



Fuzzy Logic = Computing with Words

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Abstract— *With the ever increasing demands of Computational techniques, Fuzzy logic has gained a major popularity when compared to its predecessor techniques. The main context of this paper is that words are used for computing instead of numbers and formulas. Words, in turn are used, firstly, when the available information is imprecise to abide with the usage of numbers and second when there is a tolerance for imprecision, which can be exploited to achieve robustness, tractability, low cost solution and finally a better rapport with the reality. Fuzzy logic plays a very prominent role in computing with words (CW) and vice versa. In this paper we discuss about the rules of inference that are used to transform the constraints that are implicit, from premises to conclusions. In future, this method would be implemented in a wide range of applications.*

Keywords— *Computing with words (CW), IDS (Initial Data Set), TDS (Terminal Data Set), Granulation, Explanatory Database(ED).*

I. INTRODUCTION

Fuzzy logic has now been used in wide range of applications from physical sciences to mathematical sciences. The two main questions that prevail within the technologists are 1. What is fuzzy logic? 2. How fuzzy logic excels all the other methodologies such as predicate logic, Bayesian networks and classic control? The answer to these questions can be suggested in a better way from the title of this paper. Fuzzy logic predominantly becomes the major contributor for the methodology for computing with words. No other existing methodology serves better for this purpose. Thus Computing with words (CW) becomes one of acute branches of Fuzzy Logic.

With CW, fuzzy set of points are drawn together by similarity, with a fuzzy set playing a role of fuzzy constraint on a variable. For computational purposes, propositions are expressed in terms of canonical forms which serve to place in evidence of fuzzy constraints that are implicit in the premises.

Then the rules of inference are used to transform constraints from premises to conclusions.

Why computing with words?

Computing involves mostly of symbols and numbers. Humans generally employ words in computing and reasoning, thereby aiming at conclusions expressed as words from premises expressed in a natural language. Words do have fuzzy denotations. Concept of CW has got its roots in several research papers starting with the concepts of linguistic variable and granulation and fuzzy logic seems to be a better approach to support these for meaningful representations.

Key aspect of CW is that it involves a fusion between natural languages and computation with fuzzy variables. It is this fusion that is likely to result in an evolution of CW into a basic methodology in its own right, with wide-ranging ramifications and applications.

II. LITERATURE REVIEW

In the research work [1] Computation with words has important ramification for mathematics to construct mathematical solutions of computational problems which are stated in natural language. Traditional Mathematics lacks these capabilities.

Luis Martinez and Francisco Herrera [2] suggested that the more significant and extended linguistic computing models due to its key role in linguistic decision making and a wide range of the most recent applications of linguistic decision support models.

In order to make linguistic based decision making process demands computing with words to solve related decision problems. Here decision making process can be seen as a wholesome combinations of various phases such as information gathering, analysis and selection based on several mental and reasoning process that led to choose a better alternative among the available alternatives.

In this research paper Zadeh [3] mentioned that Computers would be activated by words, which could be represented in mathematical forms using fuzzy sets. This can be otherwise mentioned as “input” words that are provided by the human is getting transformed into “output” words which are provided back to humans or to other humans. Because words can mean different things to different people, it is important to use a Fuzzy Set model that lets us capture word uncertainties.

According to Popper by falsifiability, Popper [4] meant “if a theory is incompatible with possible empirical observations it is scientific; conversely, a theory which is compatible with all such. Observations, either because, as in

the case of Marx-ism, it has . been modified solely .to accommodate such observations, or because, as in the case of psychoanalytic theories, it is consistent with all possible observations, is unscientific.”

For a theory to be called scientific it must be testable. This means that it must be possible to take measurements that are related to the theory. A scientific theory can be correct or incorrect. An incorrect scientific theory is still a scientific theory, but is one that must be replaced by another scientific theory that is itself subject to change at a later stage.

III. DEFINITIONS OF NOTATIONS AND CONCEPTS USED

We begin our exploration of CW with few definitions:

The point of departure in a CW is based on the concept of **granule**. **Granulation** is the decomposition of whole into parts. **Granules** are clump of points that are generally hierarchical in nature. For eg, for head, the granule points become the parts of head such as eyes, nose, eyebrows, lips etc.

There are many misconceptions about what Computing with Words is & what it has to offer. A very common misconception is that CW and NLP (Natural language Processing) are closely related. In fact, this is not the case. CW and NLP have different agendas and address different problems. A very simple example of a Problems in CW is the following.

Table 1: Computing with Words Vs Computing with Numbers

CN	CW
David is 24 years old	David is young
Tanya is 3 years older than David	Tanya is few years older than David
Tanya is (25+3) years older than David	Tanya is(young + few)years older than David

IV. BASIC STRUCTURE OF CW

The point of departure in CW is a question, q, of the form: What is the value of a variable, Y? The answer to this question is expected to be derived from a collection of propositions, I, I=(p1, ..., pn), which is referred to as the information set. In essence, I is a collection of question-relevant. The terminus consists of an answer of the form: Y is ans(q/I). Generally, ans(q/I) is not a value of Y but a restriction (generalized constraint) on the values which Y is allowed to take. (Zadeh 2006) Equivalently, ans (q/I) identifies those values of Y which are consistent with I. In CW, consistency is equated to possibility, with the understanding that possibility is a matter of degree.

In essence, CW is a system of computation in which the objects of computation are words, phrases and propositions drawn from a natural language. The carriers of information are propositions. It should be noted that CW is the only system of computation which offers a capability to compute with information described in a natural language

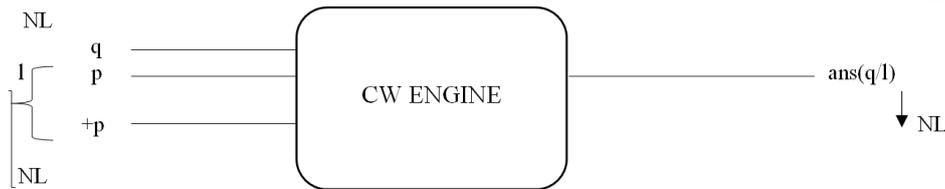


Fig 1: Basic Structure of CW

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In CW, the information is conveyed by constraining the value of variable and information is assumed to consist of a collection of prepositions expressed in a natural or synthetic language. The collection of propositions is expressed generally in terms of natural language which constitutes the Initial Data Set (IDS).From this IDS we infer the answer of a query which is expressed in terms of natural language from which the Terminal Data Set (TDS) can be derived. In order to illustrate these concepts we tend to formulate few problems:

If ‘f’ is a function such that,

f; f:U-→V,X€U,Y€V then if
 if X is small then Y is small
 if X is medium then Y is large
 if X is large then Y is small.

Function ‘f’ is approximated to f*, which is a fuzzy graph, that serves as an approximation to a function or a relation.

For example, the probability of drawing a particular colored ball from the bag. The contents of the bag, which is the verbal description are the IDS and the desired probability would be the TDS.

Considering X & Y as independent random variables taking values in a finite set V= {v1,v2,.....vn } with probabilities p1...pn and q1.....qn, where p & q are the probabilities of X &Y. The Probability distribution is described in words through fuzzy if-then rules

P: if X small p small
 X medium p large
 X large p small

Q: if Y small q is small
 Y medium q is large
 Y large q is large

Where small, medium and large are the granules. Computing with words can be viewed as combination of two related streams – fuzzy logic and test score semantics. Here the latter, is based on fuzzy logic concepts. The contact point is the collection of premises, which are assumed to be propositions expressed in natural language.

The function of canonical form is to explicitly form the implicit fuzzy constraints which are resident in the premises. The point of contact as canonical form, fuzzy constraint propagation leads to conclusions in the form of induced fuzzy constraints. Finally constraints are translated into natural language (NL) through linguistic approximations.

In computing with words, there are two core issues. They are:

1. Representation of fuzzy constraints.
2. Fuzzy Constraint Propagation.

V. REPRESENTATION OF FUZZY CONSTRAINTS AND CANONICAL FORMS

There are two observations which are in order. First, in using fuzzy constraint propagation rules in computing with words, application of the extension principle generally reduces to the solution of a nonlinear program. What we need-and do not have at present-are approximate methods of solving such programs which are capable of exploiting the tolerance for imprecision. Without such methods, the cost of solutions may be excessive in relation to the imprecision which is intrinsic in the use of words. In this connection, an intriguing possibility is to use genetic algorithm-based methods to arrive at approximate solutions to constrained maximization problems. Second, given a collection of premises expressed in a natural language, we can, in principle, express them in their canonical forms and thereby explicitate the implicit fuzzy constraints. For this purpose, we have to employ test-score semantics.

Fuzzy constraints will be based on test score semantics. In natural language, a proposition ‘p’ is considered as network of fuzzy constraints. On applying aggregation functions, the constraints which are in p result in an overall fuzzy constraint which can be represented as an expression of the form,

$$\mathbf{X \text{ is } \epsilon R}$$

where R is a constraining fuzzy relation and X is the constrained variable. The expression in question is called a canonical form. This is represented schematically as $\mathbf{p \text{ --} \rightarrow X \text{ is } R}$

in which the arrow \rightarrow denotes explicitation.

Here the meaning of p is defined in two procedures: First as Explanatory Database(ED) which is a collection of relations on which R is defined that returns the constraint variable X. The other procedure acts on ED & returns constraining relation R. ED comprises of relation names, attributes and domains with no entries in the relation. If there is an entry then we call it as EDI (which is instantiated).

The concept of a restriction: A Closer look

When asked: What is the value of a real-valued variable X? The answer is: I do not know the value precisely but I have a perception which I can express as a restriction (generalized constraint) on the values which X can take

$$8 \leq X \leq 10$$

X is small

X is normally distributed with mean 9 and variance 2.

It is likely that X is between 8 and 10.

Representation of a restriction/constraint

A restriction (generalized constraint), $\mathbf{R(X)}$, may be represented as:

$$\mathbf{R(X): X \text{ is } r R}$$

where X is the restricted (constrained) variable, R is the restricting (constraining) relation and r is an indexical variable which defines how R restricts X.

Example:

Possibility restriction (r=blank): $\mathbf{R(X): X \text{ is } A}$

where A is a fuzzy set in U with the membership function μ_A . A plays the role of the possibility distribution of X
 $\text{Poss}(X=u) = \mu_A(u)$

Probabilistic restriction

Probabilistic restriction (r=p):

$$\mathbf{R(X): X \text{ is } p P}$$

where P plays the role of the probability distribution of X.

$$\mathbf{Prob}(u \leq X \leq u+du) = p(u)du$$

where p is the probability density function of X.

In the context of natural languages, restrictions are predominantly possibilistic, probabilistic or combinations of the two.

For simplicity, the indexical variable,

r, is sometimes suppressed, relying on the context for disambiguation.

Direct and indirect restrictions:

A restriction is direct if it is of the form:

$$\mathbf{R(X): X \text{ is } r R}$$

A restriction is indirect if it is of the form:

$$\mathbf{R(X): f(X) \text{ is } r R}$$

Where, f is a specified function or functional.

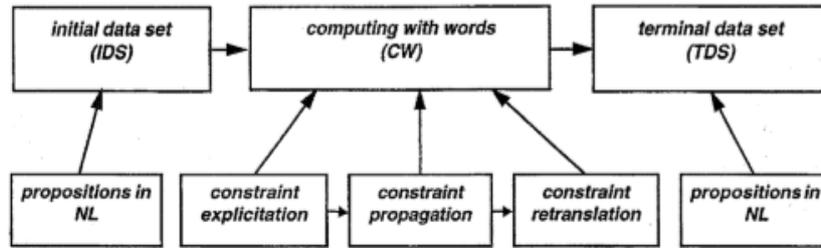


Fig 2: A more detailed structure of CW.

VI. PRECISIATION OF MEANING IN CW: A KEY IDEA

Point of departure Information=restriction

A proposition, p , is a carrier of information. In CW, a proposition, p , is precisiated by representing p as a restriction $p: X \text{ is } R$

where X , R and r are variables which typically are implicit in p . $X \text{ is } R$ is referred to as a canonical form of p .

Examples:

p : Robert is young $\text{Age}(\text{Robert}) = \text{young}$

$\begin{matrix} \uparrow & & \uparrow \\ X & & R \end{matrix}$

Both degrees of truth and [probabilities](#) range between 0 and 1 and hence may seem similar at first. For example, let a 100 ml glass contain 30 ml of water. Then we may consider two concepts: empty and full. The meaning of each of them can be represented by a certain [fuzzy set](#). Then one might define the glass as being 0.7 empty and 0.3 full. Note that the concept of emptiness would be [subjective](#) and thus would depend on the observer or [designer](#). Another designer might equally well [design](#) a set membership function where the glass would be considered full for all values down to 50 ml. It is essential to realize that fuzzy logic uses truth degrees as a [mathematical model](#) of the [vagueness](#) phenomenon while probability is a mathematical model of ignorance.

The premises are assumed to be expressed as propositions in a natural language. For purposes of computation, the propositions are expressed as canonical forms which serve to place in evidence the fuzzy constraints that are implicit in the premises. Then, the rules of inference in fuzzy logic are employed to propagate the constraints from premises to conclusions.

There are different types of uncertainties while adopting the procedure in vogue in the computation of these indices. However, it does not include expert's knowledge with a view to arrive at cause-effect relationship. We believe that the development of a method to quantify association between the pollutant and air/water-borne diseases is an important step before classifying air/water quality, either in numeric or linguistic terms. There exists an uncertainty in the pollution parametric data and epistemic uncertainty in describing the pollutants by the domain experts in linguistic terms such as *poor, good, and very good*.

Successes of probability theory have high visibility. But what is not widely recognized is that these successes mask a fundamental limitation—the inability to operate on what may be called perception-based information. A generalized constraint is represented as “ $X \text{ is } R$ ”, where is is a variable copula which defines the way in which R constrains X . More specifically, the role of R in relation to X is defined by the value of the discrete variable T . The values of T and their interpretations are defined below:

e : equal (abbreviated to $=$).

The Fuzzy Constraint propagation and the rules of inference in fuzzy logic are described below:

Conjunctive Rule 1:

$X \text{ is } A$

$X \text{ is } B$

$X \text{ is } A \wedge B$

Conjunctive Rule 2: ($X \in U, Y \in B, A \subset U, B \subset V$)

$X \text{ is } A$

$Y \text{ is } B$

$(X, Y) \text{ is } A \times B$

Disjunctive Rule 1:

$X \text{ is } A$

$X \text{ is } B$

$X \text{ is } A \cup B$

Disjunctive Rule 2: ($A \subset U, B \subset V$)

$X \text{ is } A$

$Y \text{ is } B$

$(X, Y) \text{ is } A \times V \cup U \times B$

where $A \times V$ and $U \times B$ are cylindrical extensions of A and B , respectively.

Conjunctive Rule for *isc*:

$X \text{ isc } A$

$X \text{ isc } B$

$X \text{ is } A \cup B'$

Disjunctive Rule for is:

$X \text{ is } A$

$X \text{ is } B$

$X \text{ is } A \cap B'$

Projective Rule:

$(X, Y) \text{ is } A$

$Y \text{ is } \text{proj}, A$

where $\text{proj}, A = \text{supu}A$.

Surjective Rule:

$X \text{ is } A$

$(X, Y) \text{ is } A \times V'$

A. Derived Rules

Compositional Rule:

$X \text{ is } A$

$(X, Y) \text{ is } B$

$Y \text{ is } A \circ B$

where $A \circ B$ denotes the composition of A and B .

Extension Principle (Mapping Rule):

$X \text{ is } A$ where $f: V \rightarrow V'$ and f^{-1} is defined by

$X \text{ is } A \cup B'$

Disjunctive Rule for is:

$X \text{ is } A$

$X \text{ is } B$

$X \text{ is } A \cap B'$

Conjunctive Rule for is:

$X \text{ is } A$

$X \text{ is } B$

$X \text{ is } A \cup B'$

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A. Derived Rules

Compositional Rule:

$X \text{ is } A$

$(X, Y) \text{ is } B$

$Y \text{ is } A \circ B$

where $A \circ B$ denotes the composition of A and B .

Generalized Extension Principle:

$f(X) \text{ is } A$

where

$\mu_{f(X)}(u) = \text{SUP}_{u \in f^{-1}(u)} \mu_A(g(u))$.

$\mu_{f(X)}(u) = \mu_A(f^{-1}(u))$

The generalized extension principle plays a pivotal role in fuzzy constraint propagation.

Syllogistic Rule:

$Q1A$'s are B 's

$Q2(A \text{ and } B)$'s are C 's

$(Q1 B Q2) A$'s are $(B \text{ and } C)$'s

where $Q1$ and $Q2$ are fuzzy quantifiers, A , B , and C are fuzzy relations and $Q1 B Q2$ is the product of $Q1$ and $Q2$ in fuzzy arithmetic.

Inverse Mapping Rule:

$f(X) \text{ is } A$

$X \text{ is } f^{-1}(A)$

where $\mu_{f^{-1}(A)}(u) = \mu_A(f^{-1}(u))$.

Generalized Modus Ponens:

X is A

if X is B then Y is C

Y is $A \circ ((TB) @ C)$

where the bounded sum $TB @ C$ represents Lukasiewicz's definition of implication.

However, in test-score semantics we do not presently have effective algorithms for the derivation of canonical forms without human intervention. This is a problem that remains to be addressed.

VII. CONCLUSION

It is well understood that the most complex machine that had ever been on earth is the Human brain. Many concepts of computation such as Artificial Intelligence, Computing with Words and Pattern Recognition has got its roots from the biological machine. The main objective of this note is to specify that role of fuzzy logic in computing with words is gaining key importance among technologists. It is much clear that in order to deal with the imprecision data of real world problems, CW has obtained an increased level of tolerance.

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