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Fixed Point Common Theorem in Fuzzy Metric Space

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Abstract: In this research article we are proving fixed point common theorem using occasionally Weakly Compartible Mappping in fuzzy metric space.

Keywords: Common Fixed point, Fuzzy Metric speace, Occasionally Weakly Compatible Mapping Continuous t-norm.

I. INTRODUCTION

It proved a truning point in the development of mathematics when the notion of fuzzy set was introduced by Zadeh [24] which laid the foundation of fuzzy mathematics. Kramosil and Michalek [11] introduced the notion of a fuzzy metric space by generalizing the concept of the probabilistic metric speace to the fuzzy situation. George and Veeramani [7] modified the concept of fuzzy metric spaces introduced by Kramosil and Michalek [11].There are many view points of the notion of the metric speace in fuzzy topology for instance one can refer to Kaleva and Seikkala [10], Kramosil and Michalek [11], George and Veeramani [7].

II. PRELIMINARIES

Definition 2.1 [24] Let X be any non empty set. A fuzzy set M in X is a function with domain X and values in [0,1]

Definition 2.2 [19] A binary operation $*:[0,1]\times[0,1]®[0,1]$ is a continuous t- norm if it satisfy the following conditions:

(i) * is associative and Commutative
(ii) * is continous function
(iii) $a*1=a$ for all $a\in[0,1]$
(iv) $a+b\le c*d$ whenever $a\le c$ and $a,b,c,d\in[0,1]$

Definition 2.3. [11] The - tuple (X,M,*) is called a fuzzy metric speace in the sense of Kramosil and Michalek if X is an arbitrary set, is a continuous t - norm and M is a fuzzy
set in X^2 x $[0,\infty)$ satisfying the following conditions:

(a) M (x, y, t) > 0
(b) M (x, y, t) = 1 for all t > 0 if and only if x = y
(c) M (x, y, t) = M (y , x , t)
(d) M (x, y, t) M (y, z, s) ≤ M(x, z, 1 + s)
(e) M (x, y .) $[0,\infty)\to[0,1]$ is a continuous function for all x,y,.z ∈ X and t ,s > 0

Definition 2.4 [11] Let (X , M, *) be a fuzzy metric speace. Then

(i) A sequence $\{x_n\}$ in X converges to x if and only if for each t >0 there exists $n_o\in N$ such that, lim n →∞ M (x_n x, t) = 1 for all $n > n_o$
(ii) The sequence $\{x_n\}$,$n\in N$ is called cauchy sequecne if lim n →∞ M (x_n X_{n+p}t) =1 for all t > 0 and p ∈ N
(iii) A fuzzy metric space X is called complete if every Cauchy sequence is convergent in X.

Definition 2.5 [23] Two self- mappings f and g of a fuzzy metric speace (X, M, *) are said to be weakly commuting if M (fgx, gfx t) ≥ M (fx, gx t) for each x ∈ X and for each t > 0

Definition 2.6 [5] Two self mapping f and of a fuzzy metric speace (X,M,*) are called compatible if $\lim n\to\infty M\left(fgx_n, gfx_n, t\right)=1$ whenever $\{x_n\}$ is a sequence in X such that $\lim n\to\infty fx_n=\lim n\to\infty gx_n=x$ for some x in x

Definition 2.7 [2] A pair of mappings f and g from a fuzzy metirc speace (X,M,*) into itself are weakly compatible if they commute at their coincidence points i.,e.,fx = gx implies that fgx = gfx.

Definition 2.8 Let X be a set f g selfmaps of X A point x in X is called a coicidence point of f and g i iff fx = gx.We shall call w = fx = gx a point of coincidence of and g.

Definition 2.9 [2] A pair of maps S and T is called wealky compatible pair if they commute at coincidence points.

Definition 2.10 [4] Two self maps f and g of a set X are occasionally weakly compatible (owc) iff there is a point x in X which is coincidence point of f and g at which and g commute.

A. Al - Thagafi and Naseer Shahzad [4] shown that occasionally weakly compatible is weakly compatible but converse is not true.

Lemma 2.11 [4] Le X be a set ,f,g owe self maps of X If and g have a unique point of coincidence, w = f x= gx, then w is the unique common fixed point of f and g.

III. IMPLICIT RELATIONS

(a) Let (Φ) be the set of all real continuous functions $\phi:(R^+)^5 \to R^+$ Satisfying the condition

$\phi(u,u,v,v,u) \geq 0$ imply $u \geq v$ for all $u,v \in [0,1]$

(b) Let (Φ) be the set of all real continuous functions $\phi:(R^+)^4 \to R^+$ satisfying the condition

$\phi(u,v,u,u) \geq 0$ imply $u \geq v$ for all $u,v \in [0,1]$

IV. MAIN RESULTS

Theorem 4.1: Let (X,M,) be a fuzzy metric speace with *continous t- norm. Let P,Q,R,S be self mappings of X satisying

(i) The pair (P,R) and (Q,S) be owc.

(ii) For some $\phi \in \Phi$ and for all x,y $\in$ X and every t > 0

$$\phi\{M(Px,Qy,t), M(Rx,Sy,t), M(Rx,Px,t), M(Px,Sy,T), M(Sy,Qy,t), M(Rx,Qy,t)\} \geq 0$$

then there exists a unique point w $\in$ X such that Pw = Rw= w and a unique point z$\in$ X such that Qz = Sz =z
Moreover, z = w ,so that there isa unique common fixed point of P,Q,R and S.
Proof : Let the pairs {P,R} and { Q,S} be ovc, so there are points x, y $\in$ X such that

Px =Rx and Qy = Sy

we claim that Px= Qy. If not by inequality (ii)

$$\phi\{M(Px,Qy,t), M(Px,Qy,t), M(Px,Px,t), M(Px,Qy,t), M(Qy,Qy,t), M(Px,Qy,t)\} \geq 0$$

$$\phi\{M(Px,Qy,t), M(Px,Qy,t), 1, M(Px,Qy,t), 1, M(Px,Qy,t)\} \geq 0$$

In view of Φ we get P x = Qy i.e. Px =Rx = Qy
Suppose that there is a another point z such that Pz= Rz then by (i) we have Pz= Rz = Qy = S y. so Px = Pz and w = Px = Rx is that unique point of coincidence of P and R. By Lemma 2.1.1 w is the only common fixed Assume that w #z We have.

$$\phi\{M(Pw,Qz,t), M(Rw,Sz,t), M(Rw,Pw,t), M(Pw,Sz,t), M(Sz,Qz,t), M(Rw,Qz,t)\} \geq 0$$

$$\phi\{M(w,z,t), M(w,z,t), M(w,w,t), M(w,z,t), M(z,z,t), M(w,z,t)\} \geq 0$$

$$\phi\{M(w,z,t), M(w,z,t), 1, M(w,z,t), 1, M(w,z,t)\} \geq 0$$

In view of Φ we get w = z Lemma 2.11 and z is a common fixed point of A,B, S and T. The uniqueness of the fixed point holds from (ii)

Theorem 4.2: Let (X, M *) be fuzzy metric space with * continuous t - norn. Let P,Q,R,S be self mappings of X satisfying

(i) The pair (P,R) and(Q,S) be owc.

(ii) For some $\phi \in \Phi$ and for all x,y $\in$ X and every t > 0

$$\phi\{M(Rx,Sy,t), M(Rx,Px,t), M(Rx,Qy,t), M(Px,Sy,t), M(Sy,Px,t)\} \geq 0$$

then there exists a unique point w$\in$ X such that Pw = Rw = w and a unique point z X such that Qz = Sz = z
Moreover z =w, so that there is unique common fixed point of P,Q,Rand S
Proof : Let the pairs {P,R} and {Q,S} be owc, so there are points x,y $\in$ X such that Px = Rx and Qy = Sy
We claim that Px= Qx. Ifnot by inequality(ii)

$$\phi\{M(Px,Qy,t), M(Px,Px,t), M(Px,Qy,t), M(Px,Qy,t), M(Qy,Px,t)\} \geq 0$$

$$\phi\{M(Px,Qy,t), 1, M(Px,Qy,t), M(Px,Qy,t), M(Qy,Px,t)\} \geq 0$$

$$\phi\{M(Px,Qy,t), 1, M(Px,Qy,t), M(Px,Qy,t), M(Px,Qy,t)\} \geq 0$$

In view of Φ we get Px =Qy i.e Px =Rx= Qy =Sy

Suppose that there is a another point z such that Pz = Rz then by (i) we have Pz = Rz = Qy =Sy, so Px = Pz and w = Px =Rx is the unique point of coincidence of P and R. By Lemma 2.12 w is the only common fixed point of P and R. Similary there is a unique z X suchthat z = Qz =Sz

$$\phi\{M(Px,Qy,t),M(Px,Px,t),M(Px,Qy,t),M(Px,Qy,t),M(Qy,Px,t)\}\geq 0$$

$$\phi\{M(x,y,t),M(x,x,t),M(x,y,t),M(x,y,t),M(y,x,t)\}\geq 0$$

$$\phi\{M(x,y,t),1,M(x,y,t),M(x,y,t),M(y,x,t)\}\geq 0$$

In view of Φ we get w = z by Lemma 2.11 and z is a common fixed point of P,Q,Rand S.The uniqueness of and fixed point holds from (ii)

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