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Communicating and Negotiating Proof Events in the Cyber – Physical Systems

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Abstract- In the past decade linguists, neuroscientists, cognitive scientists, computer scientists, sociologists are bringing new ideas for studying mathematics with new tools. Mathematics is not only about "Proofs and Refutations". The question "What is mathematics?" is coming into recognition as a scientific problem. "Do we really have to choose between a formalism that is falsified by our everyday experience, and a Platonism that postulates a mythical fairyland where the uncountable and the inaccessible lie waiting to be observed by the mathematician whom God blessed with a good enough intuition? - Phil Davis and Reuben Hersh, The Mathematical Experience, 1981 Engineering logic proposed by the author [16] is a blend of Computational logic and Institutional logics with the modelization of the physics of the context.

"Logic sometimes breeds monsters" - Henri Poincare

This paper proposes a method for Communicating and Negotiating Proof Events in the Cyber – Physical Systems.

Keywords- Cyber – Physical Systems; Physics; Computing; Logic; Geometric Algebra; Software Behavior; Computational Models; Communicating; Negotiating; Social Systems; Proof Events

I. INTRODUCTION

Proof is the evidence or a compelling reason for the mind to mathematically accept an assertion as true. It is valuable for validation, explanation, discovery, systemization of results, incorporation into a framework, and conveying mathematical knowledge. The process of proof explicitly introduces strategies, methods, and tools. Arguably, the first encounter with proof is the study of Geometry. Some of the benefits of proofs in the Cyber – Physical Systems [CPS] are given below:

- **Safety:** Formalize system properties that indicate Safety [What is Safe?"] and Liveness ["How to Reach a goal"?]
- **Models:** Formalize system models that clarify behavior [21]
- **Assumptions:** Make assumptions explicit rather than silent to make teams work and review
- **Constraints:** Reveal invariants, switching conditions and operating conditions
- **Design:** Invariant guidelines for safe controller design; determine design trade-offs and early feasibility
- **Constructive:** Construct some useful system models along with their proofs; turn high-level models into code & correctness monitors
- **Certification:** Provide artefacts of certification

Cyber-Physical systems are promising not only for economic growth, but also for finding solutions to major societal challenges. Their advantage is that they are grounded both in the real and in the digital world. They connect physical sensors and controls in devices, buildings, vehicles and medical equipment with communication networks. Aircrafts, trains, modern cars contain so many networked physical sensors and actuators that act in the physical world. Internet of Things is rapidly fueling further research in Cyber – Physical Systems.

However, there is a semantic gap between the virtual or mathematical world and the real world. Mathematics can often provide a proof that a particular cyber-physical system would possess a desired property specified. However, it cannot always be put it into practice in the real world due to several pragmatics. Edsger W Dijkstra defined **"intellectual distance"** as the distance between the real-world problem and the computerized solution to the problem.

"In a very real sense, the software engineer creates models of physical situations in software. The mapping between the model and the reality being modeled has been called the intellectual distance between the problem and a computerized solution to the problem. A fundamental principle of software engineering is to design software products that minimize the intellectual distance between problem and solution; however, the variety of approaches to software development is limited only by the creativity and ingenuity of the programmer. Often it is not clear which approach will minimize the intellectual distance, and often different approaches will minimize different dimensions of the intellectual distance." **- Richard E. Fairley, Software Engineering Concepts, 1985**

Edsger W. Dijkstra startled the Computing Professionals by using the terms "Taste" and "Mood" to a telling effect while introducing the idea of **"the separation of concerns"** in his widely regarded 1974 paper titled "On the role of scientific thought". In this paper Dijkstra also emphasized the need "for being one- and multiple-track minded simultaneously". "Logic" soon emerged as a valuable toolkit for dealing with situations requiring precision, and a prescription for a type of public reasoning while adhering to the separation of concerns. Logic consists of the analysis and synthesis of toolkits subsuming many other forms of precise reasoning and practice, including mathematics.

Cyber – Physical Systems is a case of multiple stakeholders and the near impossibility of exhaustive testing. Formal Verification &Validation have been mooted over the past sixty years to overcome the limitations of testing. Verification means the review activities or the static testing techniques and validation means the actual test execution activities or the dynamic testing techniques. Formal verification is the act of proving or disproving the correctness of the Cyber – Physical System. To verify mathematically the Cyber – Physical System requires an exact specification of its intended behavior.

"A mathematical system is any set of strings of recognizable marks in which some of the strings are taken initially and the remainder derived from these operations performed according to rules which are independent of any meaning assigned to the marks. [The] distinctive feature of this definition lies in the fact that it regards mathematics as dealing, not with certain denoted things - numbers, triangles, and so on - nor with certain symbolized "concepts" or "meanings", but solely with recognizable marks, and dealing with them in such wise that it is wholly independent of any question as to what the marks represent. This might be called the "external view of mathematics" or "mathematics without meaning". [W]hatever the mathematician has in his mind when he develops a system, what he does is to set down certain marks and proceed to manipulate them [...]"

- **Clarence I. Lewis, A survey of symbolic logic, University of California Press, Berkeley, 1918**

The sequence of steps are "mechanical" or syntactic applications of inference rules called axioms. "Mechanized Proof" for correctness of a sequence can be checked without understanding the meaning of the various inference rules or axioms. Often times, the associated methods tend to be clumsy, unwieldy and awkward. Also, there is a misconception that "formal" means careful and detailed, and "informal" means either sloppy or sketchy. A mathematical formalism is a notation (set of formulas) intended to aid in the precise and concise expression of with a limited scope of applicability.

"After an initial training period, using formal methods rarely costs more than using conventional methods in a development, provided the formal method has been sensibly integrated into the organisation's current development life-cycle. The usual experience is that costs are greater and time-scales longer during the earlier stages of the life-cycle, but are more than compensated by lower costs and shorter time-scales in the later stages of the development. There is no evidence that the resulting implementations are any less efficient in time or space; indeed, there is some empirical evidence showing the reverse."

- **Bernhard Aichernig, Institute of Software Technology,Graz University of Technology, Austria.**

However, formal methods are suitable when the system is large and intricate. They are found to be tricky or impractical when the depth of profound concepts are needed. Formal methods are very vulnerable where it is particularly desirable to have a corrective to human wishful thinking.

"Mathematical Proof" can thus mean different things to different people. It is veritably a "trading zone" for different communities with different traditions of demonstration to achieve a measure of practical coordination of their activities.

"Rigorous Arguments" is a category that includes all those arguments that are accepted by mathematicians (or other relevant specialists) as constituting mathematical proofs, but that are not formal proofs. Formal proof has been relatively easy to automate. Rigorous arguments are "sketches" or "outlines" of formal proofs. The arguments are usually with gaps that could be filled by sequences of applications of rules of logical inference. Rigorous arguments typically have the virtue of **"surveyability**". They are nearly always very much shorter than corresponding formal proofs, and are thus easier for human beings to read and to understand. By virtue of their appeal to the meaning of formulae, rigorous argument based proofs produce a psychological sense of conviction. Proof then becomes as social process [7] as well.

"If proof can only mean axiomatic verification with theorem provers, most of mathematics is unproven and unprovable. The 'social' processes of proof are good enough for engineers and other disciplines, good enough for mathematicians…" **- Martyn Thomas, Praxis**

Cyber – Physical Systems thus may have only "Proof Events" or Provings. In other words the mathematical notion of "Proof" is not a real thing. A "Proof Event" minimally involves a person having the relevant background and interest, and some mediating physical objects, such as spoken words, gestures, hand written formulae, 3D models, printed words, diagrams, or formulae. However, none of these mediating signs including the computer is "Proof". Also, the outcomes of "Proof Events" are not always just "true" or "false".

II. THE CONTINUOUS AND THE DISCRETE

"Only geometry can hand us the thread [which will lead us through] the labyrinth of the continuum's composition, the maximum and the minimum, the infinitesimal and the infinite; and no one will arrive at a truly solid metaphysic except he who has passed through this [labyrinth]."

- Gottfried Wilhelm von Leibniz

"Proof Event" [6] begins with a non-mathematical description such as diagrams, tables and natural language. Often times it represents a concise description of high-level behavior and properties of a system. The intelligence, dynamics and flexibility of the Cyber – Physical Systems lies in the ability to be context-sensitive and to produce software that is both dependable and adapts to real time changes.

"The idea that context consists of a set of features of the environment surrounding generic activities, and that these features can be encoded and made available to a software system alongside an encoding of the activity itself, is a common assumption in many systems…..The kind of thing that can be modeled, using the four principles above, is not the kind of thing that context is" **- Dourish P [8,9,10]**

A "Proof Event" in Cyber – Physical Systems is characterized by the following three dimensions.

1. **'Cross'-dimension:** with engineering issues such as cross-application domains, cross-technologies, cross-organizations and so on.
2. **'Self'-dimension:** with engineering issues such as self-monitoring, self-adapting, self-optimizing and so on.
3. **'Live'-dimension:** with engineering issues such as live-configuration, live-update, live-enhancement and so on.

Cyber – Physical Systems integrate computing and physical processes. Some elements are computational processes and some are physical. There are four fundamental forces in nature that govern the diverse phenomena of the macroscopic and the microscopic world. These are the "gravitational force", the "electromagnetic force", the "strong nuclear force", and the 'weak nuclear force'. Unification of different forces / domains in nature remains one of the grand challenges in physics. Physics and computing technology are related to each other.

Prima facie, "Proof Events" appear to be discrete. However, the crux is the realm of the continuous which is traditionally associated with intuition [2].

"la nature ne fait jamais des sauts" - "nature does not make jumps"

- Gottfried Wilhelm von Leibniz, New Essays, IV

The discrete is a model of tidiness in which quality is reduced to quantity and over which the concept of number reigns supreme. Any continuum admits of repeated or successive division without limit. This means that the process of dividing it into ever smaller parts will never terminate in an indivisible or an atomic part. Continua are divisible without limit or infinitely divisible. The unity of a continuum conceals a potentially infinite plurality. For several centuries continua have been represented by straight lines, plane figures, and solids. Infinitesimals began with answering the following questions.

- Is a line segment, for example, composed of an infinite number of indivisible points?
- If so, and if these infinitesimals have zero width, how does the line segment come to have a positive length? And if they have nonzero widths, why isn't the sum of their widths infinite?

An infinitesimal magnitude is usually regarded as what remains after a continuum has been subjected to an exhaustive analysis. It is a continuum "viewed in the small" or an "ultimate part" of a continuum. Since points are indivisible, it follows that no point can be part of a continuum. The continuous curves can be thought of as have "composed" of infinitesimal straight lines.

Cyber-physical systems will not be operating in a controlled environment, and must be robust to unexpected conditions and adaptable to the uncertainty emerging due to the subsystem failures. Robustness is thus an important correctness property in Cyber – Physical Systems. Continuity is one of the vital properties for robustness. Continuity and Discreteness were deemed as two opposite extremes for several centuries. Today, a continuum has no "gaps" but very cleverly connotes unity, discreteness and plurality. However, the continuum has set-theoretic foundations resulting in the abandonment of the use of the infinitesimal in mathematical analysis.

Cyber – Physical Systems warrant freedom for engineers to trade off across the cyber/physical divide, and to do so early. In Engineering, an infinitesimal is a quantity so small that its square and all higher powers can be neglected. The method of constructing infinitesimals of the kind used in nonstandard analysis depends on the model and which collection of axioms are used. Non-standard analysis reformulates the calculus using a logically rigorous notion of infinitesimal numbers. Calculus has widespread uses in science, engineering and economics and can solve many problems that algebra [19,20] alone cannot. Calculus is usually developed by working with very small quantities.

To manage design complexity and provide verification tractability, typical models of the complex Cyber-Physical Systems are hierarchically organized into layers of multiple abstraction. High-level analysis explores interactions of the system with its physical environment. The challenge is to specify a formal approach for integrating the high-level continuous time models with the lower-level discrete time models. At the system level, time is often best represented as a continuous quantity to accurately describe the inherently continuous, controlled physical environment.

The following specifications are widely used to lend expressions to "Proof Event".

- **Model Oriented:** A model of the system behavior [3,15] using mathematical objects such as sets and sequences.
- **Property Oriented:** A set of necessary properties such as axioms and rules to describe the system behavior [3,15].

"Temporal model checking" and "Automaton model checking" [11,12] are two well know approaches for model checking. Rightsizing the level of mathematical abstraction is very important for this purpose. The undecidability theorem [36] rules out the possibility of a complete algorithmic solution for any model. It further avers that even under restrictions (such as finite state spaces), the correctness problem remained computationally intractable. Model checking entails transforming the discrete time symbolic transition systems into timed automata.

"Rigorous Arguments" are axioms [11,12] that specify the Institutional logics [32] to be blended with the models indicated above to specify the physics of the context of a "Proof Event". The word probability typically evokes the nebulous concepts related to uncertainty and randomness. Probability is also a concept which is hard to characterize formally. Defining probability in terms of axioms rather than the frequency of events in repeated events enables the expressions in the realms of Institutional Logics and Algebraic statistics. Axioms in traditional thought were "self-evident truths", but that conception is problematic. At a formal level, an axiom is just a string of symbols, which has an intrinsic meaning only in the context of all derivable formulas of an axiomatic system.

"context is an emergent property" that is "continually negotiated and redefined." Furthermore, "people often find ways of using technology that are unexpected and unanticipated. ...Even when the general patterns of technology use do conform to expectations, the meaning of the technology for those who use it depends on how generic features are particularized, how conventions emerge, and so on." **– Dourish P [8,9,10]**

A "Proof Event" in the physical context is modelled by an algebraic structure which reflects the order and manner in which context is entered into and exited from.

III. COMMUNICATING PROOF [5] AND THE INFINITESIMAL [23,24,25,26]

Geometric Algebra [4] helps in visualizing the Cyber – Physical System as a model for communicating with the multiple stakeholders. One can geometrically express a complex idea (a wiggly curve) as simpler parts (rectangles). The thinner the rectangles, the more accurate is the model of the complex idea. Infinitesimals and Limits are used to create models that are simple and share the same properties as the original Cyber – Physical System. The challenge is to get slices of rectangles so thin that they are not noticed i.e give an appearance of continuum but appear large enough to "exist" for ease of analysis. The following approaches are useful for this purpose.

- **Allow another dimension:** Numbers measured to be zero in the real dimension might actually be small but nonzero in another dimension i.e. a dimension infinitely smaller than the real one. This is the Infinitesimal approach.
- **Accept imperfection:** Numbers measured to be zero are probably nonzero at a greater level of accuracy i.e something is "zero" really means "it's 0 +/- our measurement error". This is the Limit approach.

Infinitesimals are better suited for the context of Cyber – Physical Systems as seemingly small values can result in a large impact. "Non-standard" analysis uses infinitesimals and what are called "hyperreals", and most importantly, does not depend on a notion of "limit". Multidimensional co-design is ideal for integrating the computing and the physical systems that include logical and physical dynamics, continuous and discrete transitions, deterministic and random evolution and emergence resulting in hybrid behaviors [15]. Infinitesimals facilitate the seamless transition across these dimensions which helps in visualizing the physics of the context of a "Proof event" even for seemingly small but high impact values. The logic of contextual assertions, and algebras of context associated with the provings in the Cyber – Physical Systems can be communicated with clarity.

"I should like to ask the same question that Descartes asked. You are proposing to give a precise definition of logical correctness which is to be the same as my vague intuitive feeling for logical correctness. How do you intend to show that they are the same? ... The average mathematician should not forget that intuition is the final authority." **- J. Barkley Rosser**

Infinitesimals enable better comprehension of the provings among the inter-disciplinary stakeholders of a Cyber – Physical System.

IV. NEGOTIATION OF PROOF [17,18, 22]

Cyber-physical systems [CPS] enable the physical world [28] to merge with the virtual leading. The complexity resulting from their intrinsically distributed nature, the heterogeneity of the physical elements, the lack of reliability in communications, the large-scale and variability of the environments in which they are deployed, make data analysis and operation planning a very complex task. Lack of resilience, adaptability, durability, longevity and system sustainability are some of the core concerns of deploying the Cyber – Physical Systems in Social, Environmental and Technical contexts. Institutions evolve over a span of time and hence the Institutional Logic tends to adapt in response to torrents of emergent data. Longevity and Sustainability are non-functional quality attributes of the Systems imply resilience and adaptability even to seemingly small changes in the institutional capabilities and Socio-Technical processes. It is inevitable that Cyber – Physical Systems evolve over a span of time. Data facilitates the Understanding, Modeling [29, 30], Predicting, Controlling, Automating, Visualizing and Improving the process of the evolution of the system.

"[Data is] a representation of facts or ideas in a formalized manner capable of being communicated or manipulated by some process." **- IFIP Guide to Concepts and Terms in Data Processing**

The first mass data crunchers were people, not machines. Epistemologically speaking, information is made up of a collection of data and knowledge is made up of different strands of information.

"The key challenge for people with technical skills is working with management. People have to understand the business and be able to communicate effectively with non-technical people."

- Yuri Levin, Professor, Queen's University

"It is the mission of the IASC to link traditional statistical methodology, modern computer technology, and the knowledge of domain experts in order to convert data into information and knowledge."

- The International Association for Statistical Computing (IASC)

In a very short span of time, Information has gone from scarce to superabundant. Yet, the simple question "What is Data?" remains vital even after 70 years of Computing. The discipline of programming blended with Databases was a paradigm shift we witnessed several decades back. However, this was obsessed with "structuring". Gradually, databases began to shake off the "structuring" both for "right" and "wrong" reasons. Filters and Subjective interpretations began to lend meaning for "what gets structured" at some layer. The most important question is "How to unravel geometry and topology hidden in a finite set of data points? ".

The author [13,14,15,16] opines that "models and methods" [1] to make the discrete meltdown into a continuum are necessary. The word "analog" was originally used in this sense to refer to a transformed representation of a natural phenomenon. These representations consist of continuously varying properties are analogous to (or "analogs" of) the properties of the original phenomenon. Analog Data is thus recorded in a form that is similar to its original structure. The digitization of information changed data analytics in terms of the volume, variety of data and "modeling" of the real world data. However, the rigor of traditional statistical and algorithmic methodologies is very much needed to solve similar problems faced many decades ago with analog data.

In the beginning it was analog. What was it? Some said whatever progresses continuously is analog. Something like a series expansion that gradually tends to infinite terms. The most important question is "How long should one wait to get the net result?".

Theoretically the simple answer is **"until the progression ends".** In practice, a method called "Summation" is used. "Truncation" was introduced to make the method faster without losing significant data in the series. The "lossy" aspects inherent in the process began to be accepted.

Analog Computation and Simulation is a method of representing the physical system as a set of "data points". The machine is simply the "physical system" such as the meeting room or the building.

Digital age began to represent the "data points" using the binary number system. The "data points" are also discrete. "Processing" lends meaning to the sequence of "data points". "Iteration" is the typical way of getting closer to the reality. "Analog" makes the "reality" obvious unlike its digital counterpart we nudge with. Infinitesimals is a method of seamlessly including the "analog" computation.

Cyber – Physical Systems are witnessing a rapid change in both technology and the requirements for analysis of massive sets of information. More and more businesses are looking at the related issues and recognizing that patterns, trends, and analysis of these items is becoming critical to the success of the business. The database will grow in the specific areas consisting of transactional information. Some advantages of database approach are:

- Program-Data Independence: No need to rewrite programs when data is modified
- Minimal Data Redundancy
- Improved Data Consistency
- Improved Data Sharing
- Increased Application Development Productivity
- Enforcement of Standards
- Improved Data Quality (Constraints)
- Better Data Accessibility/Responsiveness
- Reduced Program Maintenance
- Security, Backup/Recovery, Concurrency

The problem remains that most institutions still regard "data" as rows and columns of numbers that can be mined and reported on using analytical tools that will end up with a graph of some sort. Unstructured data, however, is filled with ideas and emotions riddled with ambiguities and nuances.

"The repetitive processes of thought are not confined however, to matters of arithmetic and statistics. In fact, every time one combines and records facts in accordance with established logical processes, the creative aspect of thinking is concerned only with the selection of the data and the process to be employed and the manipulation thereafter is repetitive in nature and hence a fit matter to be relegated to the machine. Not so much has been done along these lines, beyond the bounds of arithmetic, as might be done, primarily because of the economics of the situation. The needs of business and the extensive market obviously waiting, assured the advent of mass-produced arithmetical machines just as soon as production methods were sufficiently advanced."

- Vannevar Bush

"How do you derive meaning from information that's as disparate and unpredictable as the people creating it?" "Knowing how to manage unstructured data is the next wave of competitive differentiation. It enables you to get closer to customers, be more agile and be more informed in your decision-making."

- Matt Malden, Vice President, Products Autonomy, HP Company

Quocirca recommends that organisations [35] look for the following characteristics:

- Can this solution deal with different data types, including text, image, video and sound?
- Can this solution deal with disparate data sources, both within and outside of my organisation's environment?
- Will the solution create a new, massive data warehouse that will only make my problems worse, or will it use metadata and pointers to minimise data replication and redundancy?
- How can and will the solution present findings back to me - and will this only be based on what has already happened, or can it predict with some degree of certainty what may happen in the future?
- How will the solution deal with back-up and restore of data, is it inherently fault tolerant and can I apply more resource easily to the system as required?

Context-sensitive [31,33,34] is a key feature, but it is difficult to develop context sensitive systems because they are complex and the developers are not familiar with the development methodology for such systems. Moreover, these systems cost much more than traditional systems because of their features including context-sensitiveness, heterogeneousness, and intelligence.

Some type of contextual information is required. Usability is a quality factor that is presently providing information about the user and the user's environment. However this is not sufficient for provings. The mechanism to reason proposed in this paper is also required.

In the case of physics of the context of a "Proof Event", it is a space-time frame that bounds the context. Some of the possible scenarios very likely to emerge are given below.

- The truth value of a statement cannot be meaningfully determined in a space-time context since it is not applicable in that context.
- The truth value of a statement is unknown in the space-time context where it is stated or queried against.
- The meaning of "true" itself may be differ from one space-time context to another.

Communicating and Negotiating "Proof Events" as mooted in this paper is important in Cyber – Physical Systems. The following six laws are very useful in the Social Negotiation of proof.

1. **Law of Reciprocity:** Offer good quality data and get good quality of data in return.
2. **Law of Commitment and Consistency:** Deliver on time each time and provide consistent results.
3. **Law of Liking:** Results that have an innate structure and symmetry are liked.
4. **Law of Scarcity:** Not much data available on the subject enhances acceptability.
5. **Law of Authority:** Proven expertise and credentials enhance acceptability.
6. **Law of Social Proof:** Credentials and Endorsements of other users of the results enhance acceptability.

V. CONCLUSIONS

Geometry [27] with its formal, logical and spatial properties is well suited to communicate and negotiate proof events modeled as mooted in this paper. The usefulness of this work in the Cyber – Physical Systems is as follows:

- **verification** (concerned with the truth of a statement);
- **explanation** (providing insight into why it is true);
- **systematisation** (the organisation of various results into a deductive system of axioms, major concepts and theorems);
- **discovery** (the discovery or invention of new results);
- **communication** (the transmission of mathematical knowledge);
- **construction** of an empirical theory;
- **exploration** of the meaning of a definition or the consequences of an assumption;
- **incorporation** of a well-known fact into a new framework and thus viewing it from a fresh perspective.

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