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A Comparison Study of Newton-Raphson and Genetic Algorithms for Change Point Problem in Non-Homogeneous Poisson Process

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Abstract—Poisson processes are prominent stochastic processes with their application in finance, environment, medicine, biology, genetic and etc. Poisson processes are generally classified as homogeneous, non-homogeneous and compound Poisson process. The most important distinctive feature of non-homogenous Poisson process from homogeneous Poisson process is the rate function which is not constant throughout time axis,but dependent on time. In the literature, several parametric forms of the rate function are available such as power-law, Musa-Okumoto, Goel-Okumato and generalized Goel-Okumato rate function.In this study, we considered an abrupt change point problem in the rate functions of the non-homogeneous Poisson process. The change point location and parameter estimates were obtained based on maximum likelihood method. Newton Raphson and genetic algorithm methodswere used in the estimation process of the parameters and the results of these methods were compared via a simulation study.

Keywords—Non-homogeneous Poisson process, Change point problem, Maximum likelihood method, Newton Raphson method, Genetic algorithm

I. INTRODUCTION

The studies on change point estimation (detection) or test procedures have extensive usagein many areas such as finance, biology, medicine, genetic, epidemiology, seismology, and industrial system processes. Statistically a change point problem is defined as follows:

Let $X_1,...,X_n$ be a sequence of random variables or time ordered variables with probability density function $f(x_i;\theta_i)$. Assume that the characteristics of distribution are altered at unknown location (or time point) in the sequence. Since the characteristics of the distribution are related with the parameters of distribution, generally a singlechange point problem is defined by:

$$\begin{aligned} X_1,...,X_\tau &\sim f(x;\theta_1) \\ X_{\tau+1},...,X_n &\sim f(x;\theta_2) \end{aligned} \quad (1)$$

where the unknown parameter τ is thechange point location in the sequence. The Eq. (1) is termed as change point problem in literature.

Firstly,reference [1] examined a change in the mean of normal distributed. Afterwardsmany authors studied on the change point problem in many distributions such as normal, binomial, Poisson and exponential. For example, reference [2], [3] and [4]studied on the change point problem in the normal distributed sequence. Reference [5] studied on the change point in the binomial distributed sequence. The change point in exponential or Poisson distributed sequence are studied by [6],[7],[8], [9] and [10]among others.

Software reliability is ever increasingly gained an importance in electronic systems. Homogeneous, non-homogeneous and compound Poisson processes (NHPP) are widely used for the software reliability modeling.Reference [11] described NHPP with intensity power law as a good model for reliability growth [12]. Logaritmic Poisson model was proposed by [13]. NHPP with the intensity based on exponential model wasintroduced by [14] and later this model is generalized by[15].As well as these models are very popular for the software reliability modeling, many other models are proposed such as Jelinski-Moranda model, Littlewood-Verrall model etc. [16].

Our interest wasa change point problem in non-homogeneous Poisson process which was described with the ratefunctions given by power law process, Musa-Okumoto process, Goel-Okumoto process and generalized Goel-Okumoto process, these processes and their propertieswere introduced in Section 2. In Section 3, the estimatorsof change point and parameters were obtained. In Section 4, Newton Raphson and Genetic Algorithm methods werepresented and then finally in Section5,a simulation study was performed in order to compare the Newton Raphson and Genetic Algorithm methods.

II. NON-HOMOGENEOUS POISSON PROCESS

As is known, in homogeneous Poisson process (HPP), λ (the number of events in the unit time)is constant throughout time, but innon-homogeneous Poisson process (NHPP) λ is a function of time.

Let $\{N_t, t \geq 0\}$ be NHPP with mean function $m(t)$. Mean function and corresponding intensity function are defined as follows:

$$\lambda(t) = \frac{d}{dt} m(t) \tag{2}$$

$$m(t) = \int_0^t \lambda(u) du \tag{3}$$

$m(1)$: expected number of events up to time 1.

Let N_t be the number of failures in the time interval $(0, t]$ and $\{N_t, t \geq 0\}$ be NHPPcounting the failures with the intensity $\lambda(t)$. The random variable N_t has a Poisson distribution and its probability distribution is given by:

$$P(N_t = i) = e^{-\int_0^t \lambda(t)} \frac{\left(\int_0^t \lambda(t)\right)^i}{i!} = e^{-m(t)} \frac{(m(t))^i}{i!}, \quad i = 0,1,...$$

In this study, $\lambda(t)$ would bespecifically considered as the forms called power-law, Musa-Okumoto process, Goel-Okumoto process and generalized Goel-Okumoto process. The processes are defined as follows:

*Power Law Process(PLP):*PLP is the most commonly used model in the software reliability.PLP is definedas follows:

$$m^{PLP}(t) = \left(\frac{t}{\varphi}\right)^{\alpha}, \quad \alpha, \varphi > 0 \tag{4}$$

$$\lambda^{PLP}(t) = \left(\frac{\alpha}{\varphi}\right)\left(\frac{t}{\varphi}\right)^{\alpha-1} \tag{5}$$

where φ is the scale parameter and α is the shape parameter.They represent the failure/repair rate in the model PLP [17].The intensity function can be a constant, decreasing or increasing form according to$\alpha = 1$, $\alpha < 1$ or $\alpha > 1$, respectively as a function of time.

*Musa-Okumoto Process(MOP):*MOP is called as Logarithmic model. In this model, the occurrences of failures follow NHPP. It is assumed that failure intensity will decrease exponentially with respect to the expected number of failures experienced whereas the exponential model assumes an equal reduction in failure intensity with each fault uncovered and corrected [18]. The intensity of the MOP:

$$m^{MOP}(t) = \varphi \log\left(1 + \frac{t}{\alpha}\right), \quad \alpha, \varphi > 0 \tag{6}$$

$$\lambda^{MOP}(t) = \frac{\varphi}{t + \alpha} \tag{7}$$

Goel-Okumoto Process (GOP): Considering the failure detection as a NHPP with an exponentially decaying rate function, the mean value and intensity functionaregiven as follows:

$$m^{GOP}(t) = \alpha\left[1 - \exp(-\varphi \mathrm{t})\right], \quad \alpha, \varphi > 0 \tag{8}$$

$$\lambda^{GOP}(t) = \alpha \varphi \exp(-\varphi \mathrm{t}) \tag{9}$$

where α is the expected total number of faults eventually detected and φ represents the fault detection rate [16]. The intensities in the MOP and GOP models present a decreasing behavior as a function of *t*.

*Generalized Goel-Okumoto Process (GGOP):*The intensity in GGOP model presents aslight increase initially and subsequently shows a decreasing behavior as functions of t [19]:

$$m^{GGOP}(t) = \alpha\left[1 - \exp(-\varphi \mathrm{t}^{\gamma})\right], \quad \alpha, \varphi, \gamma > 0 \tag{10}$$

$$\lambda^{GGOP}(t) = \alpha \varphi \gamma t^{\gamma-1} \exp(-\varphi \mathrm{t}^{\gamma}). \tag{11}$$

Parameters are same in GOP and GGOP models exceptγ parameter which is called an acceleration parameter.

III. CHANGE POINT PROBLEM IN NHPP

There are many studies considering a change point or multiple change points in HPP and NHPP. Reference [20]studied change point in Poisson process. Reference [21] studied Bayesian analysis of a Poisson process and then reference [22]suggested log-linear model for Poisson process with change point.Reference [23]investigated a change point in the HPP with an application to a worldwide earthquake dataand they also studied a test for change point in the HPP[24].Reference [19], [25] and [26] studied a change point(s) in the NHPP with an application to ozone data in Mexico City.

Assuming that there is a change point in NHPP, the intensity of the process $\lambda(t|\theta)$ is defined as:

$$\begin{aligned} &\lambda(t|\theta_1), \quad (0,\tau], \quad 0 < \mathrm{t} \leq \tau \\ &\lambda(t|\theta_2), \quad (\tau, T], \quad \tau < \mathrm{t} \leq T \end{aligned} \tag{12}$$

where τ is called as change point. Let $t_0, t_1, ..., t_n \in (0, T]$ to be the occurrence times of failures in NHPP with intensities given by Eq.(12) where $t_0 = 0,\ t_n \le T$. The likelihood function of observed data $t_0, t_1, ..., t_n \in (0, T]$ is written by using the [17]:

$$L(\theta_1, \theta_2 | t_1, t_1 ..., t_n) = L(\theta_1, \theta_2)$$
$$L(\theta_1, \theta_2) = \left[\prod_{i=1}^{N_\tau} \lambda(t_i | \theta_1)\right] \exp\left[-m(\tau|\theta_1)\right] \left[\prod_{i=N_\tau+1}^{n} \lambda(t_i|\theta_2)\right] \exp\left[-m(T|\theta_2) + m(\tau|\theta_2)\right] \tag{13}$$

Log (ℓn) likelihood function is:

$$\ell n L(\theta_1, \theta_2) = \left[\sum_{i=1}^{N_\tau} \ell n\left(\lambda(t_i|\theta_1)\right)\right] - m(\tau|\theta_1) + \left[\sum_{i=N_\tau+1}^{n} \ell n\left(\lambda(t_i|\theta_2)\right)\right] - m(T|\theta_2) + m(\tau|\theta_2). \tag{14}$$

where unknown parameter set is $(\theta_1, \theta_2, \tau)$. Maximum likelihood method can be used to estimate the parameter set.

Under the PLP model,considering a change in the scale parameter of the process, θ_1 and θ_2 would be $\theta_1 = (\alpha, \varphi_1) \in \mathbb{R}^{+2}$ and $\theta_2 = (\alpha, \varphi_2) \in \mathbb{R}^{+2}$.The likelihood function of observed data $t_0, t_1, ..., t_n \in (0, T]$ can be writtenby using the Eq.(13):

$$L^{PLP}(\alpha, \varphi_1, \varphi_2 | t_1, t_1 ..., t_n) = L^{PLP}(\alpha, \varphi_1, \varphi_2)$$
$$L^{PLP}(\alpha, \varphi_1, \varphi_2) = \left[\prod_{i=1}^{N_\tau} \lambda^{PLP}(t_i|\alpha, \varphi_1)\right] \exp\left[-m^{PLP}(\tau|\alpha, \varphi_1)\right] \times \left[\prod_{i=N_\tau+1}^{n} \lambda^{PLP}(t_i|\alpha, \varphi_2)\right] \exp\left[-m^{PLP}(T|\alpha, \varphi_2) + m^{PLP}(\tau|\alpha, \varphi_2)\right] \tag{15}$$

Using the mean and intensity functions of PLP model given Eq. (4) and Eq.(5), the log-likelihood function of the $t_0, t_1, ..., t_n$,

$$\ell n L^{PLP}(\alpha, \varphi_1, \varphi_2) = (\alpha - 1)\sum_{i=1}^{n} \ell n(t_i) + n \ell n(\alpha) - N_\tau \alpha \ell n(\varphi_1) - (n - N_\tau)\alpha \ell n(\varphi_2) - \left(\frac{\tau}{\varphi_1}\right)^\alpha - \left(\frac{T}{\varphi_2}\right)^\alpha + \left(\frac{\tau}{\varphi_2}\right)^\alpha. \tag{16}$$

To obtain the maximum likelihood estimator of the parameters α, φ_1 and φ_2 for fixed τ, the partial derivation of Eq.(16) with respect to the parameters $\alpha,\ \varphi_1$ and φ_2 are required. These derivatives are:

$$\frac{\partial \ell n L^{PLP}(\alpha, \varphi_1, \varphi_2)}{\partial \alpha} = \sum_{i=1}^{n} \ell n(t_i) + \frac{n}{\alpha} - N_\tau \ell n(\varphi_1) - (n - N_\tau)\ell n(\varphi_2) - \left(\frac{\tau}{\varphi_1}\right)^\alpha \ell n\left(\frac{\tau}{\varphi_1}\right) - \left(\frac{T}{\varphi_2}\right)^\alpha \ell n\left(\frac{T}{\varphi_2}\right) + \left(\frac{\tau}{\varphi_2}\right)^\alpha \ell n\left(\frac{\tau}{\varphi_2}\right) = 0 \tag{17}$$
$$\frac{\partial \ell n L^{PLP}(\alpha, \varphi_1, \varphi_2)}{\partial \varphi_1} = -\frac{\alpha N_\tau}{\varphi_1} + \left(\frac{\alpha}{\varphi_1}\right)\left(\frac{\tau}{\varphi_1}\right)^\alpha = 0$$
$$\frac{\partial \ell n L^{PLP}(\alpha, \varphi_1, \varphi_2)}{\partial \varphi_2} = -\frac{\alpha(n - N_\tau)}{\varphi_2} + \left(\frac{\alpha}{\varphi_2}\right)\left(\frac{T}{\varphi_2}\right)^\alpha - \left(\frac{\alpha}{\varphi_2}\right)\left(\frac{\tau}{\varphi_2}\right)^\alpha = 0$$

After solving these equations for each possible of τsimultaneously, the maximum likelihood estimator of $\tau$is obtained as:

$$\hat{\tau} = \underset{\tau \in (t_1, ... t_n)}{argmax}\left[\ell n L^{PLP}(\hat{\alpha}, \hat{\varphi}_1, \hat{\varphi}_2)\right]. \tag{18}$$

Similarly for the MOP model, considering a change in the parameter of the process, θ_1 and θ_2 would be $\theta_1 = (\alpha, \varphi_1) \in \mathbb{R}^{+2}$ and $\theta_2 = (\alpha, \varphi_2) \in \mathbb{R}^{+2}$. The likelihood function of observed data $t_0, t_1, ..., t_n \in (0, T]$ can be written by using the Eq.(13):

$$L^{MOP}(\alpha, \varphi_1, \varphi_2 | t_1, t_1 ..., t_n) = L^{MOP}(\alpha, \varphi_1, \varphi_2)$$
$$L^{MOP}(\alpha, \varphi_1, \varphi_2) = \left[\prod_{i=1}^{N_\tau} \lambda^{MOP}(t_i|\alpha, \varphi_1)\right] \exp\left[-m^{MOP}(\tau|\alpha, \varphi_1)\right] \times \left[\prod_{i=N_\tau+1}^{n} \lambda^{MOP}(t_i|\alpha, \varphi_2)\right] \exp\left[-m^{MOP}(T|\alpha, \varphi_2) + m^{MOP}(\tau|\alpha, \varphi_2)\right] \tag{19}$$

Using the mean and intensity function MOP given Eq. (6) and Eq.(7), the log-likelihood function of the $t_0, t_1, ..., t_n$,

$$\ell n L^{MOP}(\alpha,\varphi_1,\varphi_2) = -\sum_{i=1}^{n} \ell n(t_i+\alpha) + N_\tau \ell n(\varphi_1) + (n-N_\tau)\ell n(\varphi_2) - \varphi_1 \ell n\left(1+\frac{\tau}{\alpha}\right) - \varphi_2 \ell n\left(1+\frac{T}{\alpha}\right) + \varphi_2 \ell n\left(1+\frac{\tau}{\alpha}\right). \tag{20}$$

To obtain the maximum likelihood estimator of the parameters α, φ_1 and φ_2 for fixed τ, the partial derivation of Eq.(16) with respect to the parameters α, φ_1 and φ_2 are required. These derivatives are:

$$\begin{aligned} \frac{\partial \ell n L^{MOP}(\alpha,\varphi_1,\varphi_2)}{\partial \alpha} &= -\sum_{i=1}^{n} \frac{1}{(t_i+\alpha)} - \frac{\tau\varphi_1}{(1+\tau)\alpha} + \frac{T\varphi_2}{(1+T)\alpha} - \frac{\tau\varphi_2}{(1+\tau)\alpha} = 0 \\ \frac{\partial \ell n L^{MOP}(\alpha,\varphi_1,\varphi_2)}{\partial \varphi_1} &= \frac{N_\tau}{\varphi_1} + \ell n\left(1+\frac{\tau}{\alpha}\right) = 0 \\ \frac{\partial \ell n L^{MOP}(\alpha,\varphi_1,\varphi_2)}{\partial \varphi_2} &= \frac{(n-N_\tau)}{\varphi_2} - \ell n\left(1+\frac{\tau}{\alpha}\right) + \ell n\left(1+\frac{T}{\alpha}\right) = 0 \end{aligned} \tag{21}$$

After solving these equations for each possible of τ simultaneously, the maximum likelihood estimator of τ is obtained as:

$$\hat{\tau} = \underset{\tau\in(t_1,\dots t_n)}{argmax}\left[\ell n L^{MOP}(\hat{\alpha},\hat{\varphi}_1,\hat{\varphi}_2)\right]. \tag{22}$$

Using the same procedure, considering a change in the parameter of the process, θ_1 and θ_2 would be $\theta_1 = (\alpha,\varphi_1) \in \mathbb{R}^{+2}$ and $\theta_2 = (\alpha,\varphi_2) \in \mathbb{R}^{+2}$ for the GOP model. The likelihood function of observed data $t_0, t_1, ..., t_n \in (0, T]$ can be written by using the Eq.(13):

$$\begin{aligned} &L^{GOP}(\alpha,\varphi_1,\varphi_2 | t_1, t_1 ..., t_n) = L^{GOP}(\alpha,\varphi_1,\varphi_2) \\ &L^{GOP}(\alpha,\varphi_1,\varphi_2) = \left[\prod_{i=1}^{N_\tau} \lambda^{GOP}(t_i|\alpha,\varphi_1)\right] \exp\left[-m^{GOP}(\tau|\alpha,\varphi_1)\right] \times \\ &\left[\prod_{i=N_\tau+1}^{n} \lambda^{GOP}(t_i|\alpha,\varphi_2)\right] \exp\left[-m^{GOP}(T|\alpha,\varphi_2) + m^{GOP}(\tau|\alpha,\varphi_2)\right] \end{aligned} \tag{23}$$

Using the mean and intensity function GOP given Eq.(8) and Eq.(9), the log-likelihood function of the $t_0, t_1, ..., t_n$,

$$\ell n L^{GOP}(\alpha,\varphi_1,\varphi_2) = n\ell n(\alpha) + N_\tau \ell n(\varphi_1) + (n-N_\tau)\ell n(\varphi_2) - \varphi_1 \sum_{i=1}^{N_\tau} t_i - \varphi_2 \sum_{i=N_\tau+1}^{n} t_i - \alpha\left[1-exp(-\varphi_1\tau)\right] - \alpha\left[1-exp(-\varphi_2 T)\right] + \alpha\left[1-exp(-\varphi_2\tau)\right]. \tag{24}$$

To obtain the maximum likelihood estimator of the parameters α, φ_1 and φ_2 for fixed τ, the partial derivation of Eq.(24) with respect to the parameters α, φ_1 and φ_2 are required. These derivatives are:

$$\begin{aligned} \frac{\partial \ell n L^{GOP}(\alpha,\varphi_1,\varphi_2)}{\partial \alpha} &= -\frac{n}{\alpha} - \left[1-exp(-\varphi_1\tau)\right] - \left[1-exp(-\varphi_2 T)\right] + \left[1-exp(-\varphi_2\tau)\right] = 0 \\ \frac{\partial \ell n L^{GOP}(\alpha,\varphi_1,\varphi_2)}{\partial \varphi_1} &= \frac{N_\tau}{\varphi_1} - \sum_{i=1}^{N_\tau} t_i - \alpha\tau exp(-\varphi_1\tau) = 0 \\ \frac{\partial \ell n L^{GOP}(\alpha,\varphi_1,\varphi_2)}{\partial \varphi_2} &= \frac{(n-N_\tau)}{\varphi_2} - \sum_{i=N_\tau+1}^{n} t_i - \alpha T exp(-\varphi_2 T) + \alpha\tau exp(-\varphi_2\tau) = 0 \end{aligned} \tag{25}$$

After solving these equations for each possible of τ simultaneously, the maximum likelihood estimator of τ is obtained as:

$$\hat{\tau} = \underset{\tau\in(t_1,\dots t_n)}{argmax}\left[\ell n L^{GOP}(\hat{\alpha},\hat{\varphi}_1,\hat{\varphi}_2)\right]. \tag{26}$$

Using the same procedure, considering a change in the parameter of the process, θ_1 and θ_2 would be $\theta_1 = (\alpha,\varphi_1,\gamma) \in \mathbb{R}^{+3}$ and $\theta_2 = (\alpha,\varphi_2,\gamma) \in \mathbb{R}^{+3}$ for the GGOP model. The likelihood function of observed data $t_0, t_1, ..., t_n \in (0, T]$ can be written by using the Eq.(13):

$$\begin{aligned} &L^{GGOP}(\alpha,\varphi_1,\varphi_2,\gamma | t_1, t_1 ..., t_n) = L^{GGOP}(\alpha,\varphi_1,\varphi_2,\gamma) \\ &L^{GGOP}(\alpha,\varphi_1,\varphi_2,\gamma) = \left[\prod_{i=1}^{N_\tau} \lambda^{GGOP}(t_i|\alpha,\varphi_1,\gamma)\right] \exp\left[-m^{GGOP}(\tau|\alpha,\varphi_1,\gamma)\right] \times \\ &\left[\prod_{i=N_\tau+1}^{n} \lambda^{GGOP}(t_i|\alpha,\varphi_2,\gamma)\right] \exp\left[-m^{GGOP}(T|\alpha,\varphi_2,\gamma) + m^{GGOP}(\tau|\alpha,\varphi_2,\gamma)\right] \end{aligned} \tag{27}$$

Using the mean and intensity function GGOP given Eq. (10) and Eq.(11), the log-likelihood function of the $t_0, t_1, ..., t_n$,

$$\ell nL^{GGOP}(\alpha,\varphi_1,\varphi_2,\gamma)=n\ell n(\alpha)+n\ell n(\gamma)+(\gamma-1)\sum_{i=1}^{n}\ell n(t_i)+N_\tau\ell n(\varphi_1)+(n-N_\tau)\ell n(\varphi_2)- \varphi_1\sum_{i=1}^{N_\tau}t_i^{\gamma}-\varphi_2\sum_{i=N_\tau+1}^{n}t_i^{\gamma}-\alpha\left[1-exp(-\varphi_1\tau^{\gamma})\right]-\alpha\left[1-exp(-\varphi_2T^{\gamma})\right]+\alpha\left[1-exp(-\varphi_2\tau^{\gamma})\right]. \quad (28)$$

To obtain the maximum likelihood estimator of the parameters α,φ_1, φ_2, γ and for fixed τ, the partial derivation of Eq.(28) with respect to the parameters α,φ_1, φ_2, γ are required. These derivatives are:

$$\begin{aligned}
\frac{\partial \ell nL^{GGOP}(\alpha,\varphi_1,\varphi_2,\gamma)}{\partial\alpha}&=-\frac{n}{\alpha}-\left[1-exp(-\varphi_1\tau^{\gamma})\right]-\left[1-exp(-\varphi_2T^{\gamma})\right]+\left[1-exp(-\varphi_2\tau^{\gamma})\right]=0\\
\frac{\partial \ell nL^{GGOP}(\alpha,\varphi_1,\varphi_2,\gamma)}{\partial\varphi_1}&=\frac{N_\tau}{\varphi_1}-\sum_{i=1}^{N_\tau}t_i^{\gamma}-\alpha\tau^{\gamma}exp(-\varphi_1\tau^{\gamma})=0\\
\frac{\partial \ell nL^{GGOP}(\alpha,\varphi_1,\varphi_2,\gamma)}{\partial\varphi_2}&=\frac{(n-N_\tau)}{\varphi_2}-\sum_{i=N_\tau+1}^{n}t_i^{\gamma}-\alpha T^{\gamma}exp(-\varphi_2T^{\gamma})+\alpha\tau^{\gamma}exp(-\varphi_2\tau^{\gamma})=0\\
\frac{\partial \ell nL^{GGOP}(\alpha,\varphi_1,\varphi_2,\gamma)}{\partial\gamma}&=\frac{n}{\gamma}+\sum_{i=1}^{n}\ell n(t_i)-\varphi_1\sum_{i=1}^{N_\tau}t_i^{\gamma}\ell n(t_i)-\varphi_2\sum_{i=N_\tau+1}^{n}t_i^{\gamma}\ell n(t_i)-\\
&\alpha\gamma\varphi_1\tau^{\gamma}\ell n(\tau)exp(-\varphi_1\tau^{\gamma})-\alpha\gamma\varphi_2T^{\gamma}\ell n(T)exp(-\varphi_2T^{\gamma})+\alpha\gamma\varphi_2\tau^{\gamma}\ell n(\tau)exp(-\varphi_2\tau^{\gamma})=0
\end{aligned} \quad (29)$$

After solving these equations for each possible of τ simultaneously, the maximum likelihood estimator of τ is obtained as follows:

$$\hat{\tau}=\underset{\tau\in(t_1,\ldots t_n)}{argmax}\left[\ell nL^{GGOP}(\hat{\alpha},\hat{\varphi}_1,\hat{\varphi}_2,\hat{\gamma})\right] \quad (30)$$

Since the maximum likelihood estimators of parameters are not closed form, we need to use numerical methods such as Newton Raphson, Genetic algorithm, and etc.

IV. NEWTON RAPHSON AND GENETIC ALGORITHMS

A. Newton Raphson Algorithm

The Newton Raphson algorithm is a method based on Taylorapproximation and used for finding outroots of nonlinear function.To find the maximum of the log likelihood function of k parameters, the Newton Raphson algorithm is used commonly in literature. For this purpose, the algorithm is basically presented as follows:

Suppose that p is a parameter vector and $\hat{p}\in\mathbb{R}^k$ maximizes the twice continuously differentiable function $\ell nL(p):\mathbb{R}^k\to\mathbb{R}$. Let p_0 show an initial guess of the parameter vector, for next iteration, the parameter values are calculated by:

$$p_{n+1}=p_n-J^{-1}(p_n)\nabla\left(\ell nL(p_n)\right),\quad n=0,1,\ldots$$

where $J(.)$ isa matrix ofthe second partial derivatives of the log-likelihood function and $\nabla(.)$ is a vector of the first partial derivatives of the log-likelihood function with respect to each parameter. Iteration continues until convergence is satisfied withthat two consecutive solutionsare close to each other at a certain tolerance ε .

B. Genetic Algorithm

Genetic algorithm (GA) is a heuristic search method based on modeling the characteristic of chromosomes in biology. In literature, it is a well-known method to find global or approximate solution for optimization problems [28]. GA has three genetic operators (or steps): selection, crossover and mutation. GA starts with an initial population which is a possible set of values of a problemand subsequently these operators are activated to generate new populations. In GA, each individual (i.e. chromosome) which is a member of the population represents a potential solution for the problem. One of the stopping rules of the algorithm is to reach maximum population numbers. As it is known, GA has a good performance to find nearly exact solutions especially for composed problems but does not guarantee a solution for each problem. For further details about GA, readers are referred to [28], [29], [30], [31], [32] and [33].

Steps of GA for our problem are given as follows:

Step 1. An initial population is generated randomly from uniform distribution with sufficiently large interval. The population size is taken to be 20.

Step 2. Evaluation: each individual in the population is evaluated by using fitness value given Equation (14) and its values are sorted in ascending order.

Step 3. Selection: 50% of the population is selected by the elitism method which is based on the criterion: the best individuals are transferred to the next generation. The selected individuals are assigned as parents.

Step 4. Crossover: each of two parents procreates a new child by one point basic crossover operator and so new generation is equal to the initial population size.

Step 5. Mutation: to increase the diversity of the individuals, 50 percentile of the population are mutated by 10% mutation rate.

Step 6. Repeat the Step 2-Step 5 to generate the new population.

Step 7. the algorithm is stopped when the maximum population numbers has been reached (generally recommended between the 2 to 3 times of population size). The maximum population numbers are taken as40.

V. SIMULATION

In the simulation study, let $0 = t_0 < t_1 < ... < t_n \le T$ be the observed data, the observing time *T*was taken as $(10,100)$, where $T = 10$ was corresponded to the data before the change point $\tau$and $T = 100$ was corresponded to the data after the change point τ. The numbers of observations (events) $N_T = n$ were taken as 20 and 50.The numbers of observationsbefore and after τ, were taken as 10 and 25 and N_τ was also taken as 10 and 25. Thefollowing algorithm was used for simulating data [34]:

Step 1.initialize $n = 0$, $t_0 = 0$,

Step 2.while $t_n < T$, generate $X_{n+1} \sim F_{t_{n+1}}$; set $t_{n+1} = t_n + X_{n+1}$ and $n = n+1$

Step 3. end.

where F_{t_n} is:

$$
\begin{aligned}
F_{t_{n+1}}(x) &= P\left(t_{n+1} < x \middle| t_n\right) \\
&= 1 - P\left(0\, events\, in\, (t_n, t_n + x\right) \\
&= 1 - e^{\int_0^x \lambda(t_n+s)ds}.
\end{aligned}
$$

The algorithm was executed for before and after the change point. We should point out that N_τ is fixed whereas $\tau$is not a fixed time point so the estimation of the change point was evaluated over the N_τ

In the simulation, when the parameter values were generated independently, the problem was observed in the case,saying that negative values wereoccurredfor time points. Thereforeit was needed to take into account the correlations between parameters while generating the data. The parameter sets wereused for the simulation were presented in Table I, where the observing time *T*was taken as fixed values.The value of the parameters φ_1 and $\varphi_2$were determined with respect to different values of the parameterα.

Table I. Parameter's Sets Used in the Simulation Study.

n=20										
	PLP			MOP			GGOP			
T	α	φ1	φ2	α	φ1	φ2	α	φ1	φ2	γ
(10, 100)	0.5	0,025	0,25	0.5	6,57	3,77	40	0,22	0,069	0.5
	1	0,5	5	1	8,34	4,33	40	0,07	0,007	1
	2	2,24	22,36	2	11,16	5,09	40	0,01	6,931E-05	2
n=50										
	PLP			MOP			GGOP			
T	α	φ1	φ2	α	φ1	φ2	α	φ1	φ2	γ
(10, 100)	0.5	0,004	0,04	0.5	16,42	9,43	100	0,22	0,069	0.5
	1	0,2	2	1	20,85	10,83	100	0,07	0,007	1
	2	1,41	14,14	2	27,91	12,72	100	0,01	6,93E-05	2

According to the parameter sets given in Table I, simulation results for Newton Raphson algorithm and GA were summarized in Table II and Table III.

Table II. The Results of the Simulation Using Newton Raphson Algorithm.

n	*Model*	α	φ_1	φ_2	γ	$\hat{\alpha}$	$\hat{\varphi}_1$	$\hat{\varphi}_2$	$\hat{\gamma}$	$N_{\hat{\tau}}$	MSE $\hat{\alpha}$	MSE $\hat{\varphi}_1$	MSE $\hat{\varphi}_2$	MSE $\hat{\gamma}$	MSE $N_{\hat{\tau}}$	*AIC*
20	**PLP**	0,5	0,025	0,25	-	0,58	0,27	1,41	-	9,68	0,23	10,89	66,22	-	21,09	2,12E+11
		1	0,5	5	-	0,62	5,52	23,57	-	6,94	0,36	524,49	6,40E+03	-	28,19	128,92
		2	2,24	22,36	-	0,82	7,36	48,94	-	6,24	1,98	320,45	1,66E+04	-	27,96	2,14E+17
	MOP	0,5	6,57	3,77	-	0,72	9,64	5,27	-	12,41	0,28	37,70	10,32	-	53,94	-1,81
		1	8,34	4,33	-	1,39	12,38	5,91	-	12,06	0,90	60,51	12,04	-	54,30	9,13
		2	11,16	5,09	-	2,76	16,91	6,87	-	11,64	3,46	106,98	15,15	-	51,73	19,63
	GGOP	40	0,22	0,07	0,5	63,74	0,42	0,12	0,70	12,07	1693,52	0,15	0,10	0,08	13,22	-4,63E+01
		40	0,07	0,01	1	68,90	0,12	0,32	0,79	7,27	2350,99	0,05	0,51	0,38	11,78	48,83
		40	0,01	6,93E-05	2	66,72	0,01	1,38E-04	1,97	8,56	2379,48	6,29E-05	1,79E-07	0,14	4,27	64,07
50	**PLP**	0,5	0,004	0,04	-	0,51	0,43	1,26	-	11,65	0,02	10,84	98,40	-	204,13	-4,41
		1	0,2	2	-	0,56	24,53	29,08	-	22,54	0,30	3,02E+03	2,71E+03	-	256,56	285,61
		2	1,41	14,14	-	0,74	20,68	50,25	-	15,11	2,04	1,43E+03	5,04E+03	-	197,97	674,26
	MOP	0,5	16,42	9,43	-	1,03	29,65	15,91	-	30,46	0,71	415,97	136,56	-	353,88	-107,52
		1	20,85	10,83	-	1,98	39,84	17,89	-	29,78	2,20	767,06	161,74	-	370,81	-80,04
		2	27,91	12,72	-	3,92	53,90	20,74	-	29,05	8,45	1331,54	192,05	-	320,35	-50,65
	GGOP	100	0,22	0,07	0,5	155,21	0,37	0,24	0,59	27,12	8,95E+03	0,07	5,00E-01	0,04	37,77	-2,18E+02
		100	0,07	0,01	1	201,97	0,08	0,34	0,70	14,84	1,79E+04	2,77E-03	3,39E-01	0,34	114,28	2,96E+01
		100	0,01	6,93E-05	2	171,99	0,01	1,57E-04	1,92	19,84	1,55E+04	6,33E-05	2,49E-08	0,11	32,72	6,82E+01

Table III. The Results of the Simulation Using Genetic Algorithm.

n	*Model*	α	φ_1	φ_2	γ	$\hat{\alpha}$	$\hat{\varphi}_1$	$\hat{\varphi}_2$	$\hat{\gamma}$	N_{ξ}	MSE $\hat{\alpha}$	MSE $\hat{\varphi}_1$	MSE $\hat{\varphi}_2$	MSE $\hat{\gamma}$	MSE N_{ξ}	*AIC*
20	**PLP**	0,5	0,025	0,25	-	0,66	0,06	0,86	-	11,82	0,07	0,01	0,87	-	34,81	20,45
		1	0,5	5	-	1,37	0,95	10,26	-	10,37	0,45	0,82	95,98	-	3,71	60,73
		2	2,24	22,36	-	2,73	3,03	32,46	-	9,85	2,27	2,89	395,12	-	0,40	70,51
	MOP	0,5	6,57	3,77	-	1,68	18,83	8,99	-	11,62	3,51	349,35	98,41	-	49,28	-6,04
		1	8,34	4,33	-	3,28	24,93	9,88	-	11,63	12,26	566,43	106,33	-	48,76	6,48
		2	11,16	5,09	-	6,37	34,27	11,40	-	11,58	42,76	1023,85	126,67	-	46,44	18,03
	GGOP	40	0,22	0,07	0,5	59,46	0,56	0,11	0,68	12,17	2,76E+03	0,31	0,01	0,08	24,59	-47,87
		40	0,07	0,01	1	49,50	0,16	0,01	1,22	10,17	2,57E+03	0,03	1,33E-04	0,24	0,93	27,47
		40	0,01	6,93E-05	2	63,38	0,01	1,30E-04	2,06	9,94	4,87E+03	1,69E-04	3,52E-08	0,33	0,14	53,06
50	**PLP**	0,5	0,004	0,04	-	0,57	0,01	0,10	-	31,52	0,02	1,03E-04	0,01	-	141,93	-42,88
		1	0,2	2	-	1,19	0,35	3,76	-	25,11	0,15	0,10	12,68	-	3,74	57,53
		2	1,41	14,14	-	2,51	1,97	21,23	-	24,57	1,06	1,22	171,58	-	1,29	79,28
	MOP	0,5	16,42	9,43	-	1,93	51,23	21,08	-	30,40	5,13	2935,69	445,68	-	394,71	-113,60
		1	20,85	10,83	-	3,76	66,52	24,32	-	30,18	18,90	4766,63	578,03	-	389,98	-81,33
		2	27,91	12,72	-	7,03	87,48	28,53	-	29,70	65,42	8002,30	757,11	-	353,25	-51,48
	GGOP	100	0,22	0,07	0,5	136,51	0,52	0,14	0,59	31,58	12675,94	0,26	0,02	0,02	89,40	281,02
		100	0,07	0,01	1	126,16	0,16	0,01	1,11	25,06	13551,26	0,02	1,37E-04	0,09	1,13	260,33
		100	0,01	6,93E-05	2	158,11	0,01	1,35E-04	2,02	24,84	29441,70	1,79E-04	2,76E-08	0,25	0,38	324,22

In Table II, there was no significant differences between the estimations of αparameter for different sample sizes under PLP model. However, when $\alpha$was 0.05, the estimation of α parameter was found better for the sample size 50 (MSE=0.02).

According to the results of Table II and Table III, the parameter estimates were found more close to their actual values and AIC values were found lower for all parameters in GA than in Newton Raphson algorithm.

Under MOP model, the parameter estimates and AIC values were found better in Newton Raphson algorithm, whereas the estimation of change points was obtained better in the GA for sample size 20. The results were similar for the sample size 50. The estimation of change point was also similar in both GA and Newton Raphson algorithm for the same sample size.

Under GGOP model, the parameter estimates and AIC values were more satisfying in GA than in Newton Raphson algorithm. The MSE values were poor for both algorithms.

By considering the correlation between the parameters, the simulation for the case of known αwas repeated in order to be able to get better performance in the estimation of the other parameters for both algorithms. The results for that case are given in Table IV-V.

Table IV. The Results of the Simulation Using Newton RaphsonAlgorithm,whenthe Alpha is Known.

n	*Model*	α	φ_1	φ_2	γ	$\hat{\varphi}_1$	$\hat{\varphi}_2$	$\hat{\gamma}$	N_{ξ}	MSE $\hat{\varphi}_1$	MSE $\hat{\varphi}_2$	MSE $\hat{\gamma}$	MSE N_{ξ}	*AIC*
20	**PLP**	0,5	0,025	0,25	-	0,05	17,67	-	4,46	0,01	41864,24	-	49,02	25,16
		1	0,5	5	-	0,51	4,14	-	7,91	0,08	4,27	-	11,13	-136,44
		2	2,24	22,36	-	2,26	33,01	-	9,44	2,00	1,15E+05	-	1,54	-5,64E+04
	MOP	0,5	6,57	3,77	-	8,36	5,29	-	11,06	26,88	10,98	-	65,14	-4,77
		1	8,34	4,33	-	10,73	5,84	-	11,32	39,38	13,02	-	60,22	7,06
		2	11,16	5,09	-	14,49	6,56	-	11,35	65,64	15,59	-	55,21	18,02
	GGOP	40	0,22	0,07	0,5	0,56	0,16	0,58	14,42	0,36	0,02	0,08	41,73	-45,10
		40	0,07	0,01	1	0,14	0,01	1,22	10,14	0,01	2,67E-04	0,18	1,10	25,61
		40	0,01	6,93E-05	2	0,01	8,39E-05	2,16	9,98	1,50E-04	7,35E-08	0,17	0,05	50,49
50	**PLP**	0,5	0,004	0,04	-	7,25E+151	5,30E+151	-	6,08	Inf	Inf	-	404,83	48,76
		1	0,2	2	-	0,21	1,71	-	19,95	0,01	0,36	-	35,50	69,53
		2	1,41	14,14	-	1,44	15,59	-	24,11	0,10	11,44	-	2,55	79,93
	MOP	0,5	16,42	9,43	-	19,29	13,15	-	28,37	106,67	64,36	-	480,45	-112,08
		1	20,85	10,83	-	25,20	14,45	-	28,58	169,58	71,40	-	475,68	-80,88
		2	27,91	12,72	-	33,89	16,20	-	28,81	256,30	75,04	-	393,53	-51,85
	GGOP	100	0,22	0,07	0,5	0,67	0,16	0,53	35,46	3,26	0,02	0,03	168,15	-213,88
		100	0,07	0,01	1	0,14	0,01	1,13	25,07	0,01	6,21E-05	0,06	0,92	-35,36
		100	0,01	6,93E-05	2	0,01	9,40E-05	2,12	24,94	6,72E-05	7,18E-09	0,09	0,14	28,81

Table V. The Results of the Simulation Using Genetic Algorithm,When theAlpha is Known.

n	*Model*	α	φ_1	φ_2	γ	$\hat{\varphi}_1$	$\hat{\varphi}_2$	$\hat{\gamma}$	$N_{\hat{\xi}}$	MSE $\hat{\varphi}_1$	MSE $\hat{\varphi}_2$	MSE $\hat{\gamma}$	MSE $N_{\hat{\xi}}$	*AIC*
20	**PLP**	0,5	0,025	0,25	-	0,02	0,32	-	12,12	1,49E-03	0,25	-	37,37	-4,06
		1	0,5	5	-	0,52	5,27	-	10,30	0,08	7,14	-	4,00	-11841,90
		2	2,24	22,36	-	2,44	25,01	-	9,99	0,73	61,20	-	0,25	-7,62E+11
	MOP	0,5	6,57	3,77	-	9,53	9,17	-	10,93	105,71	124,02	-	57,58	-6,59
		1	8,34	4,33	-	12,27	9,55	-	11,27	167,41	138,45	-	59,18	5,85
		2	11,16	5,09	-	16,49	9,49	-	11,33	276,44	133,12	-	53,68	17,30
	GGOP	40	0,22	0,07	0,5	0,69	0,14	0,68	11,92	0,36	0,02	0,08	26,66	-49,74
		40	0,07	0,01	1	0,14	0,01	1,19	10,12	0,01	2,67E-04	0,18	1,00	25,86
		40	0,01	6,93E-05	2	0,02	1,45E-04	2,09	9,85	1,50E-04	7,35E-08	0,17	0,34	52,78
50	**PLP**		0,004	0,04	-	0,00	0,04	-	31,58	7,43E-06	2,34E-03	-	144,17	-43,99
			0,2	2	-	0,20	2,06	-	25,24	4,34E-03	0,46	-	2,85	54,96
			1,41	14,14	-	1,49	15,88	-	24,92	0,09	14,54	-	0,18	73,57
	MOP	0,5	16,42	9,43	-	20,67	18,93	-	28,37	347,25	394,97	-	427,94	-113,71
		1	20,85	10,83	-	27,37	18,98	-	29,47	607,99	400,74	-	451,37	-81,70
		2	27,91	12,72	-	37,22	20,02	-	29,33	1106,44	422,37	-	414,49	-52,16
	GGOP	100	0,22	0,07	0,5	0,56	0,15	0,59	31,48	0,15	0,01	0,02	90,05	-228,25
		100	0,07	0,01	1	0,15	0,02	1,07	24,99	0,01	2,07E-04	0,07	1,31	-34,72
		100	0,01	6,93E-05	2	0,02	1,65E-04	2,04	24,67	1,54E-04	5,77E-08	0,15	0,88	32,94

According to Table IV and Table V, the parameter estimates were found better and MSE values were found lower in GA for the sample size 20 under PLP model. For the sample size 50, the parameter estimates were obtained very poor in Newton Raphson algorithm since convergence wasn't provided.

In Table IV, parameter estimates were found generally better in Newton Raphson algorithm for all sample sizes under MOP model. But, the estimation of change point was found similar among theresults of GA and Newton Raphson algorithms.

Under GGOP model,the parameter estimates were found very similar for both Newton Raphson algorithm and GA according to the Table IV and Table V.

When α is known, the improvement was achieved in results of GA while in results of NR wasn't.

Although convergence rate was not indicated in the tables, it was very low under PLP and MOP models, while it was quite better under GGOP model.

VI. CONCLUSION

Even though Newton Raphson algorithm is a classical method based on analytical background, GA is a well-known method for the solution of composite optimization problems. Both methods have some disadvantages: for example one is that the performance of Newton Raphson method depends on choosing starting points. Another disadvantage is that the derivations of the objective function with respect to the number of unknown parameters could be obtained difficult for large number of parameters. For GA, main disadvantage is that each step of the algorithm depends on many adjustable input values. Since both algorithms have many limitations, the parameters were not able to be estimated accurately. Due to the fact that the estimation of parameters were on local optimum points, thus, new methods are needed toestimate the parameters in those models.

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