



Direction of Arrival Estimation Using Gravitational Search Algorithm for Real Antenna Array Systems

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Abstract – In this Paper, a new proposed technique is introduced to correctly estimate Direction of Arrival (DOA) spectrum of desired signals using real antenna array system. This technique firstly depends on a virtual transformation of a Real Uniform Linear Array (RULA) to another Virtual Uniform Circular Array (VUCA). To reduce the various Electromagnetic (EM) effects at the RULA signals' environment, this preprocessing stage transforms the induced voltages in the RULA to an equivalent set of produced voltages in a VUCA. In the second stage of the new technique, a recent proposed optimization algorithm called Gravitational Search Algorithm (GSA) is applied to optimize the DOA spectrum for the VUCA. The new overall technique is very simple to be applied for real antenna array systems. The simulation results show how the new combined algorithm improves the DOA estimation of the desired signals using real antenna array elements.

Keywords – Direction of Arrival (DOA); Real Uniform Linear Array (RULA); Virtual Uniform Circular Array (VUCA); Gravitational Search Algorithm (GSA)

I. INTRODUCTION

In many recent communication systems, the modern direction of arrival (DOA) estimation algorithms are widely used to correctly detect the optimized DOA^s of the desired sources located at the far field of the real antenna array systems. Many of subspace DOA estimation methods (e.g. estimation of signal parameters via rotational invariance technique (ESPRIT), multiple signal classification (MUSIC) and Root-MUSIC) succeeded in estimating the DOA spectrum of the desired signals using ideal uniform antenna arrays, but such these algorithms failed to estimate the DOA spectrum using real antenna array systems. The virtual transformation of the real antenna arrays is a good tool to reduce the induced electromagnetic (EM) effects at its signals' environment.

A new combined technique is introduced to optimize the DOA spectrums using this type of transformation and one of the most effective nature-inspired optimization algorithms (i.e. gravitational search algorithm (GSA)). In the first stage of the new technique, the virtual transformation of a real uniform linear array (RULA) to a virtual uniform circular array (VUCA) is applied to reduce the various EM effects such as the mutual coupling between the antenna elements. This electromagnetic preprocessing stage transforms the voltages that are induced in a uniformly spaced array of real antenna elements resulting by all the incident signals to an equivalent set of voltages that will be produced in VUCA containing omnidirectional isotropic point radiators by the same set of incident signals [1]. The preprocessing stage is applied using a transformation matrix. When this transformation matrix is applied to the actual measured snapshot of voltages, it yields an equivalent set of voltages that will be induced in the VUCA under the same incoming signals' scenario [1]. In the second stage, one of the nature-inspired optimization algorithms is introduced to optimize the estimated DOA^s of the desired signals. This algorithm called GSA is a new optimization technique based on the physical phenomena of the law of gravity and mass interaction [2]. In such resulting EM problems, due to using of the real antenna array systems, the problem of the DOA estimation seems to be a good application area for a GSA and other of the nature-inspired optimization algorithms [3]. The new proposed combined technique succeeded to correctly estimate the desired DOA^s with high accuracy using the real antenna array systems.

This paper is organized as follows [3]. In section 2, a virtual transformation is applied for a uniform real antenna array to have a new model of a uniform ideal circular array. In section 3, a survey of the gravitational search algorithm is presented. In section 4, A new proposed technique is introduced to correctly detect the DOA spectrum using GSA optimization algorithm and virtual transformed uniform circular array (UCA). The simulation results are discussed in details in section 4. Finally, section 5 concluded the new combined optimization technique.

II. VIRTUAL ANTENNA ARRAY TRANSFORMATION

The first stage of the new technique is a preprocessing of the output snapshots measured at the load points of a receiving RULA. This real array may be operating in the presence of mutual coupling between the antenna elements. So, the virtual array transformation is a good tool to compensate various EM effects including the mutual coupling effects in the RULA. The preprocessing stage is carried out using a transformation matrix to the measured snapshots of induced voltages in the RULA [1]. So, the goal is to select the best fit – transformation matrix, $T_{(M+1) \times (N+1)}$, between the real array

manifold vector, $A_r(\varphi)$ resulting by uniform linear distribution of real dipole elements, and the array manifold corresponding to a VUCA, $A_v(\varphi)$, such that

$$\overline{A}_v(\varphi) = \overline{T} \overline{A}_r(\varphi) \quad (1)$$

for all azimuth angles φ within a predefined sector $[\varphi_q, \varphi_{q+1}]$ for $q = 0, 1, \dots, Q-1$ where Q is the number of divided sectors of the field of view $[-180^\circ, 180^\circ]$ [1].

Where:

$$[\varphi_q] = [\varphi_q, \varphi_q + \varphi, \dots, \varphi_{q+1}] \quad (2)$$

, the angle φ represents the step size which equals in this case 1° and $Q=1$,

$$[\overline{A}_v(\varphi_q)] = [a(\varphi_q), a(\varphi_q + \varphi), \dots, a(\varphi_{q+1})] \quad (3)$$

, $a(\varphi_q)$ is the steering vector corresponding to a signal source located at φ_q angle [1] at the far field of a VUCA, such that

$$a_n(\varphi_q) = e^{j\psi_n(\varphi_q)}, \quad \psi_n = 2\pi r \cos(\varphi_q - (n-1)\varphi_0) / \lambda \quad (4)$$

Where n indicates to the index of the located antenna element within the VUCA, r is the radius of VUCA, $\varphi_0 = 2\pi/N$ is the angle between adjacent elements & N is the number of the elements of the VUCA.

By the way, the least squares method is applied to compute the transformation matrix T such that

$$\overline{T} = \overline{A}_v(\varphi_q) \overline{A}_r^H(\varphi_q) \left\{ \overline{A}_r(\varphi_q) \overline{A}_r^H(\varphi_q) \right\}^{-1} \quad (5)$$

Where the superscript H represents the conjugate transpose of a complex matrix [1]. So, the corrected voltages can be denoted as

$$\overline{X}_C = \overline{T} \overline{X}_m \quad (6)$$

Where \overline{X}_m is measured voltage vector at the loads of the real elements of the RULA.

III. GRAVITATIONAL SEARCH ALGORITHM

In this context, Gravitational search algorithm (GSA) is one of the meta-heuristic optimization methods. It is motivated by the newton's laws of gravity and mass interaction [2]. The GSA algorithm can be described within the following points [4]:

- **Taxonomy :**

GSA algorithm is a global optimization algorithm that belongs to the field of metaheuristics. Metaheuristics optimization algorithms concerned with "nature-inspired optimization algorithms", which considered its information processing from various natural fields of study, such as physics and biology [4].

- **Strategy :**

GSA algorithm considers agents as objects consisting of different masses [3]. Four parameters are concerned to each agent: position of the mass, inertia mass, active and passive gravitational mass. The collection of the mass positions of an agent at specified dimensions represents a solution of the problem. Agent's inertia mass reflects the resistance of mass movement. Both active and passive gravitational masses control the velocity of agent in specified dimensions, computed by the fitness function of the problem [3].

- **Procedure :**

For such complex and nonlinear problems, GSA algorithm could locate efficient solutions for these problems by proceeding from the following steps:

1- Definition of the search interval

2- **Initialization :** GSA algorithm generates initial positions of K numbers of agents randomly within the defined search interval as below:

$$x_i = (x_i^1, \dots, x_i^d, \dots, x_i^k), \quad \text{for } i = 1, 2, \dots, K \quad (7)$$

Where, x_i^d represents the position of the i^{th} agent in the d^{th} dimension and k is the space dimension.

3- **Agents' fitness evaluation :** at each iteration, the best and worst fitness are evaluated for all agents as below (for minimization problems):

$$\text{best}(t) = \min_{j \in \{1, \dots, N\}} \text{fit}_j(t) \quad (8)$$

$$\text{worst}(t) = \max_{j \in \{1, \dots, N\}} \text{fit}_j(t) \quad (9)$$

Where, $\text{fit}_j(t)$ indicates to fitness evaluation of the j^{th} agent at iteration t . $\text{best}(t)$ and $\text{worst}(t)$ indicate to the best and worst fitness at iteration t respectively.

4- **Gravitational constant's update:** gravitational constant G is computed at each iteration t and will be reduced with time-specified successive iterations using the following equation:

$$G(t) = G_0 e^{\left(\frac{-\alpha t}{T}\right)} \quad (10)$$

Where, G_0 is set to 100, α is set to 20 and T indicates to the total number of iterations as considered in [2].

5- **Calculations of agents' masses:** the masses of the agents (gravitational and inertia masses) can be calculated as:

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^K m_j(t)} \quad (11)$$

$$m_i(t) = \frac{\text{fit}_i(t) - \text{worst}_i(t)}{\text{best}(t) - \text{worst}(t)}, \quad (12)$$

$$M_{ai} = M_{pi} = M_{ii} = M_i; \quad i = 1, 2, \dots, K$$

Where, M_{ai} , M_{pi} , M_{ii} are the active, passive and inertia masses of the i^{th} agent respectively.

- 6- *Calculations of agents' accelerations:* The accelerations of the i^{th} agents at d -dimension and iteration t are calculated as follow:

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)} \quad (13)$$

Where, $F_i^d(t)$ is the total force acting on i^{th} agent, can be calculated as below:

$$F_i^d(t) = \sum_{j=1, j \neq i}^n \text{rand}_j F_{ij}^d(t) \quad (14)$$

Where, rand_j is a random number in the interval $[0, 1]$. $F_{ij}^d(t)$ is the force acting on agent ' i ' from agent ' j ' at d^{th} dimension and t^{th} iteration. It is computed as below [3]:

$$F_{ij}^d(t) = G(t) \frac{M_{pi}(t)M_{aj}(t)}{\|x_i(t), x_j(t)\|_2 + \epsilon} (x_j^d(t) - x_i^d(t)) \quad (15)$$

Where, M_{pi} , M_{aj} are the passive and active gravitational masses related to i and j agents respectively. $\|x_i(t), x_j(t)\|_2$ is the Euclidian distance between two agents ' i ' and ' j ' at iteration t and $G(t)$ is the computed gravitational constant at time t . ϵ is a small constant [2].

- 7- *Computation updating of agents' velocity and position:*

$$v_i^d(t+1) = \text{rand}_i v_i^d(t) + a_i^d(t) \quad (16)$$

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \quad (17)$$

- 8- *Repetition:* the steps 3 to 7 are repeated until iterations reach their maximum limit. So, the best fitness computed at final iteration is considered as a global fitness of the problem and the positions of the corresponding agents at specified dimensions as the global solution of the problem [5].

IV. NEW GSA-DOA ESTIMATION TECHNIQUE AND SIMULATION RESULTS

In this section, the new technique is proposed for correctly estimation of DOA spectrum. It includes two stages of different array processing. The first one is a virtual transformation of real antenna array of N -elements to another of virtual antenna array of M -elements. As it is illustrated in section 2, this stage is very important to have free EM effects on the antenna array's environment. Secondly, GSA optimization algorithm is applied to the output transformed snapshots of the virtual antenna array by using the maximum likelihood estimation which is called objective function in the GSA optimization algorithm of the DOA is given by [6]:

$$\theta = \arg \max_{\theta} \text{tr} \{PA(\theta) \hat{R}\} \quad (18)$$

$$, PA(\theta) = A_v(\theta) \left(A_v^H(\theta) A_v(\theta) \right)^{-1} A_v^H(\theta) \quad (19)$$

$$, \hat{R} = \frac{1}{LD} \sum_{t=1}^{LD} X_C(t) X_C^H(t) \quad (20)$$

Where \hat{R} is the sample covariance matrix and LD denotes to the number of data snapshots [3].

In the beginning of the simulation discussion, RULA of 10 elements is transformed to VUCA of 7 elements. The configurations of the two arrays are illustrated as in Figure 1.

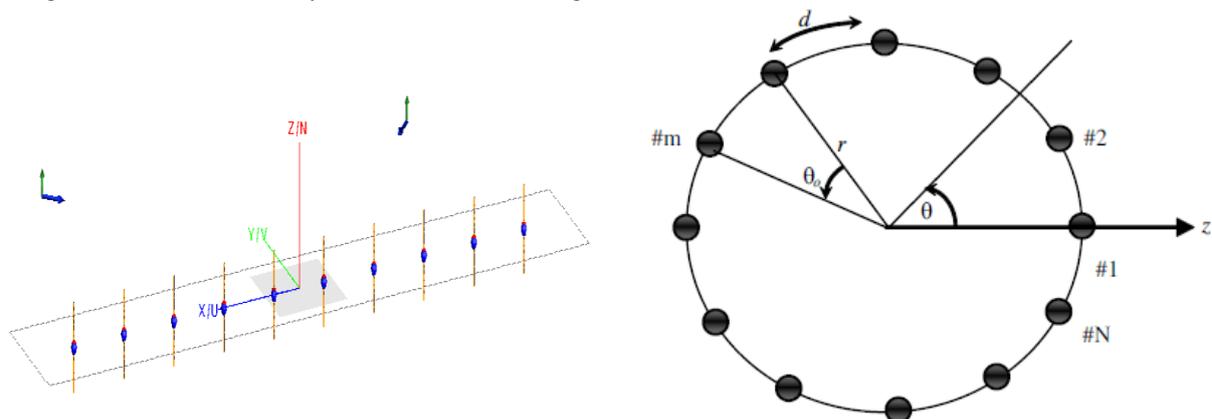


Fig. 1 Configurations of RULA (left side) and VUCA (right side)

The electrical characteristics of all elements, incident signals and its environment are summarized in Table 1.

Table 1 Electrical characteristics of RULA & VUCA

Frequency of incident signals (f)	300 MHz	Variance of noise	1
Wavelength (λ)	1 m	No. of snapshots LD	1024
Power of two incident signals	1 W	Length of z-directed wires	$\lambda/2$
First desired signal's incident angle	-45 deg.	Radius of the wires	$\lambda/200$
Two desired signal's incident angle	45 deg.	Radius of the VUCA (r)	$7\pi/4$
Loading of the center of the element	50 Ω	Inter-element displacement distance	$\lambda/2$

Using the MUSIC DOA estimation algorithm for all the following case studies and having two desired incident signals at angles -45, 45 degrees, Figure 2 illustrates the DOA spectrum at the ideal case for 10 – omnidirectional elements located at the positions of the real ones as illustrated in the left hand side of Figure 1. For scenario No. 2, Figure 3 shows a resulting distorted DOA estimation spectrum due to having no virtual transformation or any optimization processing of the output data of 10 antenna elements RULA which have electrical characteristics illustrated in Table 1. The distortion of the DOA spectrum is occurred due to various EM effects on the RULA and its signals' environment.

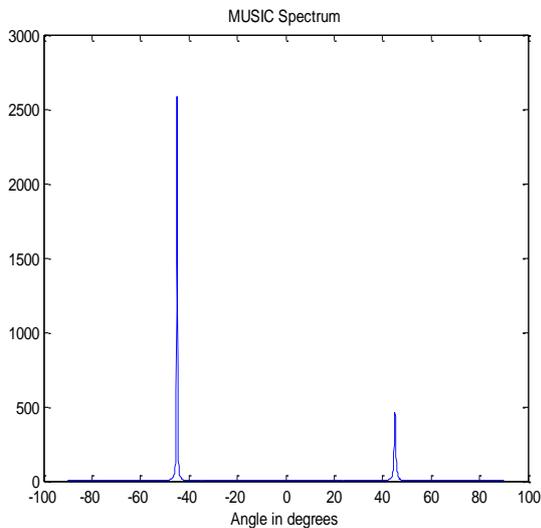


Fig. 2 DOA spectrum at the ideal case

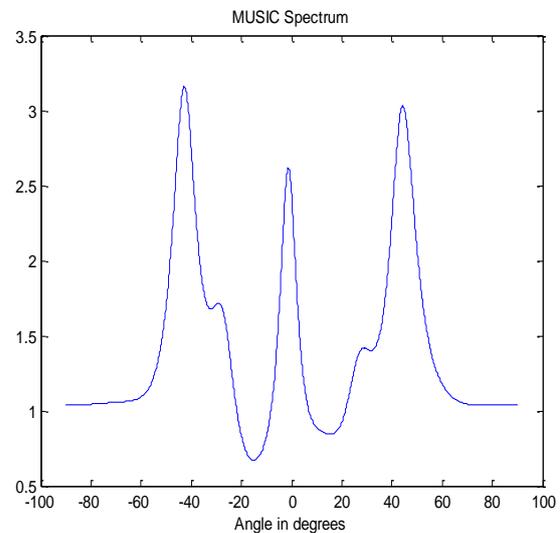


Fig. 3 DOA spectrum at the real case

For scenario No. 3, the 10 elements of RULA are transformed to 7 elements of VUCA. Figure No. 3 shows slightly changes at DOA spectrum resulting as an estimated two peaks of DOA^s at -57 and 56 degrees. This virtual transformation lonely does not achieve the correct estimation of the desired DOA^s. For scenario No. 4, the DOA spectrum is highly enhanced due to using the proposed algorithm by the successive two stages of the array processing. First stage, the virtual transformation of RULA into VUCA to compensate the various EM effects at the signals' environment. Second stage, GSA optimization algorithm is used at 100 iterations to optimize the output virtual transformed data resulting from the first stage in order to enhance the DOA estimation spectrum by using maximum likelihood as an objective function as discussed in section 4. Figure No. 5 shows the enhancement of the calibrated DOA estimation spectrum which could estimate the mostly corrected DOA^s of the real desired signals at -45.2 and 45.3 degrees.

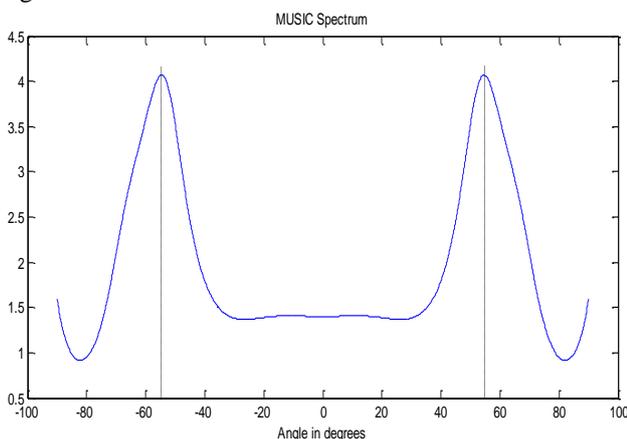


Fig. 4DOA spectrum using virtual transformation

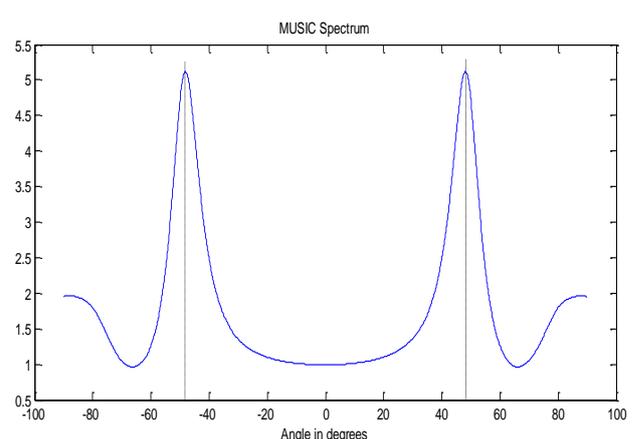


Fig. 5 Corrected DOA spectrum using new proposed technique

V. CONOLUSION

In this paper, a new combined technique is proposed to enhance the estimation of the DOA spectrums using RULA of antenna elements. It achieves better DOA estimations by using MUSIC algorithm and two successive stages of different array processing techniques. The first stage is considered as a virtual transformation of RULA to VUCA to compensate the EM effects at the signals' environment. Then, the second stage is the optimization of the transformation's output data using GSA and maximum likelihood as an efficient cost function.

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