



## Unified GSA/IWO/WDO Optimization Algorithm for Calibration of Antenna Array Systems

A. A. Yahia\*, H. M. Elkamchouchi

Department of Electronics & Communications, Faculty of Engineering, Alexandria University,  
Egypt

**Abstract**– High performance array signal processing algorithms are required for calibration of real antenna array systems. Nature-inspired optimization algorithms are efficient tools to achieve an accurate real antenna array calibration of the output data for simultaneously optimizing the array pattern synthesis and null control with minimum beam width. The proposed algorithm is called unified gravitational search algorithm, invasive weed optimization and wind driven optimization (GSA/IWO/WDO) optimization algorithm. This unique type of combined nature-inspired optimization algorithm could offer various advantages of each component algorithm separately. These advantages achieve a sufficient enhancement of side lobe level (SLL) minimization, interference minimization and beam width minimization. The simulation results could improve the whole performance of the received real antenna array for pattern synthesis resolution with null control on constraint of beam width minimization.

**Keywords**– Array pattern synthesis; gravitational search algorithm (GSA); invasive weed optimization (IWO); wind driven optimization (WDO); combined GSA/IWO/WDO algorithm; side lobe level (SLL)

### I. INTRODUCTION

Antenna array calibration is one of the most important procedures for an accurate estimation of real antenna arrays' electrical parameters. Recently, nature inspired optimization algorithms appears over traditional approaches to enhance the performance of the calibration techniques to reduce the various modeling antenna array errors. Self-calibration could use cost function according to nature inspired optimization algorithms. The main factors of the self-calibration's development are the composing of the required cost function and choice of the appropriate optimization procedure [1]. Using new modified calibration techniques, antenna array synthesis can provide side lobe level (SLL) minimization and null steering vector control [2]. Metaheuristic optimization algorithms have been proposed to enhance estimation's accuracy of the array parameters (e.g. antenna array pattern synthesis and null steering control). General purpose metaheuristic methods can be categorized into nine different groups: biology-based, physics-based, social-based, music-based, chemical-based, sport-based, mathematics-based, swarm-based and hybrid methods [3]. Hybrid methods are proposed as robust and efficient techniques for optimization problems. One of the newest methods of them is the hybrid invasive weed optimization and wind driven optimization (IWO/WDO) [2]. It could be applied to effectively synthesize antenna array patterns for SLL minimization and null control.

IWO is a plant intelligent optimization algorithm which offers good exploration and diversity properties [4]. However, this type of optimization methods is not always effective for large space problems and there is a possibility of local optima trapping [2].

WDO is a swarm-based optimization algorithm with fairly deliberate and to the point movement of the air parcel in the earth's atmosphere [5].

Unified gravitational search algorithm, invasive weed optimization and wind driven optimization (GSA/IWO/WDO) optimization algorithm is proposed to solve the problems of local optima trapping and to enhance the effectiveness of the output solution for the antenna array synthesis over other hybrid methods (e.g. IWO/WDO optimization algorithm). The additive advantage of the proposed algorithm is the high exploration for optimized solution due to the usage of the iterative variant acceleration between the seeds according to the GSA's criteria.

GSA is considered as a physics-based optimization algorithm which is based on the law of gravity, mass interaction and the iterative variant acceleration between the seeds [6]. In the unified technique, WDO technique is utilized more effectively by replacing of the gravitational constant coefficient located at the iterative velocity expression with the iterative variant acceleration between located seeds according to GSA criteria and subsequent processing is done by IWO algorithm [2]. The unified GSA/IWO/WDO optimization algorithm achieves an enhancement of SLL minimization with constraint beam width minimization to minimize interference and null control through antenna array synthesis.

This paper is organized as follows [3]. In section 2, the virtual transformation of the real uniform semi-circular array (RUSCA) to another virtual uniform linear array (VULA). Section 3 shows a detailed explanation of new unified GSA/IWO/WDO optimization algorithm. In section 4, a multi-objective cost function for antenna array synthesis and null control is discussed. The simulation results are discussed in details in section 5. Finally, section 6 concluded the new unified optimization technique and its advantages for antenna array optimization.

## II. VIRTUAL TRANSFORMATION OF REAL ANTENNA ARRAY

High performance antenna array systems are usually calibrated at every specified period of time. The pilot calibration is one of the most necessary types of antenna array calibrations. In this type of calibration, the sources have well known parameters (i.e. known locations and directions) which are used to estimate array errors and uncertainties by solving of a number of complex nonlinear equations [1]. The first step of the accurate array calibration is the virtual transformation of the real antenna array to a virtual one. The virtual transformation is a very useful tool to compensate various electromagnetic effects (e.g. mutual coupling effects between real antenna elements). The outputs of such transformations are virtual transformed voltages at a uniform linear antenna array from induced voltages at a real uniform antenna array. The following equation describes a best fit transformation matrix required to transform RUSCA to a VULA:

$$\overline{T} = \overline{A}_v(\varphi_q) \overline{A}_r^H(\varphi_q) \left\{ \overline{A}_r(\varphi_q) \overline{A}_r^H(\varphi_q) \right\}^{-1} \quad (1)$$

Where the superscript  $H$  represents the conjugate transpose of a complex matrix [7].  $\overline{A}_r(\varphi_q)$  and  $\overline{A}_v(\varphi_q)$  are the real and virtual array manifolds respectively. This step is processed after the sectorization of the field of view (i.e. azimuth angles  $\varphi$ ) to a number of predefined sectors  $[\varphi_q, \varphi_{q+1}]$  for  $q = 0, 1, \dots, Q-1$  where  $Q$  is the number of divided sectors of the field of view  $[-90^\circ, 90^\circ]$  [8]. The predefined sector is defined as :

$$[\varphi_q] = [\varphi_q, \varphi_q + \varphi, \dots, \varphi_{q+1}] \quad (2)$$

Where the angle  $\varphi$  represents the step size which in this case study equals to  $1^\circ$  and  $Q=1$ .

So, the real and virtual transformed array manifold matrix can be written as:

$$[A(\varphi_q)] = [a(\varphi_q), a(\varphi_q + \varphi), \dots, a(\varphi_{q+1})] \quad (3)$$

$$\overline{A}_v(\varphi) = \overline{T} \overline{A}_r(\varphi) \quad (4)$$

Where  $a(\varphi_q)$  represents the steering vector according to one signal source located at  $(\varphi_q)$  in the far field region of the RUSCA [8], such that

$$a_{n-RUSCA}(\varphi_q) = e^{j\psi_n(\varphi_q)} \quad , \quad \psi_n = 2\pi r \cos(\varphi_q - (n-1)\varphi_0) / \lambda \quad (5)$$

Where  $n$  indicates to the index of the located antenna element within the real uniform circular array (RUCA) configuration,  $r$  is the radius of RUCA,  $\varphi_0 = 2\pi/N$  is the angle between adjacent elements,  $N$  is the number of the elements of the RUCA which equals to the double of the number of elements of the RUSCA and  $\lambda$  is the wavelength. Also, the steering vector for the ULVA can be written as follows:

$$a_{n-ULVA}(\varphi_q) = e^{j\psi_n(\varphi_q)} \quad , \quad \psi_n = \frac{2\pi}{\lambda} (m-1)d \cos(\varphi_q) \quad (6)$$

Where  $m$  indicates to the index of the virtual element within the VULA and  $d$  is the inter-element distance between two adjacent elements of the VULA.

So, the calibrated voltages can be written as

$$\overline{X}_C = \overline{T} \overline{X}_m \quad (7)$$

Where  $\overline{X}_m$  is measured voltage vector at the loads of the real elements of the RUSCA [8].

## III. UNIFIED GSA/IWO/WDO GLOBAL OPTIMIZATION ALGORITHM

In this unique algorithm, it is exciting to combine three different optimization algorithms (i.e. GSA, IWO and GSA). This algorithm is originally based on the conventional IWO [3]. However, the colonization process is constrained by the scientific criteria of GSA and WDO efficient optimization algorithms. The new algorithm is detailed as follows:

Step 1 : Search area definition.

Step 2 : A finite number of seeds are being randomly positioned over the initialized search area.

Step 3 : Now, adaptation is provided by assuring of fitness value for each seed according to the cost function. Then, the seeds are permitted to be grown to flowering weeds.

Step 4 : Then, ranking of the generated weeds according to the fitness values and find their positions in the colony [IWO-WDO].

Step 5 : The criteria of the WDO and GSA is used to find an accurate expression for the velocity and position of the seed which are estimated by the following functions:

$$u_{new} = (1 - \alpha)u_{cur} + a_{cur}(t)x_{cur} + RT \left[ \frac{1}{z} - 1 \right] (x_{opt} - x_{cur}) + \frac{cu_{cur}^{other \ dim}}{z} \quad (8)$$

$$x_{temp} = x_{cur} + u_{new} \Delta t \quad (9)$$

Where,  $z$  represents the index number of the pressure (fitness) value at its current location [2]. As in WDO,  $\alpha$  is related to the friction coefficient and  $u_i$  represents the current velocity vector of the wind. In the new algorithm, the gravitational constant ( $g$ ) defined at WDO optimization algorithm is replaced by the iterative acceleration ( $a_{cur}$ ) of the

$i^{th}$  seeds originally defined at the GSA optimization algorithm by Eq.(10) [9].  $RT$  and  $c$  are defined as a constant and Coriolis constant respectively.  $x_i$ ,  $x_{opt}$  and  $x_{i+1}$  are assumed to be current, optimum and new positions of the seeds respectively [2]. Also,  $\Delta t$  represents the incremental time, is assumed to be 1 [2]. The main additive advantage of the new unified algorithm is the strength of the variant acceleration ( $a_{cur}$ ) at every iteration in contrast to the gravitational constant ( $g$ ) which is defined at the classical WDO. The iterative acceleration of the  $i^{th}$  seed at iteration ( $t$ ) is calculated by GSA optimization algorithm as below [9]:

$$a_{cur}^d(t) = \frac{F_i^d(t)}{M_{ii}(t)}, \quad (10)$$

$$F_i^d(t) = \sum_{i=1, j \neq 1} rand_j F_{ij}^d(t), \quad (11)$$

$$F_{ij}^d(t) = G(t) \frac{M_{pi}(t)M_{aj}(t)}{\|x_i(t), x_j(t)\|_2 + \epsilon} (x_j^d(t) - x_i^d(t)), \quad (12)$$

$$G(t) = G_o e^{\left(\frac{\beta t}{T}\right)} \quad (13)$$

$$M_{ai} = M_{pi} = M_{ii} = M_i \quad ; \quad i = 1, 2, \dots, K,$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^K m_i(t)} \quad (14)$$

$$m_i(t) = \frac{fit_i(t) - worst_i(t)}{best(t) - worst(t)}, \quad (15)$$

$$best(t) = \min_{j \in \{1, \dots, N\}} fit_j(t), \quad (16)$$

$$worst(t) = \max_{j \in \{1, \dots, N\}} fit_j(t) \quad (17)$$

All previous equations are defined in details by GSA optimization algorithm [9]. Where,  $F_i^d(t)$  is assumed to be the total incident force to  $i^{th}$  seed.  $rand_j$  represents a random number located in the interval [0, 1].  $F_{ij}^d(t)$  represents the acting pressure force from  $i^{th}$  seed from  $j^{th}$  one at the  $d^{th}$  dimension.  $\|x_i(t), x_j(t)\|_2$  is defined as the Euclidian distance between two  $i^{th}$  and  $j^{th}$  seeds at iteration ( $t$ ) [GSA1].  $G(t)$  represents the computational gravitational constant at the iteration ( $t$ ).  $G_o$  is set to 100,  $\beta$  is set to 20 and  $T$  indicates to the total number of iterations [GSA1].  $M_{ai}$ ,  $M_{pi}$  and  $M_{ii}$  represent the active, passive and inertia masses of the  $i^{th}$  seed.  $fit_j(t)$  indicates to fitness evaluation of the  $j^{th}$  seed at iteration ( $t$ ).  $best(t)$  and  $worst(t)$  indicate to the best and worst fitness at iteration  $t$  respectively [8].

Step 6: Due to the processing of the weeds' flowering, a new seed is produced according to the calculated fitness value [2],

$$S = floor \left[ S_{min} + \left( \frac{f - f_{min}}{f_{max} - f_{min}} \right) \cdot S_{max} \right] \quad (18)$$

Where,  $S_{min}$  and  $S_{max}$  represent the minimum and the maximum number of the produced seeds respectively [2].  $f$ ,  $f_{min}$  and  $f_{max}$  are defined as the current, minimum and maximum cost values respectively [2].

Step 7: Next, ( $S$ ) produced seeds are distributed normally over the search interval according to the classic IWO optimization algorithm [2]. The Standard Deviation (SD) of the new dispersion is expressed as follows [3]:

$$\sigma_{new} = \left( \frac{iter_{max} - iter}{iter_{max}} \right)^n \cdot (\sigma_{initial} - \sigma_{final}) + \sigma_{final} \quad (19)$$

Where,  $iter$ ,  $iter_{max}$  and  $n$  represent the number of current, maximum iterations and the nonlinear modulation index respectively.  $\sigma_{new}$ ,  $\sigma_{initial}$  and  $\sigma_{final}$  are defined as the new, initial and final standard deviations respectively [2].

Step 8: Then, the new seed's position is calculated by the following equation:

$$x_{new} = x_{temp} + rand \cdot \sigma_{new} \quad (20)$$

Where,  $rand$  is a distributed uniformly random variable which is located at the interval [0, 1].

Step 9: The new produced seeds are ranked with their parents according to the fitness value. Using the concept of competitive exclusion, removal of all lower ranking seeds to reach the maximum number of weeds in the colony [2].

Step 10: Generation of new seeds based on the existing seeds' ranking in the colony [2]. This generation process is continued until reaching of the global optimum within the maximum number of iterations according to the cost function criteria. It is processed by the repeating of the steps 4 to 9 [2].

#### IV. MULTI-OBJECTIVE COST FUNCTION OPTIMIZATION

In this paper, the virtual antenna array is an even number of (2M) virtual antenna elements which is symmetrically located along the x-axis as shown at the right side of Figure(1). The array factor (AF) of the ULVA can be written as follows:

$$AF(\theta) = \sum_{n=1}^{2M} a_n \cos\left(\frac{2\pi}{\lambda} d_n \cos \theta + \phi_n\right) \quad (21)$$

Where  $a_n$  and  $\phi_n$  represent the amplitude and phase coefficients respectively of the transformed voltages virtually at the VULA from the RUSCA.  $\lambda$  and  $d_n$  represent the wavelength and inter-element distance between two virtual elements of the VULA respectively [2].

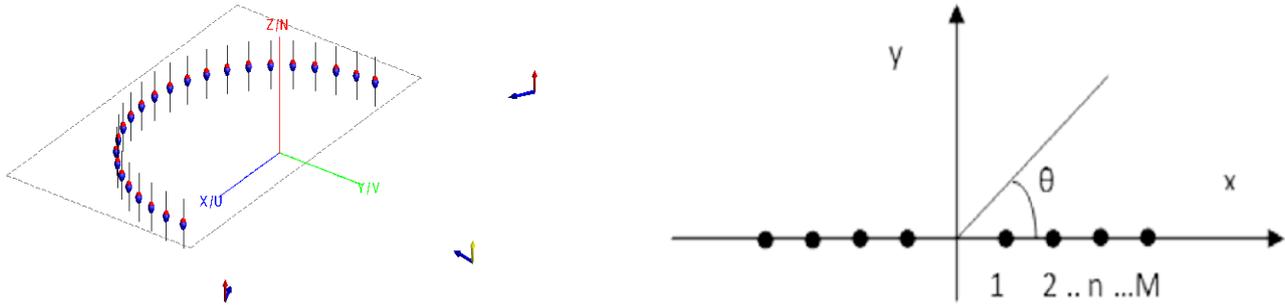


Figure. 1 Configurations of RUSCA (left side) and VULA (right side)

For calibration of the shown RUSCA at the left side of Figure (1), it is required to form an expression of a multi-objective cost function which can be optimized to achieve simultaneously minimum side lobe level (SLL) with minimum beam width and nulls' control at the desired interference directions. The multi-objective fitness function is combined of two different cost functions as shown in the following equations:

$$Fitness = Fitness I + Fitness II \quad (22)$$

$$Fitness I = \sum_{i=1}^M \frac{1}{\Delta\theta_i} \int_{\theta_{li}}^{\theta_{ui}} |AF_{\bar{x}}|^2 d\theta + \epsilon \cdot \max\{0, |BW_c - BW_d| - 1\} \quad (23)$$

Where  $Fitness I$  is a cost function which can be optimized to get minimum SLL and minimum beam width simultaneously [10].  $M$  represents the number of suppressed SLL<sup>s</sup>.  $\Delta\theta_i = \theta_{ui} - \theta_{li}$  and  $[\theta_{ui} - \theta_{li}]$  is the region of SLL suppression.  $\bar{x}$  represents the optimizing positions' vector of the elements through the representative new VULA [2].  $\epsilon$  is a constant which is equals to  $10^6$ .  $BW_c$  and  $BW_d$  represent the calculated and desired beam widths respectively [2].

$$Fitness II = \sum_{k=1}^K |AF_{\bar{x}}(\theta_k)|^2 + \epsilon \cdot \sum_{k=1}^K \max\{0, AF_{dB}(\theta_k) - C_{dB}\} \quad (24)$$

Where  $Fitness II$  is another cost function which is optimized also to minimize the interference effect by placing nulls in the desired directions and get an effective null depth levels (NDL<sup>s</sup>) at these directions [2].  $K$  represents the number of required null directions,  $\theta_k$  is the direction of  $k^{th}$  null and  $C_{dB}$  is the desired NDL in dB [2].

## V. SIMULATION RESULTS

In this paper, 24 real antenna elements which are distributed over semi-circular array of radius  $3.82 \lambda$  are transformed virtually to a ULVA of 16 ideal elements as shown in Fig.(1). In this case study, there are three incident signal sources at the angles [25, 80 and 150] degrees. This preprocessing step of virtual transformation is very important to reduce electromagnetic effects (EM) due to the mutual coupling between real antenna elements. The simulation results are considered to illustrate the efficiency of the unified GSA/IWO/WDO optimization algorithm over the hybrid IWO/WDO optimization algorithm as shown in Fig.(2). The unified technique achieves minimum SLL which equals to -21.5 dB compared to -16.01 dB at the case of hybrid method. The control parameters are the same for both algorithms as given in Table 1. The desired beam width is set to be 25 degrees which is achieved at the both algorithms. The unified technique improves the null control over the hybrid technique which achieves NDL at the interference direction 60 degree where NDL equals to -63 dB compared to -37 dB by the hybrid algorithm.

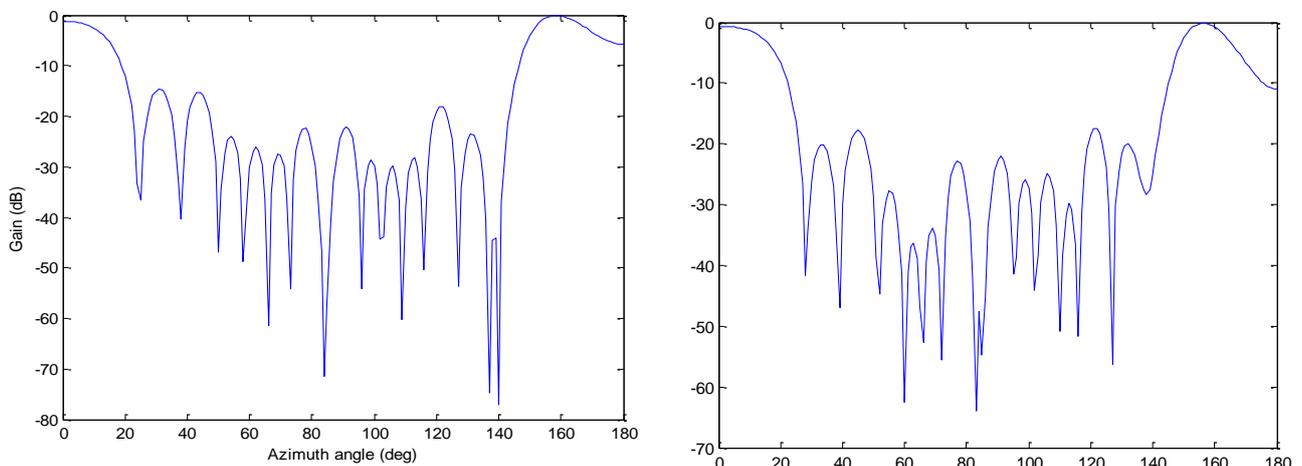


Figure 2. Normalized radiation angle pattern for VULA of 16 elements using hybrid IWO/WDO (left side) and proposed unified GSA/IWO/WDO (right side)

Table 1. Control parameters values used at the unified GSA/IWO/WDO and hybrid IWO/WDO

Symbol	Quantity	Hybrid IWO/WDO	Unified GSA/IWO/WDO
$Iter_{max}$	Maximum number of iterations	100	100
Dim	Number of antenna elements	16	16
$P_{size}$	Maximum Population	40	40
$S_{max}$	Maximum number of seeds	5	5
$S_{min}$	Minimum number of seeds	0	0
N	Nonlinear modulation index	3	3
$\sigma_{initial}$	Initial value of SD	0.5/0.015	0.5/0.015
$\sigma_{final}$	Final value of SD	05/10 <sup>5</sup>	05/10 <sup>5</sup>
$\alpha$	Friction coefficient	0.1	0.1
$g$	Gravitational coefficient	0.1	Iterative acceleration (a)
RT	Constant	2.6	2.6
C	Coriolis constant	0.4	0.4
$u_{max}$	Velocity	0.25	0.25

## VI. CONCLUSION

In this paper, a unified technique of GSA/IWO/WDO optimization algorithm is proposed for pilot and self-calibrations of RUSCA after the virtual transformation of the real array to another VULA. The major idea of the new algorithm is the optimization of the virtual elements' positions to get highest optimized multi-objective fitness function. Although, the new technique is complex, but it provides high performance of the calibration of the real antenna array to have minimum SLL with minimum beam width and effective null control through resulting array pattern synthesis.

## REFERENCES

- [1] M. Willerton, "Array Auto-Calibration", PHD Thesis, Department of Electrical and Electronic Engineering, Imperial College, London, June 2013.
- [2] S. K. Mahto and A. Choubey, "A Novel Hybrid IWO/WDO Algorithm for Interference Minimization of Uniformly Excited Linear Sparse Array by Position-Only Control", *IEEE conference*, 1536-1225, © 2015.
- [3] S. Akyol and B. Alatas, "Plant intelligence based metaheuristic optimization algorithms," *Science and Business Media Dordrecht, Springer*, 2016.
- [4] A. R. Mehrabian and C. Lucas, "A novel numerical optimization algorithm inspired from weed colonization", *Ecological Informatics, IEEE conference*, vol. 1, pp.355-366, 2006.
- [5] Z. Bayraktar, M. Komurcu, J. A. Bossard, and D. H Werner, "The wind driven optimization technique and its application in electromagnetics", *IEEE Trans. Antennas Propag.*, vol. 61, no. 3, pp.771 -779, 2013.
- [6] E. Rashedi, H. Nezamabadi-Pour, and S. Saryazdi, "GSA: A gravitational search algorithm," *Information Sciences*, Vol. 179, No. 13, 2232-2248, 2009.
- [7] T. K. Sarkar and M. S. Palma, *Smart Antennas*, John Wiley & Sons, Canada, 2003.
- [8] A. A. Yahia and H. M. Elkamchouchi, "Direction of Arrival Estimation Using Gravitational Search Algorithm for Real Antenna Array Systems", *ijarcssee*, vol. 6, Issue 11, November 2016.
- [9] A. Magdy, K. R. Mahmoud, S. G. Abdel-Gawad and I. I. Ibrahim, "Direction of Arrival Estimation Based on Maximum Likelihood Criteria Using Gravitational Search Algorithm", *PIERS Proceedings*, Taipei, March 25-28, 2013.
- [10] A. Bhargav, and N. Gupta, "Multiobjective Genetic Optimization of Nonuniform Linear Array with Low Sidelobes and Beamwidth," *IEEE Antennas wireless Propag. Letts.*, Vol. 12, pp.1547- 1549, 2013.