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Path Related Analytic Mean Square-Cordial Graphs

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Abstract*– Let $G = (V, E)$ be a graph with p vertices and q edges. An Analytic Mean Square-Cordial Labeling of a Graph G with vertex set V is a bijection from V to $\{0, 1\}$ such that each edge uv is assigned the label $f(uv) = |\lceil f(u)^2 - f(v)^2/2 \rceil|$ with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.The graph that admits a Analytic Mean Square-Cordial Labeling is called Analytic Mean Square-Cordial Graph. In this paper, we proved that path related graphs Path P_n, Fan $P_n + K_1$, Comb $P_n \odot K_1$, Ladder $P_n \times K_2$are Analytic Mean Square-Cordial Graphs.

Keywords*– Path, Comb, Fan, Ladder, Analytic Mean Square-Cordial Graph, Analytic Mean Square-Cordial Labeling. 2000 Mathematics Subject classification 05C78.

I. INTRODUCTION

A Graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair $e = \{uv\}$ of vertices in E is called edges or a line of G.In this paper, we proved that path related graphs Path P_n, Fan $P_n + K_1$, Comb $P_n \odot K_1$, Ladder $P_n \times K_2$are Analytic Mean Square-Cordial Graphs.For graph theory terminology, we follow [2].

II. PRELIMINARIES

Let $G = (V, E)$ be a graph with p vertices and q edges. A Analytic Mean Square-Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0,1} such that each edge uv is assigned the label $f(uv) = |\lceil f(u)^2 - f(v)^2/2 \rceil|$ with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by atmost 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by atmost 1.

The graph that admits a Analytic Mean Square-Cordial Labeling is called Analytic Mean Square-Cordial Graph. In this paper, we proved that path related graphs Path P_n, Fan $P_n + K_1$, Comb $P_n \odot K_1$, Ladder $P_n \times K_2$are Analytic Mean Square-Cordial Graphs.

Definition: 2.1

P_n is a path of length n - 1 .

Definition: 2.2

The join of G_1 and G_2 is the graph $G = G_1 + G_2$ with vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2 \cup \{UV: u \in V_1, v \in V_2\}$.The graph$P_n + K_1$ is called a Fan.

Definition: 2.3

The Corona $= G_1 \odot G_2$of two graphsG_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has P_1 points) and P_1 copies of G_2 and joining the i^{th} point of G_1 to every point in the i^{th}copy of G_2. The graph$P_n \odot K_1$is called a comb.

Definition: 2.4

The product $G_1 \times G_2$ of two graphs G_1 and$G_2$is defined to be the graph whose vertex set is $V_1 \times V_2$and two vertices $u = (u_1, u_2)$ and$v = (v_1, v_2)$ in $V = V_1 \times V_2$are adjacent in $G_1 \times G_2$ if either $u_1 = v_1$ and u_2 is adjacent to v_2 or $u_2 = v_2$ and u_1 is adjacent to v_1. $P_n \times K_2$is called a Ladder.

III. MAIN RESULTS

Theorem: 3.1

Path P_n is Analytic Mean Square-Cordial Graph.

Proof:

Let $V(P_n) = \{[u_i: 1 \leq i \leq n]\}$ and
$E(P_n) = \{[(u_i u_{i+1}): 1 \leq i \leq n-1]\}$.

Define $f: V(P_n) \rightarrow \{0,1\}$.

The vertex labeling are,

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

The induced edge labelling are,

$$f*[(u_i u_{i+1})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n-1$$

When n= 2m+1, m>0

$v_f(0) = \frac{n-1}{2}$

$v_f(1) = \frac{n+1}{2}$ and

$e_f(0) = e_f(1) = \frac{n-1}{2}$.

When n = 2m+2, m≥0

$v_f(0) = v_f(1) = \frac{n}{2}$ and

$e_f(0) = \frac{n-2}{2}$

$e_f(1) = \frac{n}{2}$.

Therefore, PathP_n satisfies the conditions $|v_f(0) - v_f(1) \leq 1|$and$|e_f(0) - e_f(1) \leq 1|$.

Hence, Path P_n is Analytic Mean Square-Cordial Graph.

For example, the Analytic Mean Square-Cordial Labeling of $P_6$is shown in figure 3.2.

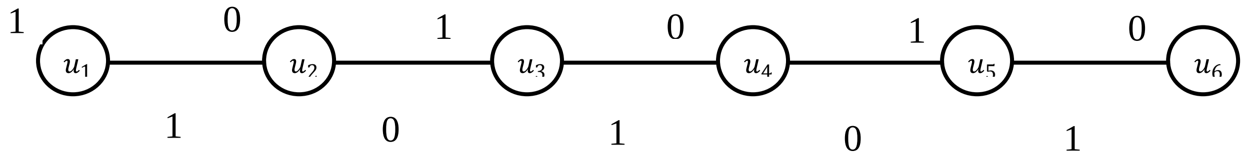

Figure 3.2: P_6

Theorem 3.3

Fan $P_n + K_1$(n-even) is Analytic Mean Square-Cordial Graph.

Proof:

Let $V(P_n + K_1)$ ={[u, u_i: 1≤i≤n]} and

$E(P_n + K_1)$ = {[uu_i: 1 ≤ i ≤ n] ∪[($u_i u_{i+1}$): 1≤i≤n-1]}.

Define $f: V(P_n + K_1) \rightarrow \{0,1\}$.

The vertex labelling are,

$f(u) = 1$

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

The induced edge labelling are,

$$f*[(uu_i)] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

$$f*[(u_i u_{i+1})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \leq i \leq n-1$$

When n = 2m+2 , m≥0

$v_f(0) = \frac{n}{2}$

$v_f(1) = \frac{n}{2} + 1$ and

$e_f(0) = n-1$

$e_f(1) = n$.

Therefore, Fan $P_n + K_1$(n-even) satisfies the conditions $|v_f(0) - v_f(1) \leq 1|$ and $|e_f(0) - e_f(1) \leq 1|$.

Hence, Fan $P_n + K_1$(n-even) is Analytic Mean Square-Cordial Graph.

For example, the Analytic Mean Square-Cordial Labeling of $P_6 + K_1$ (n-even) is shown in figure 3.4

Theorem 3.5

Fan $P_n + K_1$(n-odd) is Analytic Mean Square-Cordial Graph.

Proof:

Let $V(P_n + K_1)$ ={[u, u_i: 1≤i≤n]} and

$E(P_n + K_1)$ = {[(uu_i): 1 ≤ i ≤ n] ∪ [($u_i u_{i+1}$): 1≤i≤n-1]}.

Define $f: V(P_n + K_1) \rightarrow \{0,1\}$.

The vertex labelling are,

$f(u) = 1$

$$f(u_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \leq i \leq n$$

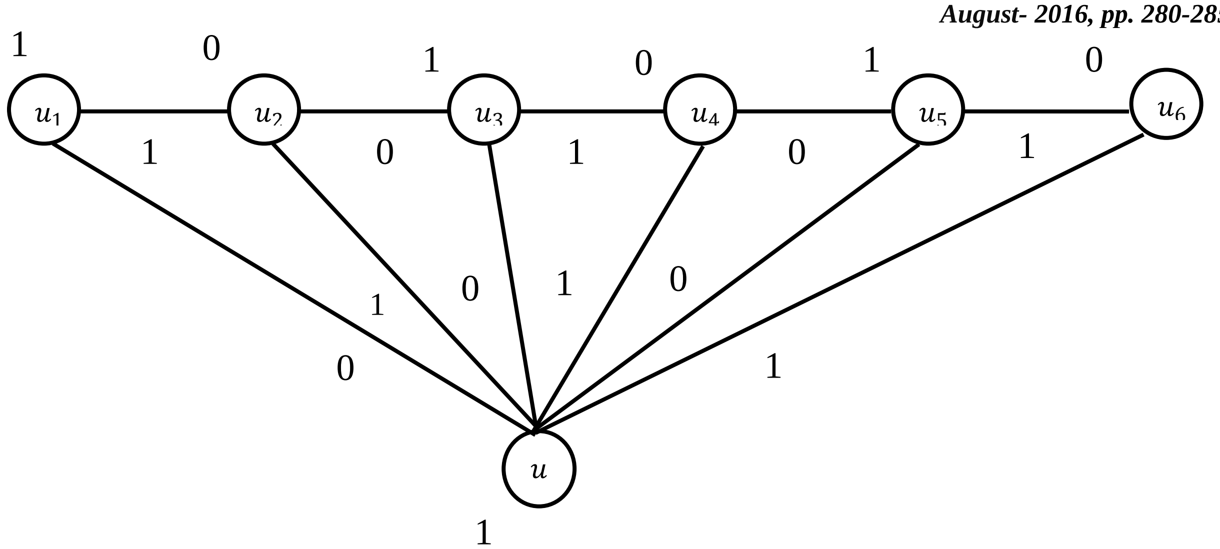

Figure 3.4: $P_6 + K_1$

The induced edge labelling are,

$f*[(uu_i)] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases}$ 1≤i≤n

$f*[(u_i u_{i+1})] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases}$ 1≤i≤n-1

When n = 2m+1, m>0

$v_f(0) = v_f(1) = \frac{n+1}{2}$ and

$e_f(0)$= n-1

$e_f(1)$= n.

Therefore, Fan $P_n + K_1$(n-odd) satisfies the conditions $|v_f(0) - v_f(1) \leq 1|$ and$|e_f(0) - e_f(1) \leq 1|$.

Hence, Fan $P_n + K_1$(n-odd) is Analytic Mean Square-Cordial Graph.

For example, the Analytic Mean Square-Cordial Labeling of$P_5 + K_1$is shown in figure 3.6

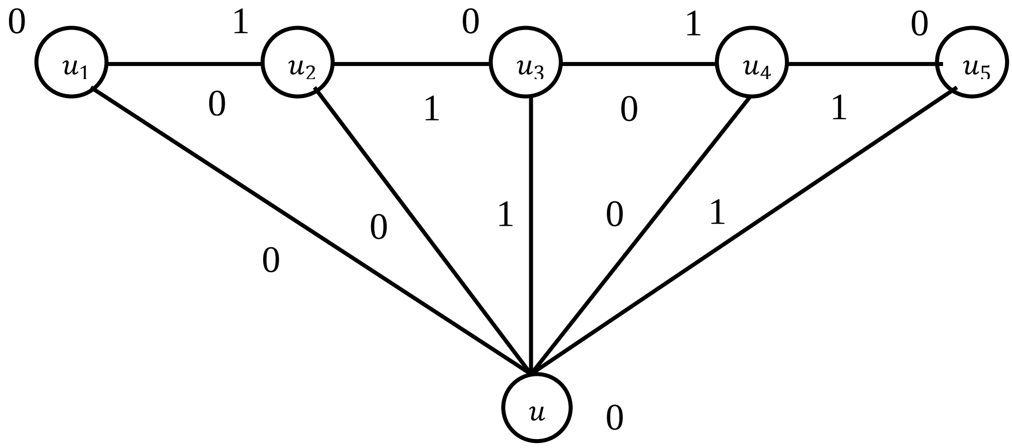

Figure 3.6: $P_5 + K_1$

Theorem: 3.7

Comb $P_n \odot K_1$ is Analytic Mean Square-Cordial Graph.

Proof:

Let $V(P_n \odot K_1)$ = $\{[u_i, v_i$: 1≤i≤n]} and

$E(P_n \odot K_1)$ = {[$(u_i u_{i+1})$: 1≤i≤n-1]∪ [$(u_i v_i)$: 1≤i≤n]}.

Define $f: V(P_n \odot K_1) \rightarrow \{0,1\}$.

The vertex labeling are,

$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases}$ 1≤i≤n

$f(v_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases}$ 1≤i≤n

The induced edge labelling are,

$f*[(u_i u_{i+1})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases}$ 1≤i≤n-1

$f*[(u_i v_i)] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases}$ 1≤i≤n

When n = 2m+1, m>0 and n = 2m+2, m≥0

$v_f(0)$=$v_f(0)$ = nand

$e_f(0)$ = n-1

$e_f(1)$ = n.

Therefore, Comb $P_n \odot K_1$ satisfies the conditions$|v_f(0) - v_f(1) \le 1|$ and$|e_f(0) - e_f(1) \le 1|$.

Hence, Comb $P_n \odot K_1$ is Analytic Mean Square-Cordial Graph.

For example, the Analytic Mean Square-Cordial Labeling of is $P_5 \odot K_1$ shown in figure 3.8.

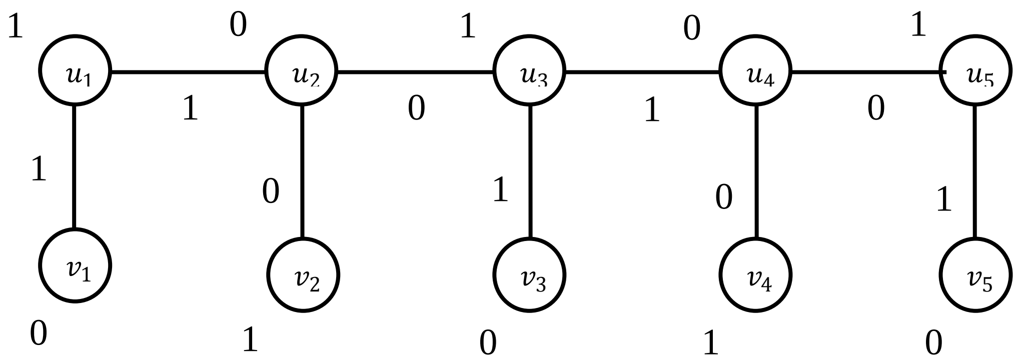

Figure 3.8: $P_5 \odot K_1$

Theorem: 3.9

Ladder $P_n \times K_2$is Analytic Mean Square-Cordial Graph.

Proof:

Let $V(P_n \times K_2)$= $\{[u_i, v_i$: 1≤i≤n]} and

$E(P_n \times K_2)$= $\{[(u_i u_{i+1}) \cup (v_i v_{i+1})$: 1≤i≤n-1]$\cup[(u_i v_i)$: 1≤i≤n]}.

Define $f : V(P_n \times K_2) \to \{0,1\}$.

The vertex labeling are,

$$f(u_i) = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n$$

$$f(v_i) = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n$$

The induced edge labelling are,

$$f*[(u_i u_{i+1})] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

$$f*[(v_i v_{i+1})] = \begin{cases} 0 & i \equiv 1 \bmod 2 \\ 1 & i \equiv 0 \bmod 2 \end{cases} \quad 1 \le i \le n-1$$

$$f*[(u_i v_i)] = \begin{cases} 0 & i \equiv 0 \bmod 2 \\ 1 & i \equiv 1 \bmod 2 \end{cases} \quad 1 \le i \le n$$

When n = 2m, m>0

$v_f(0)$ =$v_f(0)$ = n and

$e_f(0)$ = $e_f(1)$ =3m-1, m>0

When n = 2m-1, m>0

$v_f(0)$ = $v_f(1)$=n and

$e_f(0)$= 3m+1, m>0

$e_f(1)$ = 3m,m>0

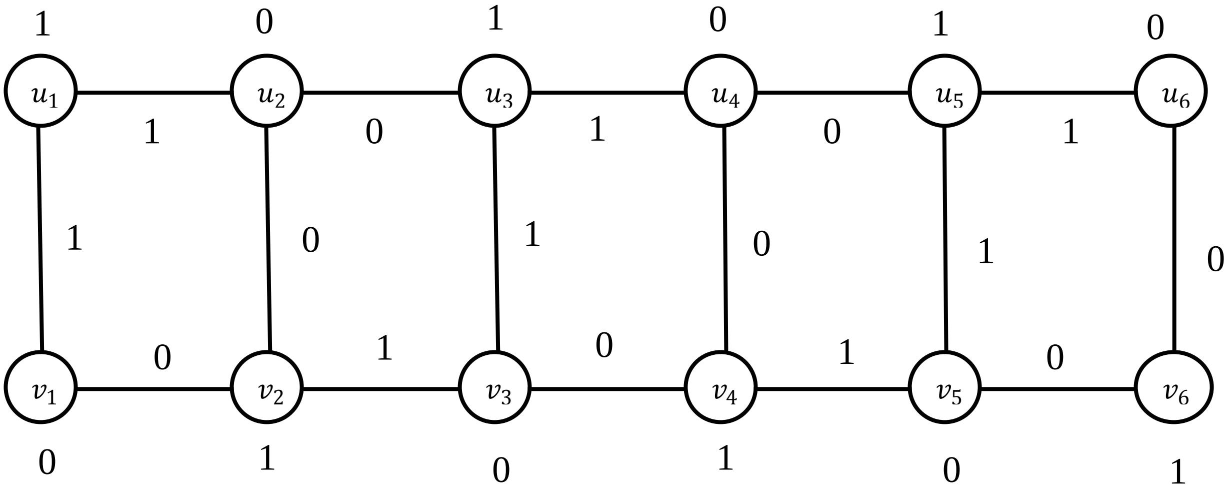

Figure 3.10:$P_6 \times K_2$

Therefore, Ladder $P_n \times K_2$ satisfies the conditions $|v_f(0) - v_f(1) \leq 1|$ and $|e_f(0) - e_f(1) \leq 1|$.
Hence, Ladder $P_n \times K_2$ is Analytic Mean Square-Cordial Graph.
For example, the Analytic Mean Square-Cordial Labeling of $P_6 \times K_2$ is shown in figure 3.10

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