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Building the Random Model to Predict the Probability of an Earthquake in Kurdistan Region

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Abstract: Poisson process is one of the most important process of stochastic process, which play an important role in our lives, And almost it's an important part in various fields, especially with regard to the natural phenomena study that suddenly occur and in the irregular times. In this paper used inhomogeneous Poisson process study where the time rate of event (rate of occurrence) is changing according to the change in time t, and it has been chosen nonlinear (exponential) function as the time rate of event and compare with linear and quadratic function with estimating their parameters by using the statistical program (Minitab 16). The study also containing realistic application and deals with the phenomenon of earthquakes in Kurdistan region within the time period 2013-2015 and Time rates of occurrence of earthquakes has been estimated in these governorates with testing the homogeneity of Poisson process.

Keyword: Earthquake, Poisson Process, Homogeneous Poisson process, Inhomogeneous Poisson process, Test of the Homogeneity.

I. INTRODUCTION

The Poisson process is one of the most important stochastic processes in probability theory. Poisson processes have been used to model random phenomena in areas such as communications, hydrology, meteorology, insurance, reliability, and seismology among others. These processes are often appropriate for modeling a series of events over time. Poisson processes are governed by an intensity function λ (t), which determines the instantaneous rate of event occurrence at time t. homogeneously, a Poisson process is also governed by the cumulative intensity function $(\Lambda(t) = \int_0^t \lambda\,(t)dt$. When the intensity function is a constant, the Poisson process is known as a homogeneous Poisson process (HPP). When the intensity function varies with time, the Poisson process is known as an Inhomogeneous Poisson process (IHPP).

Definition: A Poisson process of rate, A > 0 is an integer- valued random process (X (t); t ≥ 0) for which

$i.for\ any\ time\ points\ t_{0=}0\ <\ t_1\ <\ t_2\ <\cdots\ t_n\ \ the\ process\ increments$ $X(t_1) - X(t_0), X(t_2) - X(t_1), \ldots, X(t_n), X(t_{n-1})$ are independent random variables;

$ii.for\ s\ \geq 0, the\ random\ variable\ \ X(s+t) - X(s) has\ the\ poisson\ distribution$

$$\Pr\{\,X\,(s+t) - X\,(s) = k\} = \frac{(\lambda t)^k e^{-\lambda t}}{k\,!} \qquad for\ k = 0, 1, \ldots$$

$iii. X\,(0) = 0$

In particular, observe that if X (t) is a Poisson process of rate A > 0, then the moments are

$$\mathrm{E}[\,\mathrm{X(t)}] = \lambda t \ \text{ and } \ \mathrm{Var[X(t)]} = \sigma^2_{X(t)} = \lambda t$$

II. ASSUMPTION OF POISSION PROCESS

Given a sequence of independent events, each of them indicating the time when they occur. We assume

1. The probability that an event occurs in a time interval *I* Ç [0, +∞] does *only* depend on the length of the interval and not of where the interval is on the time axis.
2. The probability that there in a time interval of length *t* we have *at least one event*, is equal to $\lambda t + t\varepsilon(t)$, Where $\lambda > 0$ is a given positive constant.
3. The probability that we have *more than one event* in a time interval of length *t* is $t\,\varepsilon(t)$. It follows that The probability that there is *no event* in a time interval of length is given by $1 - \lambda t + t\varepsilon(t)$.
4. The probability that there is *precisely one event* in a time interval of length *t* is $\lambda t + t\,\varepsilon\,(t)$. Here $\varepsilon\,(t)$ denotes some unspecified function, which tends towards 0 for $t \to 0$.Given the assumptions on the previous page, we let $X(t)$ denote the number of events in the interval [0, *t*], and weput

$$P_k\,(t) := P\,\{X\,(t) = k\}, \qquad \text{for } k \in \mathbb{N}_0.$$

Then $X\,(t)$ is a *Poisson distributed random variable of parameter* λt. The process $\{X\,(t) \mid t \in [0, +\infty]\}$ is called a *Poisson process*, and the parameter λ is called the *intensity* of the Poisson process.

III. THE TYPE OF POISSON PROCESS

A. Homogeneous Poisson process

A homogeneous Poisson process s (HPP) is specified with a constant intensity λ and is a renewal process for which the interarrival distribution is an Exponential with mean value 1 / λ. This process is denoted HPP (λ). For a homogeneous Poisson process the number of events in disjoint time intervals are independent and Poisson distributed, i.e. when the time intervals (s, t] and (l,k] are disjoint, the random variables *N(t) – N(s)* and *N(k) – N(I)* are independent and *N(t)* has the distribution $P_0(\lambda t)$. It also holds that the increments are stationary, i.e. the number of events in a certain interval depends only on the length of the interval *N (0)* is defined to be 0, i.e. at time 0 no events has occurred. Since we know that *N (t)* ~ P_0 (λt) we can calculate the probability of *n* events occurring up to time t;

$$P(N(t) = n) = \frac{(\lambda t)^n}{n!} e^{-\lambda t} \qquad \dots (1)$$

B. Inhomogeneous Poisson process

The homogeneous Poisson process can be generalized to an inhomogeneous Poisson process (IHPP) by letting the intensity take different values at different time points. The inhomogeneous Poisson process is denoted IHPP (Λ (t)). Recall that the HPP has a constant intensity λ and that the distribution of the interarrival only depends on the length of the interval. Since the intensity varies in time for the IHPP we need to define a function that is deterministic and describes how the intensity varies. This function is called the intensity function, (λt). Then we denote the cumulative intensity function as

$$(\Lambda(t) = \int_0^t \lambda(u)du \qquad \dots\dots\dots\dots\dots\dots (2)$$

The distribution of N (t) becomes $P_0\,(\int_0^t \lambda\,(u)\,du)$ and we can calculate the probability of n events occurring up to time t;

$$P(N(t) = n) = \frac{(\Lambda(t))^n}{n!} e^{-(\Lambda(t))} \qquad \dots (3)$$

C. Difference between homogeneous and inhomogeneous Poisson process (HPP v IHPP)

A main difference between HPP and IHPP is that the distribution of the interarrival times of a IHPP not only depends on the length of the interval, but on the age of the process. Another difference is that the IHPP has “minimal repairs”, which means that when an event occurs the system restores itself to the same condition it had just before that event happened, whereas the HPP restores to the original condition.

IV. THE RATE OF OCCURRENCE IN INHOMOGENEOUS POISSON PROCESS

The cumulative function of time rate of occurrence m (t) calculated based on choosing the appropriate function of the rate of occurrence of time and determines its form building on the data type under study, There are a large number of researchers have used and suggested several forms of functions (time rate of occurrence) linear and non-linear (exponential), and assume that the function in non homogeneous Poisson process be a monotone function of time t.

In 1966, researchers (Cox and Lewis) proposed non linear function (exponential) as a time rate of occurrence, and the function is:

$$\lambda(t) = e^{(a+bt)} \qquad \dots\dots (4)$$

In 1983, the researcher (Yamada) used a function as time rate of occurrence, the function is:

$$\lambda(t) = ab^2te^{-bt} \qquad \dots.. (5)$$

And In 1984, the researcher (Musa and Okumoto) used a Fractional function as time rate of occurrence,

$$\lambda(t) = \frac{a}{t+b} \qquad \dots\dots\dots\dots (6)$$

- In the other hand, there are number of researchers used linear function as time rate of occurrence,

In 1983, the researcher (Massey et al.) used a linear function on a limited period of time in the field of applications of communications on time rate to arrive, and the function is:

$$\lambda(t) = a + bt \qquad \dots\dots (7)$$

V. SELECTION OF THE APPROPRIATE FUNCTION AS RATE OF OCCURRENCE EVENTS

The selection of the time rate function (the ratio function) for the occurrence of events in the Inhomogeneous Poisson process is based on their suitability of the data under study. Thus, the selection of time rate function in Inhomogeneous process is very important, as there are researchers used a non linear function (exponential) function, called Cox and Lewis $\lambda(t) = e^{a+bt}$, instead of a linear function $\lambda(t) = a + bt$, in order to ensure that the time rate is not negative (positive) for each values of t, without restrictions for it.

And also the researchers (Basawa and Prakasa, 1980), researchers (Snyder and Miller, 1991), confirmed that in the natural sciences be considered Poisson processes by non-linear occurrence ratios.

As a result of the above, it is very appropriate to choose the non-linear function (exponential) $\lambda(t) = e^{a+bt}$ as the time rate of occurrence events in the inhomogeneous Poisson process. The advantage of this form of the functions, it became a general form is used in many applications to homogeneous Poisson process.

VI. TEST OF THE HOMOGENEITY OF POISSON PROCESS

The Poisson process if it was the time rate of occurrence of events (λt) is a fixed amount for each values of t, that is, they are not affected in time t in its behavior, is called Homogeneous Poisson Process. But if the time rate of occurrence of events is constant for all values of t, that is, they are affected in time t in behavior that is called inhomogeneous Poisson process. To test whether the Poisson process are homogeneous or not, we use the following two hypotheses:

Null hypothesis: Poisson process is homogeneous [H_0: A = 0]

Alternative hypothesis: Poisson process is in homogeneous [H_1: A ≠ 0]

And the test statistics for testing these two hypotheses are as follows:

$$Z = \frac{\sum t_i - \frac{1}{2} n t_0}{\left(\frac{1}{12} n\, t_0^2\right)^{\frac{1}{2}}} \quad \ldots (8)$$

Where

$\sum_{i=1}^{n} t_i$: Sum of times of occurrence of events for the period of time [0, t_0]

n: Number of events that have occurred in the time period [0, t_0]

t_0: Intra accumulated periods between the occurrences of events

VII. APPLICATION OF THE PHENOMENON OF EARTHQUAKES IN KURDISTAN REGION

Earthquake is a natural phenomenon of a ground vibration due to the fast break and displacement the rocks because of the accumulation of internal stresses as a result of geological effects resulting floor plates movement. It may arise as a result of volcanic activities or landslides due to the presence in the layers of the earth. Also it leads to the cracking of the earth and depletion the springs or the emergence of new springs or occur high waves if they got under the sea (tsunami) as well as the disruptive effects of the buildings. The strength of earthquakes measured on the Richter scale, which measures the energy emitted from the epicenter. This device is a logarithmic scale of 1 to 9, where the quake, with a magnitude 7 degrees stronger ten times of the earthquake measuring 6 degrees, the more powerful 100 times earthquake measuring 5 degrees, and a stronger 1000 times of the earthquake measuring 4 degrees and so on. The estimated number of earthquakes with strength of 5 scales to 6 degrees, which occur annually in the world about 800 earthquakes while located about 50,000 earthquake of magnitude of 3 to 4 degrees per year, as one earthquake located annually measuring 8 to 9 degrees.

It is important to know where and when they may happen with knowing the scale of earthquake to estimate time rates of occurrence in order to put its future plans. It has been obtain related data to event of earthquakes in these governorates, in terms of times of occurrence and magnitude on the Richter scale from the Web site "Iraqi meteorological organization and seismology ", and selected the time period from 2013 to 2015. (Figure 1-3) shows the events of earthquakes in the three governorates in terms of times of occurrence and magnitude

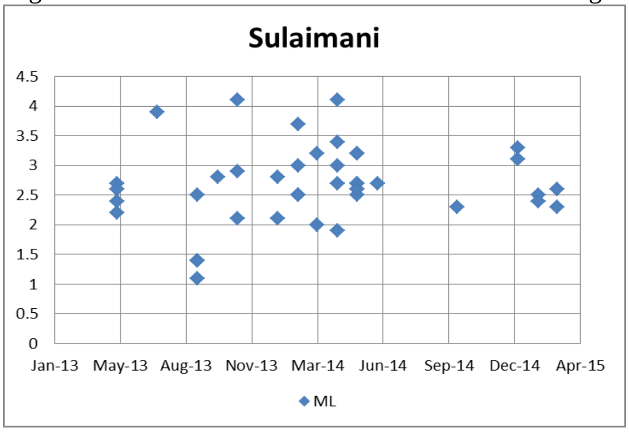

Figure1. The events of earthquakes in Sulaimani

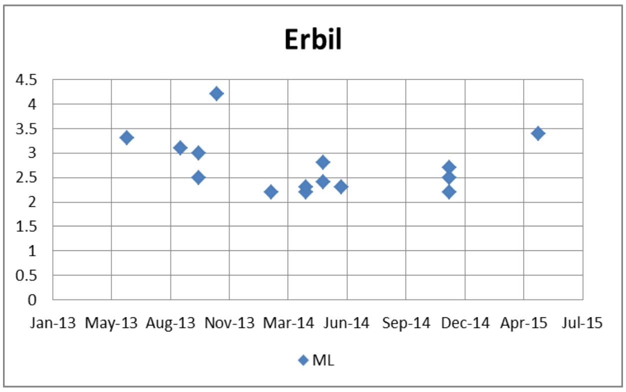

Figure 2. The events of earthquakes in Erbil

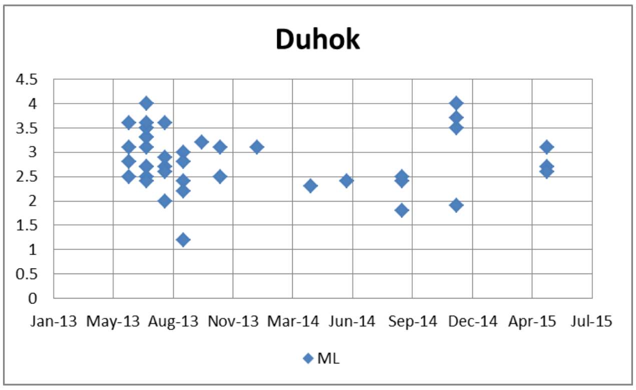

Figure 3. The events of earthquakes in Duhok

And the table 1 shows the earthquakes scale:

Table 1 Display Earthquake Sacle

Magnitude	Description	Earthquake effects
< 2.0	Micro	Not felt
2.0 – 2.9	Minor	Generally not felt, but recorded
3.0 – 3.9		Often felt, but rarely causes damage
4.0 – 4.9	Light	Noticeable shaking of indoor items, rattling noises. Significant damage unlikely
5.0 – 5.9	Moderate	Can cause major damage to poorly constructed building over small
6.0 – 6.9	Strong	Can be destructive in areas up to about 160 kilometers across in populated areas
7.0 – 7.9	Major	Can cause serious damage over larger areas
8.0 – 8.9	Great	Can cause serious damage in areas several hundred kilometers across
9.0 – 9.9		Devastating in areas several thousand kilometers across
10.0 +	Massive	Never recorded, widespread devastation across very large areas

The average depth of earth quake in Kurdistan region is 2.86 and according to table (1), the earthquake scale in all city of the region is “Minor” it means often felt but rarely causes damage.

VIII. THE RATE OF OCCURRENCE IN INHOMOGENEOUS POISSON PROCESS

Estimate the Rate of Occurrence of Earthquakes

Estimating the time rate of occurrence of earthquakes means estimate the function that have been selected to describe the time rate (daily rate) of occurrence of earthquakes for each governorate, for this purpose the statistical program (Minitab 16) has been used to estimate parameters of each selected function and the calculation of the time rate of occurrence and according to the specified time period, and the equations are shown in the below tables:

Table 2 Time Rate of Occurrence of Earthquake in Sulaimani

Sulaimani			
Model	MAPE	MAD	
Linear	20.7487	$Y_t = 2.493 + 0.00868\ t$	**0 = < t < = 1054**
Quadratic	20.4377	$Y_t = 2.216 + 0.073t - 0.000921t^2$	
Exponential	20.190	$Y_t = 2.3344 * (1.00495)^t$	

The results of the above table show that MAPE measure (mean absolute percentage error) of exponential model is less than MAPE measure for the other model, i.e. exponential model is the fit model. Similarly for the time rate of occurrence of earthquakes in other governorate, it can be used the same program and the results are shown in the following tables:

Table 3 Time Rate of Occurrence of Earthquake In Erbil

Erbil			
Model	MAPE	MAD	
Linear	12.4269	$Y_t = 3.196 + 0.067033\ t$	**0 = < t < = 1078**
Quadratic	10.1780	$Y_t = 3.547 - 0.199t - 0.00879t^2$	
Exponential	12.1274	$Y_t = 3.1474 * (0.9771)^t$	

Table 4 Time Rate of Occurrence of Earthquake in Duhok

Duhok			
Model	MAPE	MAD	
Linear	18.3840	$Y_t = 0.070 - 0.010688\ t$	**0 = < t < = 1061**
Quadratic	18.4173	$Y_t = 3.288 - 0.0403t - 0.000682t^2$	
Exponential	17.9283	$Y_t = 3.0241 * (0.99594)^t$	

Table 5 Time Rate of Occurrence of Earthquake in Kurdistan Region

Over all models		
Model	MAPE	MAD
Linear	114.968	$Y_t = 13.29 + 0.0579\ t$
Quadratic	110.663	$Y_t = 16.60 - 0.141t + 0.00201t^2$
Exponential	94.8197	$Y_t = 11.3 * (1.00283)^t$

According to the tables (3- 5) and by depending on MAPE measure, exponential model is the fit model for Time rate of occurrence of earthquake in Duhok and overall region but quadratic model is the fit model for time rate of occurrence of earthquake in Erbil.

And the time rate of occurrence of earthquakes can find from time period (01/01/2013) to (31/12/2015), when the total length of the time period t, is equal to the time of the latest event, any ($t = t_0$) in all governorate under study, as shown in the following table:

Table 6 Daily Rate of Occurrence of Earthquake

Governorate	daily rate of earthquakes
Sulaimani	2.26
Erbil	3.34
Duhok	3.01

IX. TEST OF THE HOMOGENEITY OF POISSON PROCESS

The time rate of occurrence of events in this paper is the exponential and quadratic function, where we find that the parameter A associated in time t, and so the Poisson process be homogeneous in the case of A = 0, and inhomogeneous in the case A ≠ 0, and accordingly, it is to conduct a test whether the Poisson process are homogeneous or not, we are testing the following two hypotheses:

Null Hypothesis [H_0: A = 0] versus Alternative hypothesis [H_1: A ≠ 0]

Table 7 Test of Homogeneity

Governorate	$Z_{calculate}$ value
Sulaimani	-1.081
Erbil	-6.05
Duhok	-1.024

After noticing the result of table 7, the value of calculated statistical test (Z) is less than zero which indicates that the earthquakes can be considered inhomogeneous Poisson process with time rate of occurrence of earthquakes which is decreasing with the passage of time (t).

X. CONCLUSIONS AND RECOMMENDATIONS

In this last part, will display the most important conclusions that have been reached through this study, in addition to a set of recommendations, which the study recommended.

A. Conclusions

1. Through the application on the events of earthquakes in Kurdistan region, it was noted that the Poisson process for the occurrence of earthquakes in this region through the test of homogeneity, are inhomogeneous Poisson process.
2. Through calculating the daily average of earthquakes, found that the average daily earthquakes in Duhok greater than Sulaimani and Erbil.
3. The value of the statistical test Z appeared negative for each of Sulaimani, Erbil and Duhok, which gives an indication that the time rate of occurrence is decreasing with the passage of time t.

B. Recommendations

1. They need for a systematic database in Earthquake and atmospheric circle
2. They need for more studies and research in this area because the earthquake natural disaster leads to end the lives of millions of human and destruction of cities fully
3. The use of this kind of methods for other studies, such as research in the area of floods, volcanoes and other.

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