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GPS Positioning Algorithm, its Errors and Solution

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Abstract— GPS, space-centred satellite with 24 basic satellites carrying atomic clocks navigation that deliver location and time information prudently. If three satellites are available, three spheres are known whose intersection consists of two points. The problem is to solve the three sphere equations. The receiver clock is not perfectly in sync with the satellite clock. To solve the inaccurate timing one more satellite need to be added, which brings a prodigious difference by several kilometres on the positioning. Two further problems arise when GPS is deployed, the conditioning system of equations and the transmission speed of the signals. For accuracy, we increased the satellites from four to seven. Our goal was to solve the least squares system of seven equations in four unknown variables (x, y, z, d) using Gauss-Newton iteration method. We used two types of satellites, tightly and loosely bunched. Results indicated that system becomes ill-conditioned when satellites are bunched closely in the sky.

Keywords—GPS Errors, Least Squares Systems, Gauss-Newton Iteration Method

I. INTRODUCTION

GPS (Global Positioning System) contains 24 satellites carrying atomic clocks which orbit the earth at an altitude of 20,200 km [4]. These satellites transmit carefully synchronized signals from space to GPS receivers on earth. These receivers take the signals and translate them via arithmetic and algebra to determine the (x, y, and z) coordinates of the receiver. Discussion of GPS terminology and positioning algorithms can be found, for example, in [4], [9], and [12].At a given prompt, the receiver gathers the signals from a satellite and determines the transmission time (t), the aggregate time between the signal and the receiver. In theory, the speed of the signal is accurately taken as the speed of light. To find the receiver to the satellite multiply the transmission time by the speed of light, putting the receiver on the surface of the sphere centred at the satellite position with a radius equal to the distance from the receiver to the satellite. If three satellite are used, then there are three spheres with the receiver at their intersection. However, the clocks in the receivers incline to be of truncated exactitude which means that using only three satellites can result in position errors of several kilometres. Adding a fourth satellite to this problem fixes the error but on the other hand complicates the arithmetic form to solve the equation with four equations and four unknowns. The system of equations shown below are the simplified form of the GPS problem for four satellites. The position of each satellite in the atmosphere is denoted by (Ai, Bi, Ci) in which the transmission time is denoted by ti. "d" is defined to be the difference between the synchronized time on the four satellites and the receiver clock. Then the intersection of the spheres, denoted as (x, y, z),

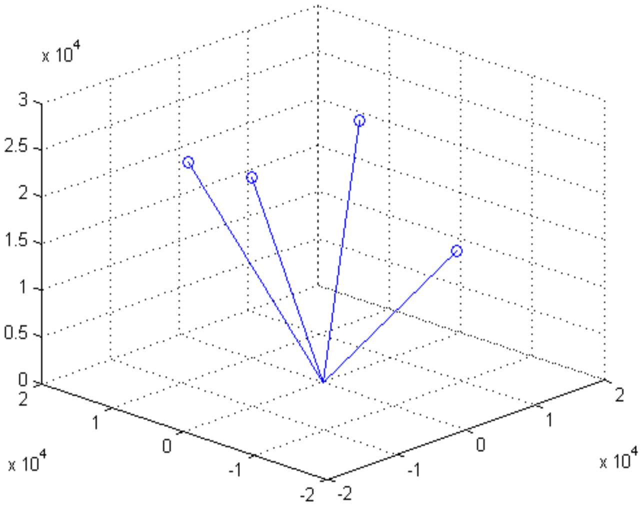
- $r1(x, y, z, d) = \sqrt[2]{(x - A1)2 + (y - B1)2 + (z - C1)2} - c\,(t1 - d) = 0$
- $r2(x, y, z, d) = \sqrt[2]{(x - A2)2 + (y - B2)2 + (z - C2)2} - c\,(t2 - d) = 0$
- $r3(x, y, z, d) = \sqrt[2]{(x - A3)2 + (y - B3)2 + (z - C3)2} - c\,(t3 - d) = 0$
- $r4(x, y, z, d) = \sqrt[2]{(x - A4)2 + (y - B4)2 + (z - C4)2} - c\,(t4 - d) = 0$

II. SOLUTION FOR THE SYSTEM OF EQUATION

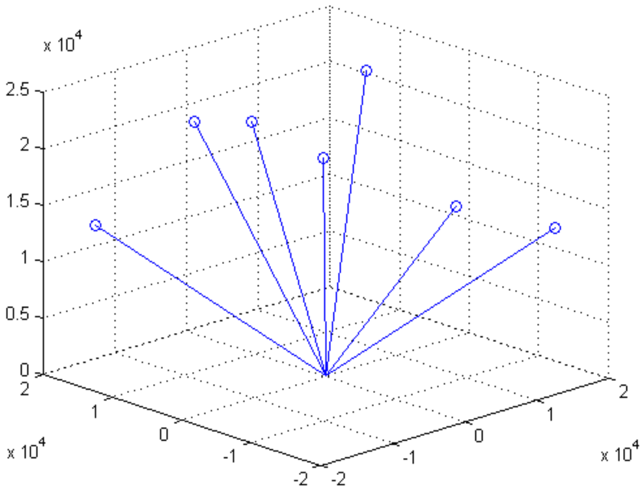
We tried to solve the system of equation by using both Multivariate Newton's Method as well as algebra, the results became accurately same on both cases. Using Multivariate Newton's Method, to find the receiver's position (x, y, z) near the earth and difference between the coordinated time on the (four) satellite clocks and the earth-constrained receiver clock d for known, simultaneous satellite positions (15600, 7540, 20140), (18760, 2750, 18610), (17610, 14630, 13480), (19170, 610, 18390) in Km, and measured time intervals 0.07074, 0.07220, 0.07690, and 0.07242 in seconds, respectively. We then set the initial vector to be (x0, y0, z0, d0) = (0, 0, 6370, 0). The approximate results are (x, y, z) = (-41.77271, -16.78919, 6370.0596) kms and d = - 3.201566 * 10-3 seconds. And using algebra, first we removed the quadratic variables by subtracting the rest of the equations from the first one. Next we are left with three equations with four unknowns, the quadratic variables will be cancelled. Then in terms of d we solved x, y and z by using the determinant equations. The two coordinates found will be (x, y, z) = (-41.772709570837364,-16.789194106526850, 6370.059559223344) and (-39.747837348142937, -134.2741443606658, -9413.624553735754). Then in order to distinguish which one to choose, we calculated the nom of each coordinates. The nom of the first coordinates is 6370.218648080797 and the second is 9414.666041613707, but only the first one is closely related to the radius of the earth. So the first coordinates is the solution and d = -0.003201566 (- 3.201566 * 10-3) seconds.

III. TESTING THE CONDITIONING OF THE GPS PROBLEM

In order test the satellite conditioning we first defined the satellite positions (X_i, Y_i, Z_i) from the spherical coordinates (ρ, ϕ, θ_i). $X_i=\rho\cos\phi_i\cos\theta_i$, $Y_i=\rho\cos\phi_i\sin\theta_i$. $Z_i=\rho\sin\phi_i$. Where $\rho=26570$ kilometres is fixed while $0\leq \phi_i \leq \pi/2$ and $0 \leq \theta_i \leq 2\pi$ for i= 1… 4 are chosen arbitrarily. The ϕ coordinate is restricted so that the four satellites are in the upper hemisphere. For the initial vector (x_0, y_0, z_0, d_0) = (0, 0, 6370, 0.0001). We defined $t= d + R_i/c$, the satellite ranges to be R1= and calculated the error magnification factor of by dividing change in position by the speed of light and multiplying it by the change in transmission time. The atomic clocks are accurate up to 10 nanoseconds. The results found for 4 satellites with $\phi=[3\pi/8,\pi/4,3\pi/8,\pi/4]$ and $\theta=[\pi/8,\pi/2,\pi,3\pi/2]$ defining the satellite's positions is:



Next we added 3 more satellites to the previous 4 satellites to reduce the position error and the condition number. We used Gauss Newton method to solve the system of equation which is as follows: The initial factor values of (x, y ,z, d)= (0, 0, 6370, 0.0001), $\phi=[3\pi/8,\pi/4,3\pi/8,\pi/4,\pi/6,\pi/3,\pi/6]$ & $\theta=[\pi/8,\pi/2,\pi,3\pi/2,3\pi/4,5\pi/4,7\pi/4]$



IV. CONCLUSION

We concluded that by adding more number of satellites to the system, the average change in position becomes very smaller compared to only using four satellites in our case, and significantly smaller than using four clustered satellites. The maximum error magnification factor we happened to find with seven satellites is close to the minimum found with four satellites initially. As we have seen the condition number of the seven satellites is much smaller than using four satellites (Condition No of seven satellites < Condition No of four satellites). Clearly when we use seven clustered satellites would produce the smallest error. So from this we can deduce that by adding more satellites the error can be reduced as much. As a conclusion more satellites reduces the error by thousands of meters.

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