



Two Stage Multi-criteria Decision Making Approach Based on Rough Set Theory

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Abstract—This paper proposes a novel two stage approach to decision making based on a combination of Rough Set Theory and Analytical Hierarchy Process (AHP) to arrive at the appropriate decision from a set of alternatives. Stage 1, leads to a classification of decision attributes into two clusters based on Rough Set Theory indicators. Stage 2, gives the convenient decision by assigning each attribute to the appropriate choice by using Analytical Hierarchy Process (AHP) technique.

Keywords— Multi-Criteria Decision Analysis, Rough Set Theory, Probabilistic Probabilities, Clustering, Analytical Hierarchy Process.

I. INTRODUCTION

Multi-criteria decision analysis (MCDA) has been recognized as an important tool in environmental decision making for formalizing and addressing the problem of competing for decision objectives. MCDA provides a framework by which different types of decisions can be made. It is both an approach and a set of techniques, with the goal of providing an overall ordering of options, from the most preferred to the least preferred option. MCDA has been utilized to assist in making complex decisions for a number of decades, as it facilitates stakeholder participation and collaborative decision-making, and does not necessarily require the assignment of monetary values to environmental or social criteria, and allows the consideration of multiple criteria in incommensurable units (i.e. combination of qualitative and quantitative criteria) [21]

One of the most important methods used in dealing with MCDM problems is Analytical Hierarchy Process (AHP) method. One of the advantage of AHP is that it illustrates how possible changes in priority at the upper levels have an effect on the priority of criteria at lower levels. Moreover, it provides the decision maker with an overview of criteria, their function at the lower levels and goals as at the higher levels. A further advantage of AHP is its stability and flexibility regarding changes within and additions to the hierarchy. In addition, the method is able to rank criteria according to the needs of the decision maker. [18]

However, AHP also has some downsides. One of these is, if more than one person is working on this method, different opinions about the weight of each criterion can complicate matters. AHP also requires data based on experience, knowledge and judgment which are subjective for each decision maker. A further disadvantage of this method is that it does not consider troubles of the attributes in the information system such as indiscernibility, reducts, inconsistency and the dependency of the attributes.

Thus, to overcome the disadvantages of the AHP method we will introduce a novel two stage multi-criteria decision-making approach by adding Rough Set Theory approach as a stage precedes AHP stage.

The main goal of the rough set analysis is to analyze information about objects described by attributes. Its methodology is concerned with the classification and analysis of imprecise, uncertain or incomplete information and knowledge. It offers mathematical tools to discover patterns hidden in the data. Also, it can be used for feature selection, data reduction, decision rule generation, and pattern extraction (templates, association rules), etc. Rough Set Theory; identifies partial or total dependencies in data, eliminates redundant data, gives an approach to null values, missing data, dynamic data and others. [10]

Data is represented in the Rough Set (RS) framework in the form of an information system or table. Each row of the table represents an object and every column represents an attribute that can be measured for each object.

II. THE PROPOSED METHOD ALGORITHM

We will outline the steps of the proposed method by the following figure;

Step 1: Using Rough Set Theory technique to deal with Information System. In this step, procedures of Rough Set Theory will be applied which are; detection indiscernibility, set approximations, determination of reducts, detection dispensable and indispensable attributes, dealing with inconsistency, check the dependency of attributes, discovering the core and get the strength, certainty and coverage indicators, as follows;

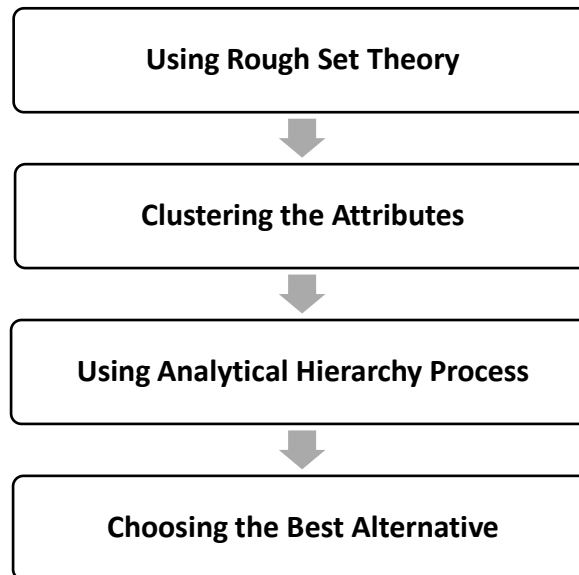


Figure 1. Steps of the Proposed Method

(a) Indiscernibility: Let $IS = (U, A)$ be an information system, where U is a non-empty finite set of objects and A is a non-empty finite set of attributes. Any subset B of A , $B \subseteq A$, determines a binary relation $IND(B)$ on U which will be called an indiscernibility relation that indicates that there is an associated equivalence relation: $IND_{IS}(B) = \{(x, x') \in U^2 \mid \forall a \in B, a(x) = a(x')\}$,

Where, $IND_{IS}(B)$ is called the B -indiscernibility relation. If $(x, x') \in IND_{IS}(B)$, then objects x and x' indiscernible from each other by attributes from B . The equivalence classes of the B -indiscernibility relation are denoted by $[x]_B$.

(b) Reducts: A reduct is a minimal set of attributes that preserve the indiscernibility relation. In any decision table, some of the attributes may be superfluous (redundant). The same or indiscernible objects may be represented several times. That is, their removal cannot worsen the classification. Keep only those attributes that preserve the indiscernibility relation and, consequently, set approximation. There are usually several such subsets of attributes and those which are minimal are called reducts.

Given an information system $IS = (U, A)$ the definitions of these notions are as follows; A reduct of A is a minimal set of attributes $R \subseteq A$ such that $IND_{IS}(A) = IND_{IS}(R)$.

(c) Inconsistency: Decisions may be inconsistent because of limited clear discrimination between criteria and because of hesitation on the part of the decision maker. These inconsistencies cannot be considered as a simple error or as noise. They can convey important information that should be taken into account in the construction of the decision making preference model. The rough set approach is intended to deal with inconsistency and this is a major argument to support its application to "Multiple criteria decision analysis".

We can deal with the inconsistency by one of the following methods,

- 1- Ask the expert what to do.
- 2- Separate the conflicting examples of different tables.
- 3- Remove the examples with less support.
- 4- Quality methods; based on lower or upper approximations.
- 5- Generating new decision attribute.

(d) Set Approximations: as a definition, lower approximation, refers to the set of observations that can all be classified into the concept. Upper approximation, refers to the set of observations that can be possibly classified into this concept. Mathematically, let $B \subseteq A$ and $X \subseteq U$. We can approximate X using only the information contained in B by constructing the B -lower and B -upper approximations of X , denoted $\underline{B}X$ and $\overline{B}X$ respectively, where;

$$\underline{B}X = \{x \mid [x]_B \subseteq X\}$$

$$\overline{B}X = \{x \mid [x]_B \cap X \neq \emptyset\}$$

B -boundary region of X ; $BN_B(X) = \overline{B}X - \underline{B}X$, consists of those objects that we cannot decisively classify into X in B .

B -Outside region of X ; $U - \overline{B}X$, consists of those objects that can be with certainty classified as not belonging to X .

A set is said to be rough if its boundary region is non-empty, otherwise the set is crisp. So there are the objects that belong to the $IND(B)$, which all are included in the set of X about the objects belonging to the lower approximation, we say, surely that they belong to a given concept (decision class),

We can test the accuracy of the approximation by the following rule;

$$\alpha_B(X) = \frac{|\underline{B}(X)|}{|\overline{B}(X)|}$$

Where $|X|$ denotes the cardinality of $X \neq \emptyset$

Obviously, $0 \leq \alpha_B \leq 1$. If $\alpha_B(x) = 1$, x is crisp with respect to B , and if $\alpha_B(x) < 1$, x is rough with respect to B .

(e) **Dependency of Attributes:** Discovering dependencies between attributes is very important. A set of attribute D depends totally on a set of attribute C, denoted by $C \rightarrow D$ is all values of attributes from D are uniquely determined by values of attributes from C. Let D and C be subsets of A. We will say that D depends on C in a degree k ($0 \leq K \leq 1$) denoted by $C \rightarrow_k D$ if,

$$K = \alpha(C, D) = \frac{|POS_B(D)|}{|U|}$$

Where, $POS_B(D) = \{X \in U / D \subseteq BX\}$ called B-positive region.

The general definition of dependency of attributes is;

$$K = \alpha(C, D) = \sum_{X \in U/D} \frac{|B(X)|}{|U|}$$

If $K = 1$ we say that D depends totally on C, and if $K < 1$ we say that D depends partially (in a degree k) on C. [7]

(f) **Core:** The set of all condition attributes indispensable in IS is denoted by CORE (C). "CORE (C) = \cap RED (C)" where, RED (C) is the set of all reducts of C.

(g) **Strength, Certainty and Coverage:** The most important feature of rough set method that it can be used to measure the strength, certainty and coverage of the attributes as follows;

$$\text{Strength of rule } i = N_i / \sum_i N_i$$

$$\text{Certainty of rule } i = \frac{|c(x) \cap D(x)|}{|C(X)|}$$

$$\text{Coverage of rule } i = \frac{|c(x) \cap D(x)|}{|D(X)|}$$

Step 2: Clustering the Attributes according to Coverage and Certainty Indicators. According to the probabilistic properties of the three indicators. Strength indicator already included in the two indicators; certainty and coverage as follows;

In an Information System, $IS = (U; C, D)$, the sequence of decision rule induced by x will be denoted by $C \rightarrow_x D$. For the strength of the decision;

$$\alpha_x(C, D) = \frac{\text{supp}_x(C, D)}{|U|}$$

Where, the number $\text{supp}_x(C, D) = |A(x)| = |C(x) \cap D(x)|$, is called a support of the decision rule $C \rightarrow_x D$.

For the certainty indicator of the decision rule, which denoted by, $\text{cer}_x(C, D)$, defined as follows;

$$\text{Cer}_x(C, D) = \frac{|c(x) \cap D(x)|}{|C(X)|} = \frac{\text{supp}_x(C, D)}{|C(x)|} = \frac{\alpha_x(C, D)}{\pi(C(X))}$$

Where, $\pi(C(x)) = \frac{|C(x)|}{|U|}$

The certainty indicator may be interpreted as a conditional probability, that y belong to c (x) symbolically $\pi(D|C)$.

Note that, if $\text{cer}_x(C, D) = 1$, then $C \rightarrow_x D$ will be called a certain decision rule; if $0 < \text{cer}_x(C, D) < 1$ the decision rule will be referred to as an uncertain decision rule.

Besides, the coverage indicator of the decision rule, denoted $\text{cov}_x(C, D)$ is defined as;

$$\text{Cov}_x(C, D) = \frac{|c(x) \cap D(x)|}{|D(X)|} = \frac{\text{supp}_x(C, D)}{|D(x)|} = \frac{\alpha_x(C, D)}{\pi(D(X))}$$

Where, $\pi(D(x)) = \frac{|D(x)|}{|U|}$

Similarly $\text{Cov}_x(C, D) = \pi_x(C|D)$

Let $C \rightarrow_x D$ be a decision rule and let $\Gamma = C(x)$ and $\Delta = D(x)$. Then the following properties are valid;

$$\sum_{y \in \Gamma} \text{cer}_y(C, D) = 1 \quad (1)$$

$$\sum_{y \in \Delta} \text{cov}_y(C, D) = 1 \quad (2)$$

$$\pi(D(x)) = \sum_{y \in \Gamma} \text{cer}_y(C, D) \cdot \pi(C(y)) = \sum_{y \in \Gamma} \alpha_y(C, D) \quad (3)$$

$$\pi(C(x)) = \sum_{y \in \Delta} \text{cov}_y(C, D) \cdot \pi(D(y)) = \sum_{y \in \Delta} \alpha_y(C, D) \quad (4)$$

$$\text{Cer}_x(C, D) = \frac{\text{cov}_x(C, D) \cdot \pi(D(x))}{\sum_{y \in \Delta} \text{cov}_y(C, D) \cdot \pi(D(y))} = \frac{\alpha_x(C, D)}{\pi(C(x))} \quad (5)$$

$$\text{Cov}_x(C, D) = \frac{\text{cer}_x(C, D) \cdot \pi(C(x))}{\sum_{y \in \Gamma} \text{cer}_y(C, D) \cdot \pi(C(y))} = \frac{\alpha_x(C, D)}{\pi(D(x))} \quad (6)$$

Any decision table satisfies equations (1) to (6). Formula (3) and (4) refer to total probability theorem. Formula (5) and (6) refer to Bayes theorem, (Herawan et al., 2010).

Thus, in order to compute the certainty and coverage factors of decision rules according to Bayes theorem, it is enough to know the strength of all decision rules only.

Accordingly, we will calculate the average for each decision attribute by using the following equation;

$$\Psi_{r,v}(x) = \sqrt{\text{cer}_x(C, D) * \text{cov}_x(C, D)} \quad (I)$$

Where, $\Psi_{r,v}(x)$ is the average of certainty and coverage indicators respectively, $cer_x(C, D)$ is the certainty indicator, and $cov_x(C, D)$ is the coverage indicator.

Then applying the following equation;

$$\mu = \frac{\sum(\max \Psi) + (\min \Psi)}{2} \quad (II)$$

Where, μ , is the mean value for the averages of the indicators. According to the value of μ we will partition the decision attributes into two clusters. We are interested only in the cluster with maximum values.

Step 3: Using Analytical Hierarchy Process (AHP) method. First, we will set scales for the alternatives according to the following table;

Table I Scales for Alternatives

Intensity of Importance	Definition	Explanation
1	Equal importance	Two activities contribute equally
3	Moderate importance	Judgment favours one activity over the other
5	Strong importance	Judgment strong favour one activity over the other
7	Very strong importance	Judgment very strong favour one activity over the other
9	Extreme importance	The evidence favouring one activity over the other

(2, 4, 6, 8 are intermediate values). [15]

Then, we will rank the alternatives according to the previous scales, normalize the values and finally get the averages for the alternatives.

Step4: Choosing the Best Alternative. In this step, we will use the neural network technique to get the relative importance of each criteria. Then, applying the following relation;

$$z_n = \begin{pmatrix} c_{11} & \dots & c_{1m} \\ \vdots & \ddots & \vdots \\ c_{n1} & \dots & c_{nm} \end{pmatrix} (RI_m) \quad (III)$$

Where; z_n is the specific value of each alternative, c_{nm} is the criterion score according to AHP, m represents criteria, n represents alternatives, RI_m is the relative importance of m criteria.

Finally, we will choose the best alternative by ordering the outcomes starting with the highest value. The greatest value indicates the best choice with best attribute.

III. ILLUSTRATIVE EXAMPLE

In this section, the proposed approach will be illustrated by a very simple tutorial example concerning purchasing a new car. Table II, represents six facts concerning six person segments. In the table, condition attributes are describing three criteria; price, size and safety of each car. The decision attributes describing the buying decision; "yes" or "no" for each person. Each row in the table determines a decision rule. E.g., row 2 determines the following decision rule: "if the price of the car is high, the size is big and the safety is medium then this person will buy the car".

Table II Information System

Persons	Price	Size	Safety	Buying decision
P ₁	High	Big	Low	No
P ₂	High	Big	Med	Yes
P ₃	High	Big	High	Yes
P ₄	Low	Big	Low	No
P ₅	Low	Small	Med	No
P ₆	Low	Big	High	Yes

The previous table represents what is known by Information System (IS), which has two classes or attributes called condition attributes and decision attributes; $IS = (U; C, D)$. Where, IS: the information system decision table, C; condition attributes, and D; decision attributes. Every $x \in U$ determines a sequence $c_1(x), c_2(x), \dots, c_n(x), d_1(x), \dots, d_m(x)$ where; $C = \{c_1, \dots, c_n\}$ and $D = \{d_1, \dots, d_m\}$, and $C \cap D = \emptyset$.

According to the proposed approach, steps will be applied as follows;

Step 1: We will use Rough Set Theory, procedures and features to deal with the data in the previous Information System (table II) as follows;

(a) **Indiscernibility:** Sets that are indiscernible are called elementary sets. Thus, the set of attributes price and size defines the following elementary sets $\{P_1, P_2, P_3\}$, $\{P_4, P_6\}$, and $\{P_5\}$. Any finite union of elementary sets is called a definable set. In the example, set $\{P_1, P_2, P_3\}$ is definable by the attributes price and size, since we may define this set by saying that any member of it characterized by the attribute price equal to "High" and the attribute size equal to "High".

(b) **Reducts:** We can define elementary sets associated with the decision as subsets of the set of all persons with the same value of the decision. In the example, such subsets will be called concepts which are $\{P_1, P_4, P_5\}$, $\{P_2, P_3, P_6\}$. The first concept $\{P_1, P_4, P_5\}$ corresponds to the set of all persons will not buy the car, the second one $\{P_2, P_3, P_6\}$ to the set of all persons will buy the car.

Now, whether we may tell who take a decision to buy a car or not on the basis of the values of attributes in table II. We may observe that in terms of rough set theory, decision "Buying a car" depends on the two criteria; price and safety, since all elementary sets of indiscernibility relation associated with {Price, Safety} are subsets of some concepts. The set {price, safety} is a reduct of the original set of attributes {price, size, safety}. The following table represents a new information table based on this reduct.

Table III Example of Reducts

Persons	Price	Size	Safety	Buying
P1	High	Big	Low	No
P2	High	Big	Med	Yes
P3	High	Big	High	Yes
P4	Low	Big	Low	No
P5	Low	Small	Med	No
P6	Low	Big	High	Yes

Persons	Size	Safety	Buying
P1	Big	Low	No
P2	Big	Med	Yes
P3	Big	High	Yes
P4	Big	Low	No
P5	Small	Med	No
P6	Big	High	Yes

Reduct 1 = {size, Safety}

persons	Price	Safety	Buying
P1	High	Low	No
P2	High	Med	Yes
P3	High	High	Yes
P4	Low	Low	No
P5	Low	Med	No
P6	Low	High	Yes

Reduct 2 = {price, Safety}

Let the set of attributes be the set {Price, Safety} and its superset be the set of all three attributes, i.e., the set {price, size, safety}. Elementary sets of the indiscernibility relation defined by the set {price, safety} are singletons, i.e., sets {P₁}, {P₂}, {P₃}, {P₄}, {P₅}, and {P₆}, and so are elementary sets of the indiscernibility relation defined by the set of all three attributes. Thus, the attribute size is redundant.

On the other hand, the set {price, safety} doesn't contain any redundant attribute, since elementary sets for attribute sets {price} and {safety} are not singletons. Such a set of attributes, with no redundant attributes, is called minimal or independent.

(c) **Inconsistency:** In the example, we will use method 5 "see section 2 (c)" to deal with inconsistency by generating new decision attribute. Suppose we have two additional person P₇ and P₈, the Information System will be as follows;

Table IV Dealing with Inconsistency by Generating New

Persons	Price	Safety	Buying
P ₁	High	Low	No
P ₂	High	Med	Yes
P ₃	High	High	Yes
P ₄	Low	Low	No
P ₅	Low	Med	No
P ₆	Low	High	Yes
P ₇	Low	Med	Yes
P ₈	Low	High	No

Decision Attributes

Elementary sets of indiscernibility relation defined by the two criteria; price and safety are {P₁}, {P₂}, {P₃}, {P₄}, {P₅, P₇} and {P₆, P₈}, while concepts defined by decision "buying" are {P₁, P₄, P₅, P₈}, {P₂, P₃, P₆, P₇}. Obviously the decision "Buying" doesn't depend on price and safety since neither concept is definable by the set {Price, Safety}.

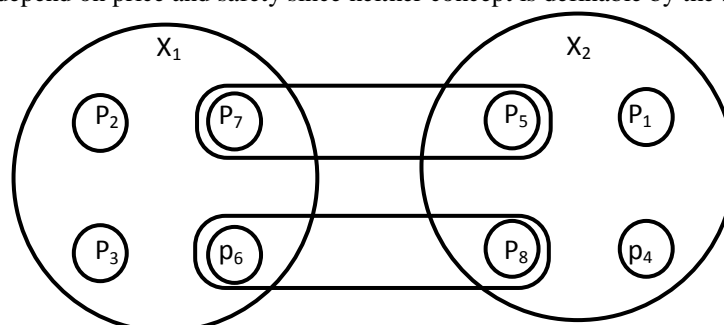


Figure 2. Inconsistency Relation

(d) **Set approximations:** According to the approximation rule, lower and upper approximations can be calculated as follows:

From the figure; $R = \{\text{Price, Safety}\}$

$P|R = \{\{P_1\}, \{P_2\}, \{P_3\}, \{P_4\}, \{P_5, P_7\}, \{P_6, P_8\}\}$

$X_1 = \{P| \text{Buying (P) = Yes}\} = \{P_2, P_3, P_6, P_7\}$

$X_2 = \{P| \text{Buying (P) = No}\} = \{P_1, P_4, P_5, P_8\}$

The approximations after generating new decision attribute P_7 and P_8 will be; lower approximations are;

$$\underline{R} X_1 = \{P_2, P_3\} \quad \underline{R} X_2 = \{P_1, P_4\}$$

Upper approximations are;

$$\bar{R} X_1 = \{P_2, P_3, P_5, P_6, P_7, P_8\} \quad \bar{R} X_2 = \{P_1, P_4, P_5, P_6, P_7, P_8\}$$

The accuracy of approximations is;

$$\alpha_B(X) = \frac{|\{P_2, P_3\}|}{|\{P_2, P_3, P_5, P_6, P_7, P_8\}|} = \frac{2}{6} = \frac{1}{3}$$

Note that, the result of the accuracy of approximations relatively low due to the small number of data. And the boundary regions are;

$$BN_R(X_1) = \{P_6, P_7, P_8, P_5\} \quad BN_R(X_2) = \{P_6, P_7, P_8, P_5\}$$

(e) **Dependency of attributes**

$$K = \frac{|\{P_2, P_3\}| + |\{P_1, P_4\}|}{|\{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}|} = \frac{4}{8} = \frac{1}{2}$$

$C \xrightarrow{K} D \quad \{\text{Price, Safety}\} \xrightarrow{0.5} \text{Buying decision.}$

(f) **Core**

$$\text{CORE} = \{\text{size, safety}\} \cap \{\text{Price, Safety}\} = \{\text{Safety}\}$$

(g) **Strength, Certainty and Coverage:** In the example, we will assume that, $\sum_i N_i = 1000$, where N is the number of similar cases, the results are presented in the following table;

Table V Strength, Certainty and Coverage

Persons	Price	Safety	Buying Decision	N	Strength	Certainty	Coverage
P ₁	High	Low	No	100	0.1	1.00	0.25
P ₂	High	Med	Yes	200	0.2	1.00	0.25
P ₃	High	High	Yes	150	0.15	1.00	0.33
P ₄	Low	Low	No	100	0.1	1.00	0.25
P ₅	Low	Med	No	100	0.1	0.5	0.25
P ₆	Low	High	Yes	150	0.15	0.6	0.25
P ₇	Low	Med	Yes	100	0.1	0.5	0.17
P ₈	Low	High	No	100	0.1	0.4	0.25

The main advantage of rough set theory that it gives us the three important results; strength, certainty and coverage. So we want to use these three indicators to help the decision maker in the situation of choosing the best alternative from a set of different alternatives.

Step 2: In this step we will apply equation (I) to get $\Psi_{r,v}(x)$. The results are represented in the following table;

Table VI Average for Certainty and Coverage

Persons	The Average
P ₁	0.5
P ₂	0.5
P ₃	0.574
P ₄	0.5
P ₅	0.354
P ₆	0.387
P ₇	0.292
P ₈	0.316

From the table; the maximum average is P_3 with value (0.574) and the minimum average is P_7 with value (0.292), so we will apply equation (II), then the mean μ will be; $[(0.574 + 0.292) / 2] = 0.433$.

This means that; the first cluster with maximum value for certainty and coverage will include the attributes for person {P₁, P₂, P₃, P₄} and the second cluster with the minimum value of certainty and coverage will include the attributes for person {P₅, P₆, P₇, P₈}.

We take the intersection between persons who say "yes" (who will surely buy a car) and the maximum values for certainty and coverage for attributes that results from the clustering step as follows;

$$\{P_1, P_2, P_3, P_4\} \cap \{P_2, P_3, P_6, P_7\} = \{P_2, P_3\}$$

This result represents the persons who are willing to buy cars with the best certainty and coverage attributes

Step 3: Using Analytical Hierarchy Process Technique

Suppose that, we have three alternatives (cars); type A, type B, and type C, that we want to choose the best one according to the two criteria (price and safety). We get the data of the three alternatives (types of cars) from website specialized in cars evaluation with link <http://www.arabsturbo.com> in January 2016.

The following table shows the price and the weight for each type of car, we used the car weight as an indicator of safety that is the heaviest weight indicates the more safety one, the following table represents the values for price and weight for each alternative.

Table VII Price and Weight for Alternatives

Alternatives	Price	Weight
Type A	84500	1464
Type B	77900	1639
Type C	89900	1561

First, we will rank the alternatives according to the scales in table I

Note that, when we deal with price, we are interested in the least price (i.e., the least price is more favour). But, when we deal with safety, we are interested in heavier weight, which indicates more safety (i.e., the heavier car is more favour). Thus, ranking for the price and safety will be as follows;

Table VIII Ranking of the Three Alternatives for Price

Alternatives	Type A	Type B	Type C
Type A	1	1/3	1/5
Type B	3	1	7
Type C	5	1/7	1
Sum	9	1.476	8.2

Table IX Normalization Step and the Average for Each Row for Price

Alternatives	Type A	Type B	Type C	Average
Type A	0.111	0.226	0.0244	0.1205
Type B	0.333	0.678	0.854	0.622
Type C	0.556	0.097	0.122	0.258

Table X Ranking of the Three Alternatives for Safety

Alternatives	Type A	Type B	Type C
Type A	1	1/5	1/3
Type B	5	1	3
Type C	3	1/3	1
Sum	9	1.53	4.33

Table XI Normalization Step and the Average for Each Row for Safety

Alternatives	Type A	Type B	Type C	Average
Type A	0.111	0.131	0.076	0.106
Type B	0.556	0.654	0.693	0.634
Type C	0.333	0.218	0.231	0.261

Then we will use "Neuroshell classifier program", to get the relative importance of the two criteria based on the data in table II, the obtained values were as follows;

Relative importance for price = 0.484

Relative importance for safety = 0.516

Note that, the result of the relative importance of each criterion shows that, safety is more important according to the data, which was the same result of the core in the rough set technique.

Step 4: Applying the relation (III) to get the specific value of each alternative, z_n , the value of type A is (0.1131), the value of type B is (0.6282) and the value of type C is (0.2595). Which means that, the best choice is type B for the two persons. Note that, for type B the value of $P_2 = 0.3141$ and the value of $P_3 = 0.3606$, which means that, the condition attribute of P_3 is better than P_2 for type B.

So that, this result could be used as an indicator of the preference of the person's attitude who are willing to buy cars, so that type B could take into consideration the condition attribute of P_3 which is "high price and high safety".

This means that the proposed method not only help the decision makers to choose among alternatives, but also could help the suppliers to know the best condition attributes to meet the preference of customers and therefore the profit will be increased.

IV. DISCUSSION

In this paper, we introduced a novel two stage approach for decision making based on a combination of Rough Set Theory and Analytical Hierarchy Process (AHP) to arrive at the appropriate decision under a set of alternatives. The proposed method overcomes the limitations in using the AHP method solely in dealing with MCDM problems. Using features of rough set theory method in stage 1, results in reduction the number of criteria from three criteria (price, size, and safety) to only two criteria (price and safety), this feature very useful in dealing with multi-criteria decision-making problems. The rough set approach is intended to deal with inconsistency and this is a major argument to support its application to multi-criteria decision analysis. Based on the probabilistic properties of rough set theory indicators we stated that the strength indicator already included in the two indicators; certainty and coverage so we used certainty and coverage indicators only in clustering step. Clustering step enables us to decrease the number of attributes. We used the neural network to get the relative importance of each criterion. In stage 2, we used AHP technique by setting scales for three alternatives depending on actual data for price and safety. Finally, according to the proposed approach we can choose the best alternative from a set of alternatives.

V. CONCLUSION

The main features of the two stage multi-criteria decision-making method that it is very useful for both the decision makers when choosing among alternatives and supplier to meet the customers' preferences. Besides, the proposed method can be used in the case of big data. Using Rough Set Theory results in improving the quality of data that will be used in the decision making process, unlike the classical MCDM methods that deal directly with the available data.

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