | 1.29678           |
| 100                 | 34.9638                                  | 20.735  | 220325          | 1.22047           | 30.5591                               | 57.17   | 231832          | 1.28604           |

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