



## Applications of Intuitionistic Fuzzy Generalized $\gamma$ Closed Sets

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**Abstract-** In this paper, we have investigated the theoretical applications of intuitionistic fuzzy generalized  $\gamma$  closed sets and obtained some important theorems.

**Keywords-** Intuitionistic fuzzy topology, Intuitionistic fuzzy generalized  $\gamma$  closed sets, intuitionistic fuzzy  $\gamma T_{1/2}$  spaces.

### I. INTRODUCTION

The notion of intuitionistic fuzzy sets was introduced by Atanassov[6] as a generalization of fuzzy sets. In 1997, Coker[2] introduced the concept of intuitionistic fuzzy topological spaces. In this paper, we have investigated the theoretical applications of intuitionistic fuzzy generalized  $\gamma$  closed sets and obtained some important theorems.

### II. PRELIMINARIES

**Definition 2.1**[6]: An intuitionistic fuzzy set (IFS in short)  $A$  is an object having the form  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$  where the function  $\mu_A : X \rightarrow [0, 1]$  and  $\nu_A : X \rightarrow [0, 1]$  denote the degree of membership (namely  $\mu_A(x)$ ) and the degree of non-membership (namely  $\nu_A(x)$ ) of each element  $x \in X$  to the set  $A$ , respectively, and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for each  $x \in X$ . Denote by  $\text{IFS}(X)$ , the set of all intuitionistic fuzzy sets in  $X$ .

An intuitionistic fuzzy set  $A$  in  $X$  is simply denoted by  $A = \langle x, \mu_A, \nu_A \rangle$  instead of denoting  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ .

**Definition 2.2**[6]: Let  $A$  and  $B$  be two IFSs of the form  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$  and  $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X \}$ . Then,

- $A \subseteq B$  in and only if  $\mu_A(x) \leq \mu_B(x)$  and  $\nu_A(x) \geq \nu_B(x)$  for all  $x \in X$ ;
- $A = B$  in and only if  $A \subseteq B$  and  $A \supseteq B$ ;
- $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle : x \in X \}$ ;
- $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X \}$ ;
- $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X \}$ .

The intuitionistic fuzzy sets  $0 \sim = \langle x, 0, 1 \rangle$  and  $1 \sim = \langle x, 1, 0 \rangle$  are respectively the empty set and the whole set of  $X$ .

**Definition 2.3**[2]: An intuitionistic fuzzy topology (IFT in short) on  $X$  is a family  $\tau$  of IFS, in  $X$  satisfying the following axioms:

- $0 \sim, 1 \sim \in \tau$
- $G_1 \cap G_2 \in \tau$  for any  $G_1, G_2 \in \tau$
- $\cup G_i \in \tau$  for any family  $\{G_i : i \in J\} \subseteq \tau$

In this case the pair  $(X, \tau)$  is called intuitionistic fuzzy topological space (IFTS in short) and any IFS in  $\tau$  is known as an intuitionistic fuzzy open set (IFOS in short) in  $X$ . The complement  $A^c$  of an IFOS  $A$  in IFTS  $(X, \tau)$  is called an intuitionistic fuzzy closed set (IFCS in short) in  $X$ .

**Definition 2.4**[5]: An IFS  $A = \langle x, \mu_A, \nu_A \rangle$  in an IFTS  $(X, \tau)$  is said to be an

- intuitionistic fuzzy  $\gamma$  closed set (IF $\gamma$ CS in short) if  $\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)) \subseteq A$
- intuitionistic fuzzy  $\gamma$  open set (IF $\gamma$ OS in short) if  $A \subseteq \text{int}(\text{cl}(A)) \cup \text{cl}(\text{int}(A))$

**Definition 2.5**[5]: Let  $A$  be an IFS in an IFTS  $(X, \tau)$ . Then the  $\gamma$ -interior and  $\gamma$ -closure of  $A$  are defined as

$$\gamma \text{int}(A) = \cup \{G / G \text{ is an IF}\gamma\text{OS in } X \text{ and } G \subseteq A\}$$

$$\gamma \text{cl}(A) = \cap \{K / K \text{ is an IF}\gamma\text{CS in } X \text{ and } A \subseteq K\}$$

Note that for any IFS  $A$  in  $(X, \tau)$ , we have  $\gamma \text{cl}(A^c) = (\gamma \text{int}(A))^c$  and  $\gamma \text{int}(A)^c = (\gamma \text{cl}(A))^c$ .

**Result 2.6**[8]: Let  $A$  be an IFS in  $(X, \tau)$ , then

- $\gamma \text{cl}(A) \supseteq A \cup (\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)))$
- $\gamma \text{int}(A) \subseteq A \cap (\text{cl}(\text{int}(A)) \cap \text{int}(\text{cl}(A)))$

**Definition 2.7**[8]: An IFS  $A$  in an IFTS  $(X, \tau)$  is said to be an intuitionistic fuzzy generalized  $\gamma$  closed set (IFG $\gamma$ CS for short) if  $\gamma \text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is an IFOS in  $(X, \tau)$ . The family of all IFG $\gamma$ CSs of an IFTS  $(X, \tau)$  is denoted by  $\text{IFG}\gamma\text{C}(X)$ .

**Definition 2.8**[9]: The complement  $A^c$  of an IFG $\gamma$ CS A in an IFTS  $(X, \tau)$  is called an intuitionistic fuzzy generalized  $\gamma$  open set (IFG $\gamma$ OS in short) in X.

The family of all IFG $\gamma$ OSs of an IFTS  $(X, \tau)$  is denoted by IFG $\gamma$ O(X).

**Definition 2.9**[10]: An IFTS  $(X, \tau)$  is said to be IFT $_{1/2}$  space if every IFGCS in  $(X, \tau)$  is an IFCS in  $(X, \tau)$ .

**Definition 2.10**[1]: An intuitionistic fuzzy point (IFP in short), written as  $p_{(\alpha, \beta)}$ , is defined to be an intuitionistic fuzzy set of X given by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p \\ (0, 1) & \text{otherwise} \end{cases}$$

An intuitionistic fuzzy point  $p_{(\alpha, \beta)}$  is said to belong to a set A if  $\alpha \leq \mu_A$  and  $\beta \geq \nu_A$ .

### III. APPLICATIONS OF INTUITIONISTIC FUZZY GENERALIZED $\gamma$ CLOSED SETS

In this section we have discussed some theoretical applications of intuitionistic fuzzy generalized  $\gamma$  closed sets.

**Definition 3.1:** An IFTS  $(X, \tau)$  is called an intuitionistic fuzzy  $\gamma$  T $_{1/2}$  (IF $\gamma$ T $_{1/2}$  in short) space if every IFG $\gamma$ CS is an IF $\gamma$ CS in X.

**Example 3.2:** Let  $X = \{a, b\}$  and  $G_1 = \langle x, (0.5_a, 0.5_b), (0.4_a, 0.5_b) \rangle$   $G_2 = \langle x, (0.4_a, 0.3_b), (0.5_a, 0.5_b) \rangle$ . Then  $\tau = \{0\sim, G_1, G_2, 1\sim\}$  is an IFT on X.

Then, IF $\gamma$ C(X) =  $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / 0 \leq \mu_a + \nu_b \leq 1 \text{ and } 0 \leq \mu_b + \nu_b \leq 1\}$ .

Thus  $(X, \tau)$  is an IF $\gamma$ T $_{1/2}$  space, as every IFG $\gamma$ CS is an IF $\gamma$ CS in  $(X, \tau)$ .

**Definition 3.3:** An IFTS  $(X, \tau)$  is called an intuitionistic fuzzy  $\gamma_g$  T $_{1/2}$  (IF $\gamma_g$  T $_{1/2}$  in short) space if every IFG $\gamma$ CS is an IFCS in X.

**Definition 3.4:** An IFTS  $(X, \tau)$  is called an intuitionistic fuzzy pre  $\gamma$  T $_{1/2}$  (IFP $\gamma$ T $_{1/2}$  in short) space if every IFG $\gamma$ CS is an IFPCS in X.

**Definition 3.5:** An IFTS  $(X, \tau)$  is called an intuitionistic fuzzy semi  $\gamma$  T $_{1/2}$  (IFS $\gamma$ T $_{1/2}$  in short) space if every IFG $\gamma$ CS is an IFSCS in X.

**Definition 3.6:** An IFTS  $(X, \tau)$  is called an intuitionistic fuzzy  $\alpha$   $\gamma$  T $_{1/2}$  (IF $\alpha$  $\gamma$ T $_{1/2}$  in short) space if every IFG $\gamma$ CS is an IF $\alpha$ CS in X.

**Remark 3.7:** Not every IF $\gamma$ T $_{1/2}$  space is an IFT $_{1/2}$  space. This can be seen easily by the following example.

**Example 3.8:** Let  $X = \{a, b\}$  and let  $\tau = \{0\sim, G_1, G_2, 1\sim\}$  be an IFT on X, where  $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) \rangle$   $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b) \rangle$ . Then  $(X, \tau)$  is an IFTS.

Then, IF $\gamma$ C(X) =  $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / \text{whenever } \mu_a \geq 0.5, \mu_b \geq 0.3, \text{ whenever } \mu_a \leq 0.4, \mu_b < 0.2 \text{ and } 0 \leq \mu_b + \nu_b \leq 1\}$ .

Since every IFG $\gamma$ CS is an IF $\gamma$ CS in  $(X, \tau)$ , it is an IF $\gamma$ T $_{1/2}$  space. Let  $A = \langle x, (0.5_a, 0.8_b), (0.5_a, 0.2_b) \rangle$ , and  $A \subseteq 1\sim$ , then  $\text{cl}(A) = G_2^c \subseteq 1\sim$ . Hence A is an IFGCS in X but  $\text{cl}(A) = G_2^c \neq A$ , A is not an IFCS in X. Therefore  $(X, \tau)$  is not an IFT $_{1/2}$  space.

**Theorem 3.9:** Every IFP $\gamma$ T $_{1/2}$  space is an IF $\gamma$ T $_{1/2}$  space but not conversely.

**Proof:** Let  $(X, \tau)$  be an IFP $\gamma$ T $_{1/2}$  space and let A be an IFG $\gamma$ CS in X. By hypothesis, A is an IFPCS in X. Since every IFPCS is an IF $\gamma$ CS[2], A is an IF $\gamma$ CS in X. Hence  $(X, \tau)$  is an IF $\gamma$ T $_{1/2}$  space.

**Example 3.10:** Let  $X = \{a, b\}$  and let  $\tau = \{0\sim, G_1, G_2, 1\sim\}$  be an IFT on X, where  $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) \rangle$ ,  $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b) \rangle$ . Then  $(X, \tau)$  is an IFTS.

Then, IF $\gamma$ C(X) =  $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / \text{either } 0.3 \leq \mu_b < 0.2 \text{ whenever } \mu_a \geq 0.5 \text{ (or) } \mu_a < 0.4 \text{ and } 0 \leq \mu_b + \nu_b \leq 1\}$ .

Since every IFG $\gamma$ CS is an IF $\gamma$ CS in  $(X, \tau)$ , it is an IF $\gamma$ T $_{1/2}$  space. Let  $A = \langle x, (0.4_a, 0.3_b), (0.5_a, 0.7_b) \rangle$  be an IFS in X. Now since  $\text{cl}(\text{int}(A)) = \text{cl}(G_2) = G_1^c \not\subseteq A$ , A is not an IFPCS in X. Therefore  $(X, \tau)$  is not an IFP $\gamma$ T $_{1/2}$ .

**Theorem 3.11:** Every IF $\alpha$  $\gamma$ T $_{1/2}$  space is an IF $\gamma$ T $_{1/2}$  space but not conversely.

**Proof:** Let  $(X, \tau)$  be an IF $\alpha$  $\gamma$ T $_{1/2}$  space and let A be an IFG $\gamma$ CS in X. By hypothesis, A is an IF $\alpha$ CS in X. Since every IF $\alpha$ CS is an IF $\gamma$ CS[2], A is an IF $\gamma$ CS in X. Hence  $(X, \tau)$  is an IF $\gamma$ T $_{1/2}$  space.

**Example 3.12:** Let  $X = \{a, b\}$  and let  $\tau = \{0\sim, G_1, G_2, 1\sim\}$  be an IFT on X, where  $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) \rangle$   $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b) \rangle$ . Then  $(X, \tau)$  is an IFTS.

Then, IF $\gamma$ C(X) =  $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / \text{either } 0.3 \leq \mu_b < 0.2 \text{ whenever } \mu_a \geq 0.5 \text{ (or) } \mu_a < 0.4 \text{ and } 0 \leq \mu_b + \nu_b \leq 1\}$ .

Since every IFG $\gamma$ CS is an IF $\gamma$ CS in  $(X, \tau)$ , it is an IF $\gamma$ T $_{1/2}$  space. Let  $A = \langle x, (0.4_a, 0.3_b), (0.5_a, 0.7_b) \rangle$  be an IFS in X. Now since  $\text{cl}(\text{int}(\text{cl}(A))) = \text{cl}(\text{int}(G_1^c)) = \text{cl}(G_1) = G_1^c \not\subseteq A$ , A is not an IF $\alpha$ CS in X. Therefore  $(X, \tau)$  is not an IF $\alpha$  $\gamma$ T $_{1/2}$ .

**Theorem 3.13:** Every IFS $\gamma$ T $_{1/2}$  space is an IF $\gamma$ T $_{1/2}$  space but not conversely.

**Proof:** Let  $(X, \tau)$  be an IFS $\gamma$ T $_{1/2}$  space and let A be an IFG $\gamma$ CS in X. By hypothesis, A is an IFSCS in X. Since every IFSCS is an IF $\gamma$ CS[2], A is an IF $\gamma$ CS in X. Hence  $(X, \tau)$  is an IF $\gamma$ T $_{1/2}$  space.

**Example 3.14:** Let  $X = \{a, b\}$  and let  $\tau = \{0\sim, G_1, G_2, 1\sim\}$  be an IFT on X, where  $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b) \rangle$   $G_2 = \langle x, (0.4_a, 0.2_b), (0.6_a, 0.8_b) \rangle$ . Then  $(X, \tau)$  is an IFTS.

Then, IF $\gamma$ C(X) =  $\{0\sim, 1\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] / \text{either } 0.3 \leq \mu_b < 0.2 \text{ whenever } \mu_a \geq 0.5 \text{ (or) } \mu_a < 0.4 \text{ and } 0 \leq \mu_b + \nu_b \leq 1\}$ .

Since every IFG $\gamma$ CS is an IF $\gamma$ CS in  $(X, \tau)$ , it is an IF $\gamma$ T $_{1/2}$  space. Let  $A = \langle x, (0.4_a, 0.3_b), (0.5_a, 0.7_b) \rangle$  be an IFS in X. Now since  $\text{int}(\text{cl}(A)) = \text{int}(G_1^c) = G_1 \not\subseteq A$ , A is not an IFSCS in X. Therefore  $(X, \tau)$  is not an IFS $\gamma$ T $_{1/2}$ .

**Theorem 3.15:** An IFTS  $(X, \tau)$  is an IF $\gamma$  T $_{1/2}$  space if and only if IFG $\gamma$ O(X) = IF $\gamma$ O(X)

**Proof: Necessity:** Let  $A$  be an IFG $\gamma$ OS in  $(X, \tau)$ , then  $A^c$  is an IFG $\gamma$ CS in  $X$ . By hypothesis  $A^c$  is an IF $\gamma$ CS in  $(X, \tau)$ . Hence  $A$  is an IF $\gamma$ OS in  $(X, \tau)$ . Then IFG $\gamma$ O( $X$ ) = IF $\gamma$ O( $X$ ).

**Sufficiency:** Let  $A$  be an IFG $\gamma$ CS in  $(X, \tau)$ . Then  $A^c$  is an IFG $\gamma$ OS in  $(X, \tau)$ . By hypothesis  $A^c$  is an IF $\gamma$ OS in  $(X, \tau)$ . Therefore  $A$  is an IF $\gamma$ CS in  $(X, \tau)$ . Hence  $(X, \tau)$  is an IF $\gamma$  T $_{1/2}$  space.

**Theorem 3.16:** An IFTS  $(X, \tau)$  is an IF $\gamma_g$  T $_{1/2}$  space if and only if IFG $\gamma$ O( $X$ ) = IFO( $X$ )

**Proof: Necessity:** Let  $A$  be an IFG $\gamma$ OS in  $(X, \tau)$ , then  $A^c$  is an IFG $\gamma$ CS in  $X$ . By hypothesis  $A^c$  is an IFCS in  $(X, \tau)$ . Hence  $A$  is an IFOS in  $(X, \tau)$ . Then IFG $\gamma$ O( $X$ ) = IFO( $X$ ).

**Sufficiency:** Let  $A$  be an IFG $\gamma$ CS in  $(X, \tau)$ . Then  $A^c$  is an IFG $\gamma$ OS in  $(X, \tau)$ . By hypothesis  $A^c$  is an IFOS in  $(X, \tau)$ . Therefore  $A$  is an IFCS in  $(X, \tau)$ . Hence  $(X, \tau)$  is an IF $\gamma_g$  T $_{1/2}$  space.

**Theorem 3.17:** Let  $(X, \tau)$  is an IF $\gamma_g$  T $_{1/2}$  space. Then

- (i) Any union of IFG $\gamma$ CS is an IFG $\gamma$ CS
- (ii) Any intersection of IFG $\gamma$ OS is an IFG $\gamma$ OS

**Proof:** (i) Let  $\{A_i\}_{i \in J}$  be any collection of IFG $\gamma$ CSs. Since  $(X, \tau)$  is an IF $\gamma_g$  T $_{1/2}$  space, every IFG $\gamma$ CS is an IFCS. But any union of IFCSs is an IFCS,  $\bigcup_{i \in J} A_i$  is also an IFCS in  $(X, \tau)$ . Therefore  $\bigcup_{i \in J} A_i$  is an IFG $\gamma$ CS in  $X$ , as every IFCS is an IFG $\gamma$ CS.

(ii) It can be proved that taking complement in (i).

**Theorem 3.19:** Let  $(X, \tau)$  be an IF $\gamma_g$  T $_{1/2}$  space. Then  $A$  is an IFG $\gamma$ OS in  $X$  if and only if for every IFP  $p_{(\alpha, \beta)} \in A$ , there exists an IFG $\gamma$ OS  $B$  in  $X$  such that  $p_{(\alpha, \beta)} \in B \subseteq A$ .

**Proof: Necessity:** If  $A$  is an IFG $\gamma$ OS in  $X$ , then we can take  $B = A$ , so that  $p_{(\alpha, \beta)} \in B \subseteq A$  for every IFP  $p_{(\alpha, \beta)} \in A$ .

**Sufficiency:** Let  $A$  be an IFS in  $(X, \tau)$  and assume that there exists an IFG $\gamma$ OS in  $X$  such that  $p_{(\alpha, \beta)} \in B \subseteq A$ . Since  $X$  is an IF $\gamma_g$  T $_{1/2}$  space,  $B$  is an IFOS in  $X$ . Then  $A = \bigcup_{p_{(\alpha, \beta)} \in A} p_{(\alpha, \beta)} \subseteq \bigcup_{p_{(\alpha, \beta)} \in A} B \subseteq A$ . Therefore  $A = \bigcup_{p_{(\alpha, \beta)} \in A} B$  which is an IFOS. Then  $A$  is an IFG $\gamma$ OS in  $X$ . [9]

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