Volume 6, Issue 2, February 2016 ISSN: 2277 128X
International Journal of Advanced Research in Computer Science and Software Engineering
Research Paper
Available online at: www.ijarcsse.com

Duality in Soft Erosion in Multi Scale Soft Morphological Environment

Kompella Venkata Ramana
Associate Professor, Dept. of Computer Science & Systems Engineering, Andhra University,
Andhra Pradesh, India

Abstract: In this paper, duality is discussed in soft-erosion in multi scale environment .In mathematical morphological environment, erosion and dilation are dual to each other; open and close are dual to each other. But in soft morphology, the duality is not discussed thoroughly. It is going to be discussed in this paper in detail.

Key Words: Duality, Mathematical morphology, Mathematical soft morphology, Soft morphology, Erosion, Dilation, Soft erosion, Soft dilation, Primitive morphological operation.

I. INTRODUCTION

The human beings have the desire of recording incidents ,through images. It has started from early cavemen also. Later, so many techniques, to get the images and so many techniques, to process the images are developed. After assembling of computers, image processing was expanded.
In 1964 G. Matheron was asked to investigate the relationships between the geometry of porous media and their permeability. At the same time, J. Serra was asked to quantify the petrography of iron ores, in order to predict their milling properties. (1, 2) At the same time a centre was developed to study mathematical morphological techniques in Paris school of mines, France. Mathematical morphology can provide solutions to many tasks, where image processing can be applied, such as in remote sensing, optical character recognition, Radar image sequence recognition, medical image processing etc.,
The image processing algorithms or techniques can be classified in to two categories.
1) Linear methods
2) Non Linear methods.
The non linear methods will provide best results, compared to linear methods. The mathematical morphological methods / filters will come under the category of non linear methods/filters. In mathematical morphological operations, Erosion and Dilation are primitive operations (3,4). But there will exist some type of rigidity in mathematical morphological operations. That rigidity is relaxed and the morphological operations are redesigned to overcome some inconveniences, as well as to get some advantages. The primitive operations, Erosion and Dilation, now are called Soft Erosion and Soft Dilation.
The properties of soft morphological operations are not discussed thoroughly , till now.
That gap is tried to be filled in this paper.

II. DEFINITIONS

2.1 Soft Erosion:

In some papers, researchers proposed soft morphology using two sets of structuring elements.
A) The core
B) The soft boundary [7,8,9 etc].
But, in some papers [5] they proposed soft morphology, by counting logic. They have done the counting of ones, in the particular sub image, chosen. Then they have applied threshold value, for soft Erosion .
Soft Erosion may be defined as (5)

$$(I \ominus S^{(m)})[x, y] = 0 \text{ If } |\bar{I} \cap S_{(x,y)}| \geq m$$
$$= 1 \text{ otherwise. } \bar{I} = \text{inversion of I; m= threshold} \leq |S|.$$

2.2 Duality:

Two operations *, . are duals to each other if $(A*B)=(A^C.B)^C$ or $(A.B)=(A^C*B)^C$
In Mathematical morphology, dilation (⊕) and erosion (⊖) are duals. . i.e. $(I \oplus S) = (I^C \ominus S)^C$ or $(I \ominus S) = (I^C \oplus S)^C$
In Soft morphology, the duality will exist in a different way because depending upon threshold values, many soft morphological operations will exist. . It is explained in the following sections. Duals are proposed for soft erosion, in this paper, in multi scale environment. Self duals are also proposed. At the ending of this paper the duals are discussed for a general case, assuming w/w S.E. size.

III. BACKGROUND

3.1 Duality:

Applying of dual pairs for noise elimination is studied in (10,11) for better results. These can be extended to soft morphology and muti scale soft morphology. Dual pairs can be constructed for statistical soft morphological as well as Fuzzy soft morphological operators and above methodology (10,11) can be extended to these areas also. Especially idempotency and duality properties are used for speckle noise removal in radar images (10). These can be studied in soft morphological domain and multi scale soft morphological domains.
BOUAYNAYA, N; etc. (10) proposed another morphological algorithm using IDEMPOTENCY and DUALTY property for elimination of speckle noise in radar images. [In this paper the importance of duality & idempotency properties are understood]. LEI, T; FAN, Y. Shown (11) elimination of impulse noise by a pair of morphological **dual** operators. They have shown that, this **dual** pairs provides better results for image smoothing.
JIANN–JONE CHEN etc. extended the MSMM to 3D segmentation, using **dual** (MS morphological) concepts (G_{30}). the following way, in a very simple manner.

3.2 Soft Morphology:

In primitive morphological operation, dilation, isolated pixels, even though, they are irrelevant to the image's content, significantly affect the output of the transformation. The net effect is an increased number of large spurious particles, increasing the confusion in the dilated image. So, noise will be added, which may be named as additive noise. (12).
But, many applications require more tolerance to noise than is provided by erosion and dilation. Soft morphological operators possess many of the characteristics, which are desirable, perform better in noisy environments. (12)
So, the soft morphological filters, improve the behavior of standard morphological filters, in noisy environment. The soft morphological filters are better compared to mathematical morphology in small detail preservation and impulse noise. In soft morphology, it preserves details, by adjusting its parameters (24). It can be designed in such a way that, it performs well in removal of salt – and – pepper noise as well as Gaussian noise, simultaneously. (25)
A soft morphological filter can be designed in such a way that, it reduces periodic noise also (26). A filter designed in frequency domain, can function better for smoothening & edge enhancement, according to our requirements. The reason is that by tuning its frequency. But the design involves complex computation. But using soft morphological filters, using very simple computations we can achieve the quality of image processing, to that of filters in frequency domain, which involves complex computations (27). So, we can conclude that soft morphological filters perform excellent, compared to morphological filters.
The idea of soft morphological operations is to relax, the standard morphological definition, a little, in such a way that, a degree of Robustness is achieved, While, most of the desirable properties of standard morphological operations are maintained. The soft morphology was introduced by KOSKINEN etc, and developed by a few researchers (6) .
MICHAEL A. Z MODA and LOUIS. A. TAMBURINO discussed (12) morphological operations, soft morphological operations in detail. In this paper they discussed the definitions of Erosion, Dilation on the basis of methodology like counting, which is suitable to extend to soft morphological operations, by fixing threshold values. They discussed some more algorithms for implementation of soft morphological operations, properties up to some extent. PAULI KUOSMANERI etc. (13) have discussed about statistical properties of soft morphological ops. They discussed about noise reduction using soft morphological ops, with detail preservation, in this research paper. The above authors discussed in another research paper (14) about the relation in between soft morphology ops as well as stack filters.
SHIH, F.Y. etc. discussed (15) soft morphological properties are discussed up to some extent. Some of the properties are stated and idem potency is discussed up to some extent. They discussed about, soft morphology op's in gray scale, using threshold super position theorem. They discussed about implementation of soft morphology op's, using logic gates also.
PU, C.C. discussed about (16) implementation of soft morphological op's in gray scale. They integrated super position property and stacking to extend soft morphology from binary scale to gray scale.
PAULI KUOSMANEN & JAAKKO ASTOLA (H_6) also discussed, statistical properties, of soft morphology op's, up to some extent, with connection to stack filters.
GASTERATOS discussed (18) a new technique, for the realization of soft morphology op's basing upon majority gate algorithm. MICHAEL A. ZMUDA (19) proposed an algorithm for implementation of soft morphology ops. Normally voting logic also may be used, across neighborhoods, defined by the S.E.
But, in this algorithm instead of processing all the votes, a few votes may be chosen randomly and the service of FSM also, will be taken, in implementation of this algorithm. It is faster than conventional algorithms. Accuracy: more than 90%.
ZHAL CHUNHUI (20) designed soft morphological filter, using genetic algorithm. It is in optimized and improved algorithm. PERTTIT. KOI VISTO, etc. (21) also concentrated and discussed improved algorithms for soft morphological ops, using genetic algorithms.
M. VARDA VOULIA etc. (22) designed algorithms for small detail preservation and impulse noise suppression, using soft morphological op's [soft vector morphology] in color environment and shown better results compared to algorithm designed, based on morphological operators. [Mathematical vector morphology]. A. GASTERATOS, etc. (23) discussed about structuring element decomposition, in soft morphological environment.
In this way soft morphology was discussed by a few researchers.

3.3 Multiscale Morphology

In the process of understanding the objective world, the appearance of an object does not depend only on the object itself, but also on the scale that the observer used. It seems that appearance under a specific scale does not give sufficient information about the essence of the percept, we want to understand. If we use a different scale, to examine this percept, it will usually have a different appearance. So, this series of images and its changing pattern over scales reflect the nature of the percept.

Till now, some amount of research is done in this area, and it is applied in so many areas. In mathematical morphology also, a new area multi scale mathematical morphology is developed, and applied in so many areas like smoothening, edge enhancement, analysis of radar imagery, remote sensing, medical image processing etc.

PETROS MARAGOS entered into multi scale morphology, in addition to other areas. He explained about changes of shapes, as the scale is changed. He explained the applications of MSMM, and back ground mathematics. He explained about application of MSMM in skeletenization also. He extended these concepts to gray scale, also (28). MING – HUA CHEN & PING – GAN YAN explained (G_2) Erosion, Dilation, Open, Close in multi scale environment, with diagrams (results), mathematical analysis, as well as symbolic conventions.

(35) provided one type of analysis in MSMM. They discussed how to relate the results of one scale with the results at different scale. They have provided this analysis with good examples, using Erosion/Dilation morphological operations. This paper discussed the B.G. theory, in one angle, relating to MSMM.

KUN WANG etc. proposed an algorithm, for edge detection in the presence of Gaussian noise & salt – pepper noise in multi scale morphological environment. The experimental results are better than that of conventional algorithms (30). The same authors KUNWANG etc. proposed another algorithm for edge detection (31) which will function better in Gaussian, salt - paper noise environment, in MS morphological approach.

KIM WANG and others discussed an edge detection algorithm, in multi scale environment, which is suitable to apply on brain MRI, in noisy environment. (32).

ZENG PINGPING etc. proposed another algorithm, for edge enhancement (33) in multi scale morphological approach, using order morphology also, which is suitable to apply in noisy environment also. ZHEANHUA LI; & others (34) discussed another technique for edge enhancement, in MS morphological environment.

The above discussion tells that multi scale morphology has entered in to various areas of image processing. The above research can be extended to multi scale soft morphology also. So the discussion of soft erosion has to be done in multi scale environment. So it is done in this paper.

IV. DISCUSSION ON DEFINITION OF SOFT EROSION

4.1. $3/3$ Structuring Elements

Soft Erosion may be represented symbolically as $E_{(n)}$ where n is threshold value.

If threshold value= 1 then $E_{(1)}$ may be defined as

$(I\ominus S^{(1)})$ [x , y]=0 If $|\bar{I} \cap S_{(x,y)}| \geq 1$
= 1 other wise.

Here, $S^{(1)}$ means, threshold value=1, in the $3/3$ sub image which is chosen, from the image.

If threshold value= 2 then $E_{(2)}$ may be defined as

$(I\ominus S^{(2)})$ [x, y]= 0 If $|\bar{I} \cap S_{(x,y)}| \geq 2$
= 1 other wise.

Here, $S^{(2)}$ means, threshold value=2, in the $3/3$ sub image which is chosen, from the image.

If threshold value= 9 then $E_{(9)}$ may be defined as

$(I\ominus S^{(9)})$ [x, y]=0 If $|\bar{I} \cap S_{(x,y)}| \geq 9$
= 1 other wise.

Here, $S^{(9)}$ means, threshold value=9, in the $3/3$ sub image which is chosen, from the image.

4.2. $5/5$ Structuring Elements

For $5/5$ Structuring Elements, the thresholds are 1 to 25. The soft erosions may be defined as follows.

If threshold value= 1 then $E_{(1)}$ may be defined as

$(I\ominus S^{(1)})$ [x, y]=0 If $|\bar{I} \cap S_{(x,y)}| \geq 1$
= 1 other wise.

Here, $S^{(1)}$ means, threshold value=1, in the sub image chosen, which is having dimension $5/5$ from the image.

If threshold value= 2 then $E_{(2)}$ may be defined as

$(I\ominus S^{(2)})$ [x, y]=0 If $|\bar{I} \cap S_{(x,y)}| \geq 2$

= 1 other wise.

Here, $S^{(2)}$ means, threshold value=2, in the sub image chosen, which is having dimension $5/5$ from the image.

If threshold value= 3 then $E_{(3)}$ may be defined as

$(I\ominus S^{(3)})$ [x, y]=0 If $|\bar{I} \cap S_{(x,y)}| \geq 3$

= 1 other wise.

Here, $S^{(3)}$ means, threshold value=3, in the sub image chosen, which is having dimension $5/5$ from the image.

In the same way $E_{(4)}$, $E_{(5)}$, $E_{(6)}$, $E_{(7)}$, $E_{(8)}$, $E_{(9)}$.

If threshold value= 10 then $E_{(10)}$ may defined as

$(I\ominus S^{(10)})$ [x, y]=0 If $|\bar{I} \cap S_{(x,y)}| \geq 10$

= 1 other wise.

Here, $S^{(10)}$ means, threshold value=10, in the sub image chosen, which is having dimension $5/5$ from the image.

If threshold value= 25 then $E_{(25)}$ may be defined as

$(I\ominus S^{(25)})$ [x , y]=0 If $|\bar{I} \cap S_{(x,y)}| \geq 25$

= 1 other wise.

Here, $S^{(25)}$ means, threshold value=25, in the sub image chosen, which is having dimension $5/5$ from the image.

4.3. $7/7$ Structuring Elements

For $7/7$ Structuring Elements, where m=1 to 49, the Soft Erosions are $E_{(1)}$, $E_{(2)}$, $E_{(3)}$,…………………$E_{(49)}$.

They may be defined, as described in the above sections.

In the same way, the size of Structuring Elements may be extended to $9/9$, $11/11$, $13/13$ $15/15$, $17/17$, $19/19$ ……. to any dimension, according to our requirement.

V. DISCUSSION ON DUALITY OF SOFT EROSION IN MULTI SCALE ENVIRONMENT

5.1. $3/3$ Structuring Element

By means of the definition of soft erosion the following tables may be constructed. Table 1 will have soft erosion values if the structuring element dimension is $3/3$. So, the thresholds are from 1 to 9.

Table 1 Number of 1's of sub image

	0	1	2	3	4	5	6	7	8	9
$E(1)$	0	0	0	0	0	0	0	0	0	1
$E(2)$	0	0	0	0	0	0	0	0	1	1
$E(3)$	0	0	0	0	0	0	0	1	1	1
$E(4)$	0	0	0	0	0	0	1	1	1	1
$E(5)$	0	0	0	0	0	1	1	1	1	1
$E(6)$	0	0	0	0	1	1	1	1	1	1
$E(7)$	0	0	0	1	1	1	1	1	1	1
$E(8)$	0	0	1	1	1	1	1	1	1	1
$E(9)$	0	1	1	1	1	1	1	1	1	1

The duality may be explained for soft erosion, in this way.

For threshold 1, $E(1)$ is soft erosion. Here number of 1's of sub image having $3/3$ dimension is taken. If number of 1's are 0 then output = 1 otherwise output = 0.

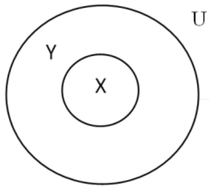
For threshold 9, $E(9)$ is soft erosion. If number of 1's in sub image = 9 then output = 0 otherwise output = 1.

It can be represented as in the following Tabular form.

Number of 1's of sub image

	9	8	7	6	5	4	3	2	1	0
$E(1)$	0	0	0	0	0	0	0	0	0	1
$E(9)$	0	1	1	1	1	1	1	1	1	1
	A	B								C

It can be divided into three blocks A, B, C. For example we take image X; its compliment with respect to 'U' is Y.



For the operations specified in blocks A, C. For $E(1)$, $E(9)$ we get same output.

For operations specified in block B. For $E(1)$ we get output=0, where as For $E(9)$ we get output=1. So if $E(1)$ is applied in X it will shrink, which is equal to applying $E(9)$ in Y, where Y will expand. So applying $E(1)$ in X is equal to applying $E(9)$ in Y which is complement of X.

In the same way applying $E(9)$ in X (it expands) is equal to applying $E(1)$ in complement of X, which is Y (it shrinks). So $E(1), E(9)$ are dual operations. In the same way we study the case of $E(2)$ and $E(8)$ It can be represented in tabular form as

Number of 1's of sub image

	9	8	7	6	5	4	3	2	1	0
$E(2)$	0	0	0	0	0	0	0	0	1	1
$E(8)$	0	0	1	1	1	1	1	1	1	1
	A		B						C	

Here Block A, Block C will provide same output, on operations $E(2)$ and $E(8)$. But Block B provides one output in $E(2)$ and another output in $E(8)$. So $E(2)$ is applied in X, it is equal to applying $E(8)$ in Y. if $E(8)$ is applied in X, it is equal to applying $E(2)$ in Y. In both the situations if one expands the other will shrink. So both $E(2)$ and $E(8)$ are dual operations. In the same way for $E(3)$, $E(7)$. It can be represented as

Number of 1's of sub image

	9	8	7	6	5	4	3	2	1	0
$E(3)$	0	0	0	0	0	0	0	1	1	1
$E(7)$	0	0	0	1	1	1	1	1	1	1
	A			B				C		

By way of same logic, we can say that $E(3), E(7)$ are duals. In the same way $E(4)$, $E(6)$

Number of 1's of sub image

	9	8	7	6	5	4	3	2	1	0
$E(4)$	0	0	0	0	0	0	1	1	1	1
$E(6)$	0	0	0	0	1	1	1	1	1	1
	Block A				Block B		Block C			

By way of same logic, we can say that $E(4), E(6)$ are duals. In the same way for $E(5)$.

Number of 1's of sub image

	9	8	7	6	5	4	3	2	1	0
$E(5)$	0	0	0	0	0	1	1	1	1	1
$E(5)$	0	0	0	0	0	1	1	1	1	1
	A					C				

Here we can say that $E(5)$ is self dual.

So, $E(1), E(9)$ are duals. $E(2), E(8)$ are duals. $E(3), E(7)$ are duals.
$E(4), E(6)$ are duals. $E(5), E(5)$ are duals.
$E(5)$ is self dual.

In general, $\left(E(m)\right)^d = E(10 - m)$ where $m = 1$ to 9
$m = 5$ … … self dual.

5.2. $\frac{5}{5}$ Structuring Element

The following table will give soft erosion values if structuring element size is 5/5 . Here the threshold values are starting from 1 to 25

Table 2 Number of 1's of sub image

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
$E(1)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
$E(2)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
$E(3)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
$E(4)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
$E(5)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
$E(6)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1
$E(7)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
…	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$E(16)$	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$E(17)$	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$E(18)$	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$E(19)$	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$E(20)$	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$E(21)$	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$E(22)$	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$E(23)$	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$E(24)$	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$E(25)$	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

The duality may be explained for soft erosion, in this way. In the case of $\frac{5}{5}$ sub image there will be 25 threshold values run from 1 to 25.

So erosions are $E(1), E(2), E(3), \ldots \ldots E(25)$, where $E(1)$ is erosion at threshold 1, $E(2)$ is erosion at threshold 2, $E(3)$ is erosion at threshold 3,… … $E(25)$ is erosion at threshold 25.

The $E(1), E(25)$ may be represented separately, in a tabular form as…

Number of 1's of sub image

	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
$E(1)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
$E(25$	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	A	B																								C

In Block A and C, The output is same, But for B output is quite opposite. (i.e.) if $E(1)$ is applied, the output is 0 ; if $E(25)$ is applied, output is 1.

So if $E(1)$ is applied on X, it will shrink, which is equal to expand of Y. But Y will be expanded, by applying $E(25)$ on Y. In the same way, if $E(25)$ is applied on X, it will expand, which is equal to shrinking of Y . the same shrinking of Y is achieved by applying $E(1)$ on Y.

So, we can conclude that $E(1)$ & $E(25)$ are duals. Now, we discuss $E(2)$ & $E(24)$, when they are applied, on the figure which is mentioned above.

First we represent $E(2)$ & $E(24)$ in the tabular form.

Number of 1's of sub image

	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
$E(2)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
$E(24)$	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	A		B																						C	

In this situation also, blocks A and C provide same output for $E(2)$ & $E(24)$ (in A the output = 0 and in C the output = 1). (But it is for both $E(2)$ & $E(24)$). When Block B is applied, for $E(2)$ the output = 1 where as for $E(24)$ the output = 0.

So, if $E(2)$ is applied on X, it will shrink. So, automatically Y will expand. It is equal to applying $E(24)$ on Y. In the same way , if $E(24)$ is applied on X, it will expand, which is equal to shrink of Y, which is equal to applying $E(2)$ on Y. So, both $E(2)$ & $E(24)$ will work in a dual manner. So $E(2)$ & $E(24)$ are duals.

By same logic we can prove that $E(3)$ & $E(23)$ are duals, $E(4)$ & $E(22)$ are duals, and so on.

So, $E(1), E(25)$ are duals. $E(2), E(24)$ are duals. $E(3), E(23)$ are duals.
$E(4), E(22)$ are duals. $E(5), E(21)$ are duals. $E(6), E(20)$ are duals.
$E(7), E(19)$ are duals. $E(8), E(18)$ are duals. $E(9), E(17)$ are duals.
$E(10), E(16)$ are duals. $E(11), E(15)$ are duals. $E(12), E(14)$ are duals.
$E(13)$ is self dual.

In general, $\left(E(m)\right)^d = E(26-m)$ where $m = 1$ to 25
$m = 13$ … … self dual.

5.3. $7/7$ Structuring Element

Now let $7/7$ sub image. There will be 49 threshold values, running from 1 to 49. So soft erosions are $E(1), E(2), E(3), E(4), E(5), \ldots \ldots E(49)$, Where $E(1)$ is erosion at threshold 1, $E(2)$ is erosion at threshold 2, $E(3)$ is dilation at threshold 3, $E(4)$ is erosion at threshold 4, … … …

$E(49)$ is erosion at threshold 49. The number of 1's may be (in a sub image having dimensions $7/7$), is 0 to 49.

The following table will give soft erosion values if structuring element size is 7/7. Here the threshold values are starting from 1 to 49.

umber of 1's in sub image $7/7$ dimensions

	49	48	47	46	45	44	43	42	41	… … …								10	9	8	7	6	5	4	3	2	1	0
E(1)	0	0	0	0	0	0	0	0	0									0	0	0	0	0	0	0	0	0	0	1
E(2)	0	0	0	0	0	0	0	0	0									0	0	0	0	0	0	0	0	0	1	1
E(3)	0	0	0	0	0	0	0	0	0									0	0	0	0	0	0	0	0	1	1	1
E(4)	0	0	0	0	0	0	0	0	0									0	0	0	0	0	0	0	1	1	1	1
E(5)	0	0	0	0	0	0	0	0	0	… … …								0	0	0	0	0	0	1	1	1	1	1
E(6)	0	0	0	0	0	0	0	0	0									0	0	0	0	0	1	1	1	1	1	1
E(7)	0	0	0	0	0	0	0	0	0									0	0	0	0	1	1	1	1	1	1	1
..	..	..	..	..	.	.	.	.	.	.	.	.	.	.	.	:	.	.	.	.	.	.	.	.	.	.	.	.
E(42)	0	0	0	0	0	0	0	0	1									1	1	1	1	1	1	1	1	1	1	1
E(43)	0	0	0	0	0	0	0	1	1									1	1	1	1	1	1	1	1	1	1	1
E(44)	0	0	0	0	0	0	1	1	1									1	1	1	1	1	1	1	1	1	1	1
E(45)	0	0	0	0	0	1	1	1	1	… … …								1	1	1	1	1	1	1	1	1	1	1
E(46)	0	0	0	0	1	1	1	1	1									1	1	1	1	1	1	1	1	1	1	1
E(47)	0	0	0	1	1	1	1	1	1									1	1	1	1	1	1	1	1	1	1	1
E(48)	0	0	1	1	1	1	1	1	1									1	1	1	1	1	1	1	1	1	1	1

E(49)	0	1	1	1	1	1	1	1	1									1	1	1	1	1	1	1	1	1	1	1

For threshold 1: $E(1)$ is soft erosion. Here, if sub image having dimension $7/7$, if number of 1's are ≥1. (i.e., minimum single 1 is maintained in sub image). Then output =1 other wise output = 0. For $E(49)$, the output = 1. For number of 1's ≥ 49 other wise output = 0. These $E(1)$ and $E(49)$ may be represented separately, in a tabular form as

	49	48	47	46	45	44	43	42	41	... ...	10	9	8	7	6	5	4	3	2	1	0
E(1)	0	0	0	0	0	0	0	0	0	... ...	0	0	0	0	0	0	0	0	0	0	1
E(49)	0	1	1	1	1	1	1	1	1	... ...	1	1	1	1	1	1	1	1	1	1	1
	A	B																			C

In Block A, C the output is same for $E(1)$ and $E(49)$.
But for B, the output is quite opposite. i.e., if $E(1)$ is applied the output =0. If $E(49)$ is applied the output =1.
So if $E(1)$ is applied on X, it will shrink which is equal to expand of Y.
But Y will be expanded, by applying $E(49)$ on Y.
In the same way, if $E(49)$ is applied on X. It will expand, which is equal to expansion of Y, the same shrinking of Y is achieved, by applying $E(1)$ on Y. So, we can conclude that, $E(1)$ and $E(49)$ are duals.
By means of the same logic we can say that $E(2)$ and $E(48)$ are duals, $E(3)$ and $E(47)$ are duals, $E(4)$ and $E(46)$ are duals and so on…

So, $E(1), E(49)$ are duals. $E(2), E(48)$ are duals. $E(3), E(47)$ are duals.
$E(4), E(46)$ are duals. $E(5), E(45)$ are duals. $E(6), E(44)$ are duals.
$E(7), E(43)$ are duals. $E(8), E(42)$ are duals. $E(9), E(41)$ are duals.
$E(10), E(40)$ are duals. $E(11), E(39)$ are duals. $E(12), E(38)$ are duals.
$E(13), E(37)$ are duals. $E(14), E(36)$ are duals. $E(15), E(35)$ are duals.
$E(16), E(34)$ are duals. $E(17), E(33)$ are duals. $E(18), E(32)$ are duals.
$E(19), E(31)$ are duals. $E(20), E(30)$ are duals. $E(21), E(29)$ are duals.
$E(22), E(28)$ are duals. $E(23), E(27)$ are duals. $E(24), E(26)$ are duals.
$E(25)$ is self dual.
In general, $\underline{(E(m))^d = E(50-m)}$ where $m = 1$ to 49. $m = 25$ ... ... self dual.

5.4. $9/9$ Structuring Element

Now let $9/9$ sub image. There will be 81 threshold values, running from 1 to 81.
So soft erosions are $E(1), E(2), E(3), E(4), E(5), \ldots \ldots E(81)$, Where $E(1)$ is erosion at threshold 1, $E(2)$ is erosion at threshold 2, $E(3)$ is erosion at threshold 3, $E(4)$ is erosion at threshold 4, ... $E(81)$ is erosion at threshold 81. The number of 1's may be (in a sub image having dimensions $9/9$), 0 to 81.

Now by means of same logic provided for $3/3$, $5/5$, $7/7$, we can prove that
So, $E(1), E(81)$ are duals. $E(2), E(80)$ are duals. $E(3), E(79)$ are duals.
$E(4), E(78)$ are duals. $E(5), E(77)$ are duals. $E(6), E(76)$ are duals.
$E(7), E(75)$ are duals. $E(8), E(74)$ are duals. $E(9), E(73)$ are duals.
$E(10), E(72)$ are duals. $E(11), E(71)$ are duals. $E(12), E(70)$ are duals.
$E(13), E(69)$ are duals. $E(14), E(68)$ are duals. $E(15), E(67)$ are duals.
$E(16), E(66)$ are duals. $E(17), E(65)$ are duals. $E(18), E(64)$ are duals.
$E(19), E(63)$ are duals. $E(20), E(62)$ are duals. $E(21), E(61)$ are duals.
$E(22), E(60)$ are duals. $E(23), E(59)$ are duals. $E(24), E(58)$ are duals.
$E(25), E(57)$ are duals. $E(26), E(56)$ are duals. $E(27), E(55)$ are duals.
$E(28), E(54)$ are duals. $E(29), E(53)$ are duals. $E(30), E(52)$ are duals.
$E(31), E(51)$ are duals. $E(32), E(50)$ are duals. $E(33), E(49)$ are duals.
$E(34), E(48)$ are duals. $E(35), E(47)$ are duals. $E(36), E(46)$ are duals.
$E(37), E(45)$ are duals. $E(38), E(44)$ are duals. $E(39), E(43)$ are duals.
$E(40), E(42)$ are duals. $E(41)$ is self dual.
In general, $\underline{(E(m))^d = E(82-m)}$ where $m = 1$ to 81
$m = 41$ ... ... self dual.
By means of same logic we can prove duality for $11/11$, $13/13$, ….

5.5. W/W Structuring Element

Now for general case W/W , where w is sub image size, we can show that

$E(1), E(w^2)$ are duals. $E(2), E(w^2-1)$ are duals. $E(3), E(w^2-2)$ are duals.
$E(4), E(w^2-3)$ are duals. $E(5), E(w^2-4)$ are duals. $E(6), E(w^2-5)$ are duals.
$E(7), E(w^2-6)$ are duals. $E(8), E(w^2-7)$ are duals. $E(9), E(w^2-8)$ are duals.
$E(10), E(w^2-9)$ are duals. $E(11), E(w^2-10)$ are duals. $E(12), E(w^2-11)$ are duals.

… … …

… … …

$\mathrm{E}\left(\frac{w^2+1}{2}-5\right), \mathrm{E}\left(\frac{w^2+1}{2}+5\right)$ are duals. $\mathrm{E}\left(\frac{w^2+1}{2}-4\right), \mathrm{E}\left(\frac{w^2+1}{2}+4\right)$ are duals.
$\mathrm{E}\left(\frac{w^2+1}{2}-3\right), \mathrm{E}\left(\frac{w^2+1}{2}+3\right)$ are duals. $\mathrm{E}\left(\frac{w^2+1}{2}-2\right), \mathrm{E}\left(\frac{w^2+1}{2}+2\right)$ are duals.
$\mathrm{E}\left(\frac{w^2+1}{2}-1\right), \mathrm{E}\left(\frac{w^2+1}{2}+1\right)$ are duals. $\mathrm{E}\left(\frac{w^2+1}{2}\right)$ … … self dual.

In general,

- $\left(E(m)\right)^d = E(w^2+1-m)$ where $m = 1$ to w^2.

$m = \frac{w^2+1}{2}$ … … self dual.

VI. CONCLUSION

The dual of soft morphological operations are having wide range of applications. So in this paper Dual of soft erosion in multi scale environment is discussed in detail. The concept of self dual is also discussed.

REFERENCES:

[1] J. Serra-Image Analysis and Mathematical Morphology.

[2] A$_2$. Matheron - Mathematical Morphology

[3] Robert. M. Haralick, Stanley R. Sternberg, Xinhua Zhuang (July 1987) *Image Analysis using Mathematical Morphology*, IEEE Transactions on PAMI, Vol. 9, No. 4.

[4] A$_4$. H. J. A. M. Heijmans and C. Ronse. (1990) *The Algebraic Basis of Mathematical Morphology. 1. Dilations and Erosions*, IEEE Transactions on Computer Vision, Graphics and Image Processing

[5] H$_1$. Michel A. Zmoda and Louis. A.Tamburino – Efficient algorithms for the soft morphological operators. --- IEEE Tr. of Pami – Vol. 18 - No.11 November 1996.

[6] A generalized approach for handling equality and duality properties in soft morphology…PhD thesis of KOMPELLA VENKATA RAMANA

[7] Shih, F. Y; Pu. C. C; ---Analysis of the properties of soft morphological filtering using threshold decomposition.-- IEEE Tr. of S.P. – Vol. 43 - No.2 November 1995.

[8] Pu. C. C; - Threshold decomposition algorithm for gray scale soft morphological operations --- II –Erosion;--- 5th International conference on Image processing and its applications – 1995.

[9] Pauli Kuosmanen & Jaakko Astola. – Soft morphological filtering --- Journal of mathematical imaging and vision 1995.

[10] Bouaynaya,N; Charif-chef caaouni.M, schonfeld, D. M- idempotent & self-dual morphological filters. IEEE Trans. of Pami – Vol. 34. No-4. -2012---pages 805 to 813.

[11] Lei,T; Fan,Y.- Noise graditent reduction based on morphological dual operators. IET Trans. of Image Processing. – Vol. 05. No-1. - -2011.---pages 1-17.

[12] Michel A. Zmoda and Louis. A.Tamburino – Efficient algorithms for the soft morphological operators. --- IEEE Tr. of Pami – Vol. 18 - No.11 November 1996

[13] Pauli Kuosmaneri; Lasse Koskinen; Jaakko T. Astola; -- Analysis and extensions of soft morphological filters. Proceedings of SPIE – 1902, 41 (1993)---January 2005.

[14] Pauli Kuosmane N; Lasse Koskinen; Jaakko Astola;---The connection between generalized soft morphological operations and stack filters. --- IEEE winter workshop on nonlinear digital signal processing 1993.

[15] Shih, F. Y; Pu. C. C; ---Analysis of the properties of soft morphological filtering using threshold decomposition.-- IEEE Tr. of S.P. – Vol. 43 - No.2 November 1995.

[16] Pu. C. C; - Threshold decomposition algorithm for gray scale soft morphological operations --- II –Erosion;--- 5th International conference on Image processing and its applications – 1995.

[17] Pauli Kuosmanen & Jaakko Astola. – Soft morphological filtering --- Journal of mathematical imaging and vision 1995.

[18] Gasteratos, A; --- Realization of soft morphological filters. IEEE proceedings on CDS – June 1998.

[19] Michael A. Zmuda – Stochastic algorithm for approximating soft morphological operators. --- Journal of Optical engineering –Opt. Eng. – 40, 2001.

[20] Zhao Chun Hui; --- Optimization design of soft morphological filters, based on improving genetic algorithm. -- Proceedings of the Int. Conf. on Neural networks and signal processing – Vol. 1 – 2003.

[21] Pertti T. Koi Visto; Heikki Huttunen; Pauli Kuosmanen; --- Optimal compositions of soft morphological filters.

[22] M. Varda Voulia, I. Andreadis and Ph. Tsali Des.--- Soft morphology in the HSB colour space definitions and implementation.

[23] A. Gasteratos, I. Andreadis and Ph.Tsalides --- soft morphological structuring element decomposition. – Mathematical morphology and its applications to image and signal

[24] Koivisto, P; Huttunen; H. Kuosmanen, P;--- Optimal soft morphological filtering under breakdown probability constraints --- IEEE Int. Sym. On C & S – 1996.

[25] Lipeng Wang; Hanbin Wang; -- Implementation of a soft morphological filter based on GPU frame work – Int. Conf. on Bioinformatics and Biomedical Engineering 2011.
of IEEE – EURASIP workshop, Italy – 2003.

[26] Zhen Ji; Zhong Ming; Qi Li and Qinghua Wu; -- Reducing periodic noise using Soft morphology filter. – Journal of electronics, China – March 2004.

[27] Zhen Ji; Huilian Liao; Xijun Zhang; Wu, Q.H. – Simple and efficient Soft morphological filter in periodic noise reduction.—IEEE Region 10 Conference – TENCON – 2006.

[28] PETROS; MARAGOS; Pattern spectrum and multi scale shape representation. - IEEE Trans. of Pami – Vol. 11. No.-7. July -1989

[29] MING – HUA CHEN &PING – FAN.YAN. --- A Multi scaling approach based on morphological filtering. - IEEE Trans. of Pami – Vol. 11. No-7. July -1989

[30] KUN WANG; JIANHUA WU; LIQUN GAO; ZHAOYU PIAN; LI GUO--- Magnetic resonance images, edge detection , based on multi – scale morphology. --- INT. CONF. ON – COMPLEX MEDICAL ENGINEERING – CME – IEEE – 2007

[31] KUN WANG; LIQUN GAO; ZHENGANG SHI; LI GUO; ZHAOYA PIAN. – An edge detection algorithm based on “multi scale morphology” --- IEEE CONF. ON INDUSTERIAL ELECTRONICS & APPLICATIONS. – ICIEA – 2007

[32] Kim Wang; Jianhga Wu; Zhaoyu pian: --- Edge detection algorithm for magnetic resonance images based on multi – scale morphology. Int. Conf. on Control and Automation 2007.

[33] Zeng Ping Ping; Zeng Binyang --- A new algorithm based on multi scale order morphology for gray image edge detection --- 8th International conference on electronic measurement and instruments --- ICEMI -- 2007.

[34] Zhenhua Li; Ying ping yang; Wei jiang --- Multi scale morphologic tracking approach for edge detection --- 4th International Conf. on image and graphics – ICIG – 2007.

[35] Paul.T.Jackway, Mohamed Deriche. – Scale Space Properties of the multi scale Morphological Dilation – Erosion.- IEEE Tr. of Pami – Vol. 18 - No.1 January 1996.

ABOUT AUTHOR

The author **Kompella Venkata Ramana** has done his B.E(E.C.E) and M.E(COMPUTER ENGINEERING) from ANDHRA UNIVERSITY, VISAKHAPATNAM, INDIA. He has started his carrier as LECTURER in N.I.T. (W). Later he shifted to ANDHRA UNIVERSITY. At present he is working as ASSOCIATE PROFESSOR in the department of computer Science &Systems Engineering in ANDHRA UNIVERSITY. His areas of interest are Image Processing, Formal languages and Automata theory, compiler Design and Systems Programming. He has written books on the above areas. He has experience of more than twenty five years in teaching and guided more than one hundred Thesis in M.Tech. Level ,majority thesis in Image Processing.