



An Effective and Efficient Solution to Tail Assignment Problem

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Abstract– Air transport is the fastest growing mode of transportation, whose services contribute much to both domestic as well as international transport system. The tail assignment problem is one of the classical planning problems in the operation of an airline. Tail Assignment optimizes flights and maintenance of aircrafts together, while taking operational costs and constraints into consideration. Flight scheduling is of paramount importance for every airline as each instance of a flight schedule affects the revenue of an airline. Connection flights contribute much in minimizing operational costs. Solution to Tail Assignment Problem is a procedure to develop a robust flight schedule for a particular airline company. Given a set of flights and a set of aircraft, the tail assignment problem involves finding large sequence of legs for each aircraft to fly so that each flight is covered exactly once incurring minimum cost and satisfying the constraints. In this thesis, the problem is modeled as a multi objective optimization problem, optimizing the operational cost by scheduling connection flights that satisfies all the constraints and assigning aircrafts to each flight, considering the maintenance constraints. The problem is solved using multi-objective optimization using genetic algorithm and the results are compared with Lagrangian relaxation formulation.

Keywords– MySQL, JAVA, MOGA(Multi-objective Optimization using Genetic Algorithm), QSI(Quality of Service Index), MNL(Multinomial Logit Model), LP(Linear Programming), GA(Genetic Algorithm).

I. INTRODUCTION

Air transport facilitates widen business communications and is a key component in the growth of tourism, now one of the world’s major employment sectors. An airline is a company that provides air transport services for travelling passengers and freight. The prime challenge faced by an airline industry is the economic prosperity. The economic impact of the airline industry purely depends on the business structure.

Business structure of an airline industry can be divided into two sections; Planning section and Operating section. Planning section is the long term preparation of the activities to be executed. It consists of fleet assignment, route selection, flight schedule, aircraft routing and maintenance routing. Operation section is the short term arrangements for the successful execution of the planning which aims to manage the business. It consists of crew scheduling, gate assignment, airline irregular operations and other real time challenges. The process is illustrated in Figure 1.1. The efficiency of the planning phase determines the success of the operation phase.

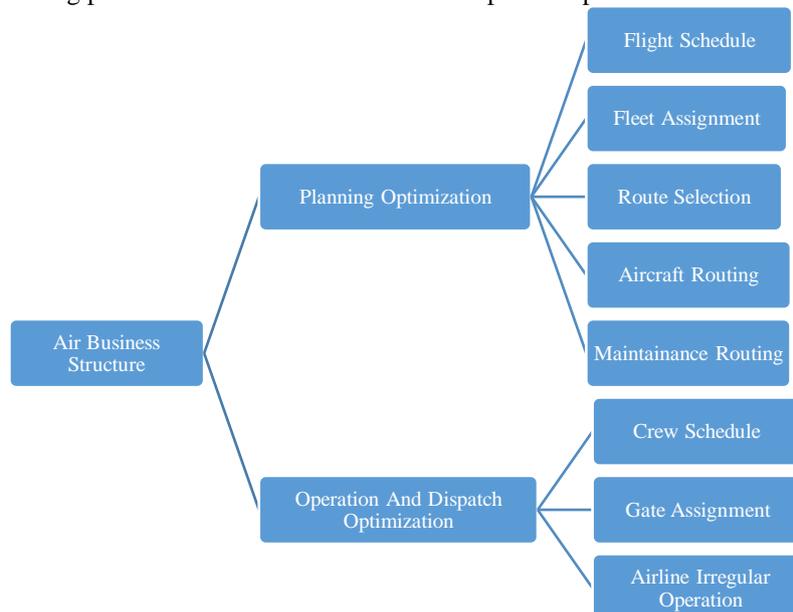


Figure 1.1: Air business Structure

Planning Phase -

Flight Scheduling: Flight schedule is a list of flight legs developed by each airline company satisfying the market demand. A non-stop flight from a source to destination with specific departure and arrival time is known as a flight leg. An airline company plans its schedules for a fixed period of time, usually up to three months and generates the schedules months ahead of the actual trip. Schedules are only a timetable which has the source, destination, ground time, departure and arrival time along with assigned fleet.

Fleet Assignment: Next step is the efficient allocation of resources to fly each leg in the schedule. If a large aircraft is assigned to a leg with a little demand, it is wastage of revenue in the form of fuel and high operation cost. Fleet assignment is performed based on the match of capacity and demand for optimal revenue.

Route selection: Routes can be either point-to-point or can be connection route which is highly economical for an airline company. Connection flight is an ordered sequence of legs that connects a particular source to another selected destination via few intermediate stations. Creating long connection routes which can be satisfied with an aircraft minimizes the total operational cost. So, the aim is to connect several legs sharing stations in common to create a routing model consisting of time disjoint legs. Each leg should neither have time overlapping nor have huge gulf between the arrival and departure time at intermediate nodes. Also, the minimum ground time specified in the flight schedule must be satisfied. That is when an aircraft arrives at that gate, there should be sufficient time for the ground personnel to service the aircraft, transfer baggage before the plane leaves for its next leg, allow sufficient time for passengers to move out of the plane and allow time for the next group of passengers to move in. Connection flights have to be planned well in advance which comprises of maximum number of possible legs from the schedule and assign the exact aircraft to each flight.

Aircraft and Maintenance Routing: Aircraft routing is assigning a particular aircraft for a particular connection flight. i.e., longest connection flights are created and are assigned with a tail number. Tail number is a unique identification number of an aircraft, given at its tail. Aircrafts are assigned only if they satisfy availability, i.e., both position and time, for each connection flight. For safety reasons, aircrafts must be regularly maintained, thus maintenance must be embedded within the aircraft routes. For each airplane in the fleet, separate maintenance-routing plan must be drawn up. Maintenance of airplanes requires fixed stations for periodic mechanical checks. With the available set of aircrafts, airlines deal with the rotation problem through maximizing aircraft utilization. Routing plans must be coordinated to provide the best overall service and so finally, tail numbers are assigned to each aircraft rotations only on satisfying maintenance routing.

Operational Phase –

Crew scheduling is crew selection, where number of crews depends on the aircraft type. After selecting the crew, crew scheduling needs to be performed. Crew scheduling is allocating routes for each crew. A factor like labor and contractual rules, crew expenses etc plays a major role in scheduling.

Gate assignment, the task of assigning arriving flights at an airport to the available gates, is a key activity in airline station operations. Different objectives like minimizing total waiting time for the aircraft after landing before the gate is free or minimizing total towing actions need to be satisfied.

Airline irregular operation is one of the major challenges that an airline company face. It includes uncontrollable causes like weather, security, medical etc and controllable causes like mechanical delay, aircraft substitution, catering delay etc.

II. OBJECTIVE

In airline industry, major focus is on the planning phase that involves long term operations are considered in this thesis, as it is an optimization phase which can directly influence the performance and revenue of an airline company. For a particular airline company, given a set of legs and a set of aircrafts, the tail assignment problem involves finding the possible largest sequence of legs for each aircraft to fly so that each airport is covered exactly once in a trip, incurring minimum cost and satisfying the restrictions. Assigning aircrafts to a route which consists of large sequence of legs (long connection flights) will help to cover more number of legs with a specific aircraft. Prime importance is given to maintenance schedule of aircrafts.

III. PROBLEM DEFINITION

The tail assignment problem is assigning an aircraft to a route consisting of largest sequence of legs, incurring minimum cost. Route has to be created by combining all the possible legs such that each airport is covered exactly once. Prime importance is given to maintenance schedule of aircrafts. Tail assignment is more relevant as the aircrafts have to be utilized efficiently. Aircraft rotation will influence its proper utilization as each aircraft have to fly for a fixed duration before it is maintained. Fleet assignment is not considered and assumed that all the schedules are satisfied by aircrafts of same fleet type. Main inputs to the system are-

- Flight Schedule
- Maintenance Timetable
- Aircraft Inventory

The flight schedule lists all the required legs for the airline based on the market demand. It gives the details about the departure as well as arrival stations and their date and time. It also has the ground time, which is the minimum

time that an aircraft need to spend in the arrival station assigned to the leg. These schedules cannot be modified or dropped. All the schedules need to be satisfied. The flight schedule considered here are for same fleet.

The airline keeps an inventory of aircraft (leased or owned) along with their seating and other configuration information. Each aircrafts in the inventory need to be utilized efficiently. No request for additional aircrafts can be performed during the planning or operation phase. Each flight has to be thoroughly utilized before maintenance.

The aviation department of a country, the aircraft manufacturer etc. has stringent rules on the required maintenance of the aircraft, how often and when it has been performed etc. In addition, there can be corrective maintenance to repair a defect. These requirements cause the aircraft to be grounded for repair and maintenance. Aircrafts in the maintenance list could not be used for operating a flight on those days and so the maintenance planned for the aircraft in the inventory list needs to be considered before assigning.

- Modeling the tail assignment problem is to
- Decide which legs should be included in a route
- Decide which path should be served first
- Decide which aircraft to be used for each route,
- Decide how to satisfy the maintenance constraint

Aim of the design is to assign the best aircraft from the inventory list for scheduling a suitable connection flight without violating the maintenance timetable and other constraints.

IV. SOLUTION APPROACH

Multi-Objective optimization using Genetic Algorithm

Being a population-based approach, GA is well suited to solve multi-objective optimization problems. A generic single-objective GA can be modified to find a set of multiple non-dominated solutions in a single run. The ability of GA to simultaneously search different regions of a solution space makes it possible to find a diverse set of solutions for difficult problems with non-convex, discontinuous, and multi-modal solution spaces. The problem is a bi-objective problem having two objective functions and a few set of equality and inequality constraints. The problem results with a few set of possible solutions rather than a single solution.

Scalarizing a multi-objective optimization problem is formulating a single-objective optimization problem such that optimal solutions to the single-objective optimization problem are Pareto-optimal solutions to the multi-objective optimization problem. Weighted sum method is the approach for scalarizing. In this method a set of objectives are scalarized into a single objective by adding each objective pre-multiplied by a user supplied weight.

Problem Modeling using MOGA

Based on the methodology the objective functions can be converted to a single objective function as

$$Z = \min[\gamma_i(\sum_{(i,j) \in E} l_{ij} * R) - \gamma_j(\sum_{(i,j) \in E} w_{ij} * l_{ij})] \quad (5.9)$$

Where,

$$\gamma_i, \gamma_j > 0 \quad (5.10)$$

$$\gamma_i > \gamma_j \quad (5.11)$$

as more importance is given to the cost factor than length.

Here we propose an algorithm for generating flight schedule based on the concepts of MOGA. Initialization is performed by selecting a set of possible solutions based on the objective function. Solutions are possible connection flights that form a route. Algorithm for route selection is given in Algorithm 3.

Once the routes are fixed each route is assigned with an aircraft that satisfies all the constraints and follow maintenance timetable. Algorithm for flight selection is as in Algorithm 4.

Algorithm 3: Route Selection	
Input:	Specific source, N_s and destination, N_d
Output:	Possible route
1	Initialize Route = 0
2	Enter N_s and N_d
3	Select leg (N_s, N_e)
4	Repeat
5	Assign N_e to N_b and N_c
6	$N_e = \text{NULL}$
7	$N_b.\text{visit} = 1$
8	Select next leg (N_b, N_e)
9	If ($N_e.\text{visit}$)! = 1
10	If satisfies Date Check (constraint 4.9), add to Route
11	Goto 4
12	Else Goto 8
13	until $N_e = N_d$
14	Return Route

<p>Algorithm 4: Flight Selection</p> <p>Input: Possible routes Output: Assign a particular aircraft from the inventory for given route</p> <ol style="list-style-type: none"> 1 Select Route(N_s, N_d) 2 For each A_i 3 if(base station of $A_i == N_s$) 4 Assign A_i for the route 5 Else: 6 Assign A_i with earliest maintenance date & least flying count

Size of the population is predefined as N. Each solution is represented as chromosome by using permutation encoding method. Length of the chromosomes is fixed to a particular size in order to restrict routes of infinite length. Sample chromosome is as in figure 3.2. The chromosome consists of two parts; first is the list of nodes that will be visited during a travel and the second part represents the flight assigned to the route. Identification number provided for the cities and aircrafts are used in permutation encoding.

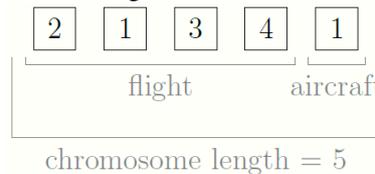


Figure 3.2: Chromosome Structure

Once the initial population is created, elicit members are identified by the selection process. Roulette Wheel Selection is a genetic operator used for selecting potentially useful solution for recombination. It is also called stochastic sampling with replacement. Here, parents are selected according to their fitness i.e., each individual is selected with a probability proportional to its fitness value. In the problem fitness value is evaluated based on the objective function (equation 5.9). In other words, depending on the percentage contribution to the total population fitness, string is selected for mating to form the next generation. This way, weak solutions are eliminated and strong solutions survive to form the next generation. The process is repeated until the desired number of individuals is obtained (called mating population). This technique is analogous to a roulette wheel with each slice proportional in size to the fitness.

<p>Algorithm 5: Roulette Wheel Selection</p> <p>Input: List of possible solutions Output: N Elicit members from the population</p> <ol style="list-style-type: none"> 1 for all members of population 2 sum+=fitness of the individual 3 end for 4 for all members of population 5 Probability = (fitness/sum) 6 sum of probabilities += probability 7 end for 8 loop until next generation is created 9 number = random number generated 10 for all members of the population 11 if sum of probabilities > number 12 select the member to the population 13 end for 14 end loop

Crossover is explorative; it makes a big jump to an area somewhere in between two (parent) areas. Method adopted for performing crossover is single point crossover. It is also known as potential bias where genes that are near to each other are kept together. Here genes from opposite ends of a chromosome can never be kept together. Method adopted for performing crossover is as in Algorithm 6.

<p>Algorithm 6: Single point Crossover</p> <p>Input: 2 randomly selected members from population Output: New siblings</p> <ol style="list-style-type: none"> 1 Generate a random number, R_n between 1 and the population size 2 Select the route with id = R_n as first parent 3 Select the route with id = $R_n + 1$ as second parent 4 To create siblings, interchange nodes of parents from second position to last 5 Select the siblings to next generation on satisfying 6 a. Date check of route selection 7 b. Flight selection

Mutation results with siblings of random small diversions, thereby staying near the parent and maintain diversity in population set. Algorithm 7 explains Mutation.

Algorithm 7: Mutation	
Input:	Random member selected from population
Output:	New siblings
1	Randomly select a route
2	Generate a random position in the route
3	Replace it with a new node id from the list
4	Select the siblings to next generation on satisfying
5	a. Date check of route selection
6	b. Flight selection

Termination occurs on converging the solution to a particular one or after 50 iterations.

Algorithm 8: MOGA	
Input:	Specific source and destination
Output:	Optimal path
1	Initial population: Generate N possible solutions using 'Route Selection' and 'Flight Selection'
2	Evaluation: Fitness of each chromosome in the Initial population is evaluated using objective function
3	Parent Selection: Select N members from initial population using 'Roulette wheel selection'
4	Crossover: For crossover probability
5	Create offspring using 'Single point Crossover'
6	Add offspring to initial population if it satisfies all constraints
7	Mutation: For mutation probability
8	Create offspring using 'Mutation'
9	Add offspring to initial population if it satisfies all constraints
10	If termination condition is satisfied end algorithm, else goto step 2

V. IMPLEMENTATION AND PERFORMANCE EVALUATION

In this chapter, we present the evaluation methods carried out to assess the feasibility and accuracy of the two proposed approaches and compare them.

PLATFORM

The new traffic management method was implemented in Java 1.6 with the support of Mysql 5.5.16. The experiments were conducted in an Intel core i3 2.00GHz processor with a 64-bit Windows 8.1 operating system.

Table 1: Sample Aircraft Inventory

Id	ARegistration		Start		Available_date	Status	Date	Count
	tion	Atype	Tseats	Place				
1	HUH	321	128	TVM	30/04/2016 00:58:00 AM	Pending	30-04-2016	114
2	HUI	321	128	KOC	30/04/2016 00:59:00 AM	Pending	29-04-2016	17
4	IJN	321	128	IAD	28/04/2016 3:40:00 AM	Pending	28-04-2016	99
5	ILO	321	128	MUB	30/04/2016 23:59:00 PM	Pending	30-04-2016	98
6	ILP	321	128	TVM	28/04/2016 22:15:00 PM	Pending	28-04-2016	12
7	ITN	321	128	JFK	27/04/2016 20:00	Pending	0	98
8	IXD	321	128	AUH	28-04-2016 23:50	Pending	0	97
9	JMD	321	128	TVM	27/04/2016 21:20	Pending	29-04-2016	9
10	JDR	321	128	IAD	28-04-2016 02:30	Pending	0	99
11	JEJ	321	128	ORD	27/04/2016 00:00	Pending	26-11-2013	98
12	JGS	321	128	BOM	28-04-2016 02:03	Pending	0	98

DATA SETS

To illustrate and compare the LR and MOGA approaches, a simple network of 30 city examples is considered. This network consists of 75 flight legs from one node to another, which can be combined to form connection flights. 12 aircrafts are considered in the inventory. Initial population considered is 10. Crossover probability is 80% and mutation probability is 10%. 10% of elitist parents are also selected for the next generation.

Table 2: Sample Flight Legs

Flight Date	Departure	STD	Arrival	STA	Ground Time	Distance	Cost
27-04-2016	TVM	27-04-2016 04:20	AUH	27-04-2016 07:10	35	1610	5
27-04-2016	AUH	27-04-2016 10:45	JFK	27/04/2016 10:45	90	6257	2
28-04-2016	JFK	28-04-2016 01:30	IAD	28-04-2016 10:30	40	1585	4
27-04-2016	TVM	27-04-2016 05:45	AUH	27-04-2016 08:30	35	6898	7
27-04-2016	AUH	27-04-2016 09:20	ORD	27-04-2016 23:00	90	1661	5
28-04-2016	ORD	28-04-2016 00:45	IAD	28-04-2016 09:00	40	4356	4
28-04-2016	TVM	28-04-2016 07:30	CHN	28-04-2016 09:00	30	2154	2
28-04-2016	CHN	28-04-2016 10:45	JFK	29-04-2016 01:30	90	2000	5
29-04-2016	JFK	29-04-2016 01:30	IAD	29-04-2016 10:30	40	500	4
28-04-2016	TVM	28-04-2016 05:30	DEH	28-04-2016 09:00	30	900	5
28-04-2016	DEH	28-04-2016 10:45	JFK	28-04-2016 23:00	90	900	2
29-04-2016	JFK	29-04-2016 04:00	IAD	29-04-2016 12:00	40	200	4
29-04-2016	TVM	29-04-2016 04:20	AUH	29-04-2016 07:10	35	300	2
29-04-2016	AUH	29-04-2016 10:45	JFK	29-04-2016 09:10	90	344	4
30-04-2016	JFK	30-04-2016 01:30	IAD	30-04-2016 10:30	40	345	5
29-04-2016	TVM	29-04-2016 05:45	AUH	29-04-2016 08:30	35	897	8
29-04-2016	AUH	29-04-2016 09:20	ORD	29-04-2016 23:00	90	235	9
30-04-2016	ORD	30-04-2016 00:45	IAD	30-04-2016 09:00	40	897	7
28-04-2016	KOC	28-04-2016 08:45	KOL	28-04-2016 09:00	20	897	4
28-04-2016	KOL	29-04-2016 01:30	MUB	29-04-2016 10:30	40	235	5
29-04-2016	MUB	29-04-2016 11:10	DEH	29-04-2016 12:10	20	345	2
29-04-2016	KOC	29-04-2016 08:30	HBD	29-04-2016 09:10	20	111	2
29-04-2016	HBD	29-04-2016 09:30	BGR	29-04-2016 10:10	20	200	5
29-04-2016	BGR	29-04-2016 10:15	DEH	29-04-2016 11:30	20	111	2

Table 3: Sample Maintenance Timetable

Aircraft	Activity	Start_time	End_time	Location
JDR	CHECK1	28-04-2016 00:20	28/04/2016 23:59	IAD
ITN	CHECK2	02-12-2013 12:00	13/12/2013 12:00	TVM
ILO	CHECK3	05-12-2013 00:15	05-12-2013 03:15	BCN
IXD	CHECK4	14-11-2013 03:50	14-11-2013 06:50	IAD
IXD	CHECK5	15-11-2013 22:50	15-12-2013 23:50	JFK
JNI	CHECK6	26-11-2013 00:00	26-11-2013 03:00	TVM
JEJ	CHECK7	26-11-2013 00:00	15-12-2013 00:00	BCN
JGS	CHECK8	26-11-2013 00:03	26-11-2013 02:03	IAD
JMR	CHECK9	26-11-2013 21:57	27-11-2013 00:57	TVM
JDM	CHECK10	26-11-2013 22:00	26-11-2013 23:58	TVM
JEJ	CHECK11	26-11-2013 22:00	27-11-2013 01:00	BCN
IGK	CHECK12	26-11-2013 22:00	27-11-2013 01:00	IAD
JGS	CHECK13	26-11-2013 22:00	27-11-2013 01:00	BCN
JZM	CHECK14	26-11-2013 22:00	27-11-2013 01:00	IAD
JLI	CHECK15	26-11-2013 22:01	27-11-2013 01:01	JFK
JDR	CHECK16	26-11-2013 22:02	27-11-2013 01:02	TVM
JDM	CHECK17	26-11-2013 23:58	27-11-2013 03:01	CHN
JMR	CHECK18	27-11-2013 12:00	15-12-2013 12:00	ORD
IGK	CHECK19	29-11-2013 18:00	01-12-2013 06:30	MUM

VI. RESULTS

The flight schedule given as input contains several paths that connect a particular source and destination. From these paths one feasible path is selected.

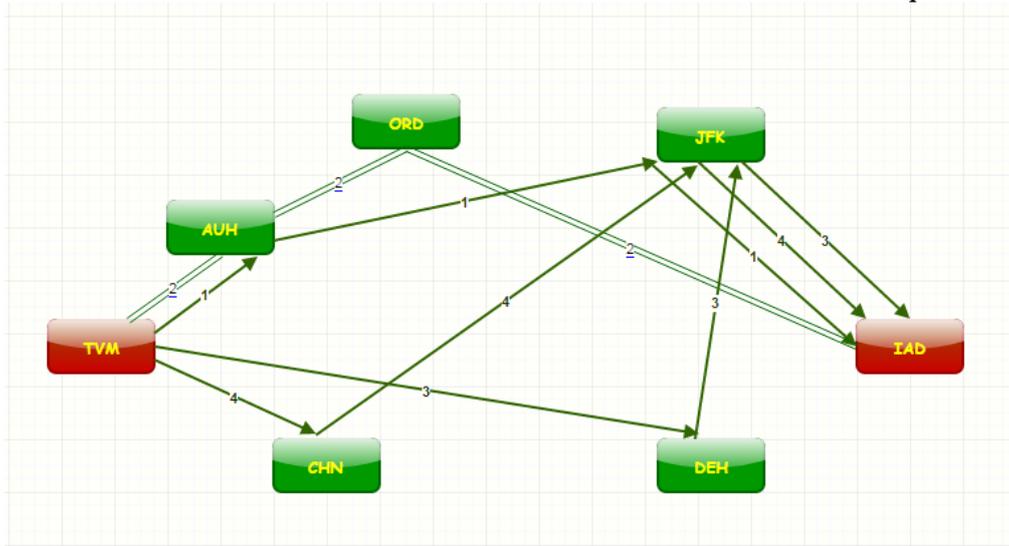


Figure 2: Sample Network Model of Routes

Figure 6.1 represents a sample network model of connection flights from source, TVM to destination IAD. There are four paths from which the path 2 is selected as the optimal path. Similarly all paths are identified to create the timetable. Table 6.4 and Table 6.6 shows that both approaches result with similar output ensuring the generation of most optimized routes.

Table 4: LR Result

Source	Destination	Feasible Paths
KOC	DEL	KOC-KOL-MUB-DEH
CHI	NYR	CHI-ABD-WAN-NYR
XXS	VIE	XXS-ORY-BCN-VIE
FCO	LIN	FCO-VCE-PRG-LIN
TVM	IAD	TVM-AUH-ORD-IAD

Table 5: Sample intermediate result of Genetic Algorithm

Route_number representation	Route Names	Fitness	Cs	Ps
25 29 27 28	FCO PRG DKR LIN	431	0.899859287	0.342166979
18 5 23 21	XXS ORY BCN VIE	1587	0.280657103	0.420869968
9 15 16 7 6	MUB PTN BPL DEH	450	0.077701149	0.802988506
9 15 16 7 6	MUB PTN BPL DEH	450	0.077701149	0.802988506
8 10 9 7 6	KOC KOL MUB DEH	1475	0.660250671	0.660250671
8 11 12 7 9	KOC HBD BGR DEH	420	0.188003581	0.848254252
8 10 9 7 6	KOC KOL MUB DEH	1475	0.660250671	0.660250671
1 2 5 4 1	TVM AUH ORD IAD	12911	0.403330105	0.698478648
1 2 5 4 1	TVM AUH ORD IAD	12911	0.403330105	0.698478648
1 2 3 4 1	TVM AUH JFK IAD	9448	0.295148543	0.295148543
1 2 5 4 1	TVM AUH ORD IAD	12911	0.403330105	0.698478648
1 2 5 4 1	TVM AUH ORD IAD	12911	0.403330105	0.698478648
1 2 5 4 1	TVM AUH ORD IAD	12911	0.403330105	0.698478648

Table 6: MOGA Result

Source	Destination	Feasible Paths
KOC	DEL	KOC-KOL-MUB-DEH
CHI	NYR	CHI-ABD-WAN-NYR
XXS	VIE	XXS-ORY-BCN-VIE
FCO	LIN	FCO-VCE-PRG-LIN
TVM	IAD	TVM-AUH-ORD-IAD

PERFORMANCE EVALUATION

By evaluating the execution time of the two approaches it is found that MOGA is faster than LR approach.

Table 7: LR Execution time

Source	Destination	Feasible Paths	Time(in nanoseconds)
KOC	DEL	KOC-KOL-MUB-DEH	81000
CHI	NYR	CHI-ABD-WAN-NYR	51236
XXS	VIE	XXS-ORY-BCN-VIE	96542
FCO	LIN	FCO-VCE-PRG-LIN	89061
TVM	IAD	TVM-AUH-ORD-IAD	50739

Table 8: MOGA Execution time

Source	Destination	Feasible Paths	Time(in nanoseconds)
KOC	DEL	KOC-KOL-MUB-DEH	12155
CHI	NYR	CHI-ABD-WAN-NYR	27378
XXS	VIE	XXS-ORY-BCN-VIE	36250
FCO	LIN	FCO-VCE-PRG-LIN	40144
TVM	IAD	TVM-AUH-ORD-IAD	44303

VII. CONCLUSION

The solution procedure that was developed in the study to improve the robustness of a flight schedule was implemented on a set of flight data to minimize the operational cost and maximize flight distance. For the test data, it was shown that the proposed solution procedure using MOGA is capable of obtaining flight schedules faster than the LR approach. MOGA also resulted with set of feasible paths from a source to a destination, from which the most apt route is considered to the final timetable. This provides the airline industry a reserved set of optimal paths that can be utilized on facing operational issues.

In the evaluation process, only planning phases are considered. Considering the crew assignment along with this will help the airline industry to attain a better solution.

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