Volume 6, Issue 10, October 2016 ISSN: 2277 128X
International Journal of Advanced Research in Computer Science and Software Engineering
Research Paper
Available online at: www.ijarcsse.com

Generalized Fuzzy Neutrosophic Soft Sets and Its Application in Decision Making

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Abstract— ***In this paper, generalized fuzzy Neutrosophic soft sets and relations on generalized fuzzy Neutrosophic soft sets are defined and a few of their properties are studied. An application of generalized fuzzy Neutrosophic soft sets in decision making with respect to degree of preference is investigated.***

Keywords— Soft sets, Neutrosophic soft sets, fuzzy soft sets, , decision making.

I. INTRODUCTION

In real life situation, most of the problems in economics, social science, medical science, environment etc. have various uncertainties. However, most of the existing mathematical tools for formal modelling, reasoning and computing are crisp, deterministic and precise in character. There is theories viz. theory of probability, evidence, fuzzy set, intuitionistic fuzzy set, vague set, interval mathematics, rough set for dealing with uncertainties. These theories have their own difficulties [8].In 1999, Molodtsov [8] initiated a novel concept of soft set theory, which is completely new approach for modelling vagueness and uncertainties. Soft set theory has a rich potential for application in solving practical problems. The intutionistic fuzzy set handle the incomplete information considering the truth membership value and falsity value. Smarandache introduced the concept of neutrosophic set, which handles indeterminate and imprecise data [11]. A new notion of Fuzzy Neutrosophic soft set and its basic operations and results in Fuzzy Neutrosophic soft spaces are obtained.[1]. In this paper, we have introduced generalized Fuzzy neutrosophic soft set. Our definition is more realistic since it contains a degree of preference corresponding to each parameter. Relations on generalized fuzzy neutrosophic soft set are defined and a few of its properties are studied. An application of generalized fuzzy neutrosophic soft set in decision making is presented.

II. PRELIMINARIES

Definition 2.1:[8] Let U be the initial universe set and E be a set of parameters. Let P(U) denote the power set of U. Consider a non-empty set A , $A \subset E$ .A pair (F,A) is called a soft set over U, where F is a mapping given by $F: A \to P(U)$

Definition 2.2:[12] A Neutrosophic set A on the universe of discourse X is defined as
A=$\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$, Where $T, I, F: X \to]^-0,1^+[$,
$^-0 \le T_A(x)+I_A(x)+F_A(x) \le 3^+$

Definition 2.3: [1] A Fuzzy Neutrosophic set A on the universe of discourse X is defined as
A= $\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F: X \to [0, 1]$ and $0 \le T_A(x)+I_A(x)+F_A(x) \le 3$.

Definition 2.4: [1] Let U be the initial universe set and E be a set of parameters. Consider a non-empty set A, $A \subset E$. Let P (U) denotes the set of all fuzzy neutrosophic sets of U. The collection (F, A) is termed to be the fuzzy neutrosophic soft set over U, where F is defined by $F: A \to P(U)$.

Definition 2.5: [1] A fuzzy neutrosophic soft set A is contained in another fuzzy neutrosophic soft set B. (i.e.,) $A \subseteq B$ if $\forall x \in X, T_A(x) \le T_B(x), I_A(x) \le I_B(x), F_A(x) \ge F_B(x)$.
is continuous t - norm if $*$ satisfies the following conditions :
(i) $*$ is commutative and associative.(ii) $*$ is continuous ,
(iii) $a * 1 = a \ \forall a \in [0, 1]$,(iv) $a * b \le c * d$ whenever $a \le c$, $b \le d$ and $a , b , c , d \in [0,1]$.
Examples of continuous t-norm are $a * b = ab$, $a * b = \min\{a, b\}$,$a * b = \max\{a + b - 1, 0\}$.

Definition 2.6 [11] A binary operation $\diamond : [0, 1] \times [0, 1] \to [0, 1]$
is continuous t-conorm if $\diamond$ satisfies the following conditions :
(i) $\diamond$ is commutative and associative.(ii) $\diamond$ is continuous,(iii) $a \diamond 0 = a \ \forall a \in [0, 1]$,
(iv) $a \diamond b \le c \diamond d$ whenever $a \le c$, $b \le d$ and $a , b , c , d \in [0, 1]$.
Examples of continuous t-conorm are $a \diamond b = a + b - ab$, $a \diamond b = \max\{a, b\}$, $a \diamond b = \min\{a + b,1\}$.
We further assume that (C_1) $a * a = a$, (C_2) $a \diamond a = a$.

III. GENERALIZED FUZZY NEUTROSOPHIC SOFT SETS

Definition 3.1: Let U be the universal set and E be the set of parameters. Let $A \subseteq E$ and $F : A \rightarrow P(U)$ and α be a fuzzy subset of A i.e., $\alpha : A \rightarrow [0, 1]$, where P(U) is the collection of all Fuzzy neutrosophic subset of U. Let $F_\alpha : A \rightarrow P(U) \times [0, 1]$ be a function defined as follows: $F_\alpha(a) = [\{x, T_{F(a)}(x), I_{F(a)}(x), F_{F(a)}(x)\}, \alpha(a)]$ where T, I, F denotes the Truth value, indeterminate value and false value. Then F_α is called a generalized fuzzy neutrosophic soft set over (U,E). Here for each parameter ei, $F_\alpha(ei)$ indicates not only degree of belongingness of the elements of U in F(a) but also degree of preference of such belongingness which is represented by α(ei).

Example 3.2. Let $U = \{S_1, S_2, S_3, S_4\}$ be the set of students under consideration for the best student of an academic year with respect to the given parameters $A \subseteq E$ and
A = {r = "result", c = "conduct", g ="games and sports performances"}.
Let $\alpha : A \rightarrow [0, 1]$ be given as follows: α(r) = 0.7, α(c) = 0.5, α(g) = 0.6
We define Fα as follows:
$F_\alpha(r) = [\{(S_1, 0.8, 0.1,0.7), (S_2, 0.9, 0.05,0.6), (S_3, 0.85, 0.1,0.5), (S_4, 0.75,0.2, 0.2)\}, 0.7]$
$F_\alpha(c) = [\{(S_1, 0.6, 0.3,0.3), (S_2, 0.65,0.25, 0.2), (S_3,0.5, 0.7, 0.2), (S_4, 0.35,0.65, 0.2)\}, 0.5]$
$F_\alpha(g) = [\{(S_1, 0.75, 0.4,0.2), (S_2,0.4, 0.5, 0.3), (S_3, 0.5, 0.4,0.3), (S_4, 0.7, 0.6,0.2),\}, 0.6]$
Then F_α is an generalized fuzzy neutrosophic soft set.

Definition 3.3. Let F_α and G_β be two generalized fuzzy neutrosophic soft set over (U,E). Now F_α is called a generalized fuzzy neutrosophic soft set of G_β if
(i) α is a fuzzy subset of β, (ii) $A \subseteq B$,
(iii) $\forall a \in A$, F(a) is fuzzy neutrosophic subset of G(a) i.e., $T_{F(a)}(x) \leq T_{G(a)}(x)$, $I_{F(a)}(x) \leq I_{G(a)}(x)$ and $F_{F(a)}(x) \geq F_{G(a)}(x)$ and $\forall x \in U$ and $a \in A$.We write $F_\alpha \tilde{\subseteq} G_\beta$.

Example 3.4: Let G_β be a generalized fuzzy neutrosophic soft set defined as follows:
$G_\beta(r) = (\{(S_1, 0.85,0.2, 0.05), (S_2, 0.9, 0.6,0.025), (S_3, 0.9, 0.1,0.4), (S_4, 0.8,0.4, 0.1)\}, 0.75)$
$G_\beta(c) = (\{(S_1, 0.7,0.4, 0.2), (S_2, 0.7,0.4, 0.15), (S_3, 0.75,0.8, 0.2), (S_4, 0.65, 0.7,0.15)\}, 0.6)$
$G_\beta(g) = (\{(S_1, 0.8,0.5, 0.2), (S_2, 0.6,0.7, 0.3), (S_3, 0.7, 0.6,0.2), (S_4, 0.7,0.6, 0.1)\}, 0.65)$
and consider the a generalized fuzzy neutrosophic soft set in Example 3.2. Then F_α is a generalized fuzzy neutrosophic soft subset of G_β.

Definition 3.5: The intersection of two generalized fuzzy neutrosophic soft sets (F , A) and (G , B) is denoted by (F , A) ∩ (G , B) and defined by a fuzzy neutrosophic soft set $H : A \cap B \rightarrow P(U)$ such that for each $e \in A \cap B$, H (e)=($\{\langle x, T_{H(e)}(x), I_{H(e)}(x), F_{H(e)}(x)\rangle, x \in \}, \delta(e)$) where

$$T_{H(e)}(x) = T_{F(e)}(x) * T_{G(e)}(x),\ I_{H(e)}(x) = I_{F(e)}(x) * I_{G(e)}(x),\ F_{H(e)}(x) = F_{F(e)}(x) \diamond F_{G(e)}(x)$$

$\delta(e) = \alpha(e) * \beta(e)$.

Definition 3.6: The union of two generalized fuzzy neutrosophic soft sets F_α and G_β is denoted by (F , A) $\tilde{\cup}$ (G , B) and defined by a generalized fuzzy neutrosophic soft set $H : A \cup B \rightarrow P(U)$ such that for each $e \in A \cup B$

$H(e) = \{\langle x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)\rangle, x \in U, \alpha(e)\}$ if $e \in A - B$

$= \{(x, T_{G(e)}(x), I_{G(e)}(x), F_{G(e)}(x))\text{: } x \in U, \beta(e)\}$ if $e \in B - A$

$= \{(x, T_{H(e)}(x), I_{H(e)}(x), F_{H(e)}(x)) : x \in U, \delta(e)\}$ if $e \in A \cap B$ where

$T_{H(e)}(x) = T_{F(e)}(x) \diamond T_{G(e)}(x)$, $I_{H(e)}(x) = I_{F(e)}(x) \diamond I_{G(e)}(x)$, $F_{H(e)}(x) = F_{F(e)}(x) * F_{G(e)}(x)$, $\delta(e) = \alpha(e) \diamond \beta(e)$.

Theorem 3.8. Let F_α, G_β and H_δ be any three generalized fuzzy neutrosophic soft set over (U,E), then the following holds:
(i) $F_\alpha \tilde{\cup} G_\beta = G_\beta \tilde{\cup} F_\alpha$, (ii) $F_\alpha \tilde{\cap} G_\beta = G_\beta \tilde{\cap} F_\alpha$.
(iii) $F_\alpha \tilde{\cup} (G_\beta \cup H_\delta) = (F_\alpha \tilde{\cup} G_\beta) \cup H_\delta$. (iv) $F_\alpha \tilde{\cap} (G_\beta \tilde{\cap} H_\delta) = (F_\alpha \tilde{\cap} G_\beta) \tilde{\cap} H_\delta$.

Remark 3.9: Let F_α, G_β and H_δ be any three generalized fuzzy soft sets over (U,E). If we consider $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$ then the following holds:
(i) $F\alpha \cap (G_\beta \cup H_\delta) = (F_\alpha \cap G_\beta) \cup (F_\alpha \cap H_\delta)$ (ii) $F_\alpha \cup (G_\beta \cap H_\delta) = (F_\alpha \cup G_\beta) \cap (F_\alpha \cup H_\delta$.
But in general above relations does not hold.

IV. RELATIONS ON GENERALIZED FUZZY NEUTROSOPHIC SOFT SETS

Definition 4.1: Let U be an initial universal set and E be the set of parameters. Let $A, B \subseteq E$ and (F , A), (G ,B) be two generalized fuzzy neutrosophic soft sets over (U, E). Then the Cartesian product of (F , A) and (G , B) is denoted by (F , A) × (G , B) = (H, C) where $C = A \times B$ and $H : C \rightarrow P(U)$ is defined as H(a, b) = (F , A) $\tilde{\cap}$ (G , B), $(a, b) \in C$.

Definition 4.3: Let (F , A), (G , B) be two generalized fuzzy neutrosophic soft sets over (U, E). Then a generalized fuzzy neutrosophic soft relation from (F , A) to (G , B) is a generalized fuzzy neutrosophic soft subset of (F , A)×(G,B).

Definition 4.6: Let R be a generalized fuzzy neutrosophic soft relation from (F , A) to (G,B) then R^{-1} is defined as $R^{-1}(a, b) = R(b, a)$, $\forall (a, b) \in C \subseteq A \times B$

Proposition 4.7: If R is a generalized fuzzy neutrosophic soft relation from (F , A) to (G,B) then R^{-1} is a generalized fuzzy neutrosophic soft relation from (G , B) to (F,A).

Proposition 4.8: If R_1 and R_2 be two generalized fuzzy neutrosophic soft relations from (F, A) to (G , B) then (i) $(R_1^{-1})^{-1} = R_1$ and (ii) $R_1 \subseteq R_2 \Rightarrow R_1^{-1} \subseteq R_2^{-1}$.

Definition 4.9: The composition $\circ$ of two generalized fuzzy neutrosophic soft relations R_1 and R_2 is defined by $(R_1 \circ R_2)(a, c) = R_1(a, b) \tilde{\cap} R_2(b, c)$ where R_1 is a generalized fuzzy neutrosophic soft relation form (F , A) to (G , B) and R_2 is a generalized fuzzy neutrosophic soft relation from (G, B) to (H, C).

Proposition 4.10: If R_1 and R_2 be two generalized fuzzy neutrosophic soft relations from (F , A) to (G , B) then $(R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1}$.

Theorem 4.11: R_1 is a generalized fuzzy neutrosophic soft relation form (F , A) to (G , B) satisfying (C_1) and (C_2) and R_2 is a generalized fuzzy neutrosophic soft relation from (G , B) to (H, C) satisfying (C_1) and (C_2) then $R_1 \circ R_2$ is a generalized fuzzy neutrosophic soft relation from (F , A) to (H, C).

Definition 4.10: A generalized fuzzy neutrosophic soft relation R on (F, A) is said to be generalized fuzzy neutrosophic soft symmetric relation if $R(a, b) = R(b, a), \forall a, b \in A$.

Definition 4.11: A generalized fuzzy neutrosophic soft relation R on (F , A) is said to be generalized fuzzy neutrosophic soft transitive relation if $R \circ R \subseteq R$.

Definition 4.12: A generalized fuzzy neutrosophic soft relation R on (F, A) is said to be a generalized fuzzy neutrosophic soft reflexive relation if $R(a, b) \subseteq R(a, a)$ and $R(b, a) \subseteq R(a, a), \forall a, b \in A$.

Definition 4.13: A generalized fuzzy neutrosophic soft relation R on (F , A) is said to be a generalized fuzzy neutrosophic soft equivalence relation if it is symmetric, transitive and reflexive.

Proposition 4.14: If R is symmetric if and only if R^{-1} is so.

Proposition 4.15: R is symmetric if and only if $R = R^{-1}$.

Proposition 4.16: If R_1 and R_2 are symmetric relations on (F , A) then $R_1 \circ R_2$ is symmetric on (F , A) if and only if $R_1 \circ R_2 = R_2 \circ R_1$.

Corollary 4.17: If R is symmetric then R^n is symmetric for all positive integer n, where R^n is $R \circ R \circ \cdots \circ R$ (n times).

Proposition 4.18: If R is transitive then R^{-1} is also transitive.

Proposition 4.19: If R is reflexive then R^{-1} is so.

V. AN APPLICATION OF GENERALIZED FUZZY NEUTROSOPHIC SOFT SET IN DECISION MAKING

There are many applications of fuzzy neutrosophic soft set theory to deal with uncertainties. Here we present such an application for solving a socialistic decision making problem.

Suppose there are six colleges in the universe U = { C_1, C_2, C_3, C_4, C_5, C_6 } and the parameter set E={e_1, e_2, e_3, e_4, e_5, e_6 }, where , $1 \leq i \leq 6$ indicates an attribute of C_j ,$1 \leq i \leq 6$. **e_1 stands for percentage, e_2 stands for excellence in sports, e_3 stands for regularity in attendanc,e_4 communication skill, e_5 stands for technical skill, e_6 stands for logical skill.**

Suppose a concern X wants to sponsor candidates from a single college Y and their wishing parameters are A = { e_1, e_2, e_3, e_5} and $A \subseteq E$.

Let the preference of the criterions for the company is been described by the fuzzy subset α: A ----> [0,1] of A as follow $\alpha(e_1) = 0.8$ $\alpha(e_2) = 0.7$ $\alpha(e_3) = 0.5$ $\alpha(e_5) = 0.6$

Consider the generalized FNSS F_α as the collection of

F_α (e_1) = [{(C_1 ,0.2,0.3,0.7) (C_2,0.6,0.4,0.1) (C_3,0.2,0.1,0.0) (C_4,0.9,0.9,0.4) (C_5,0.6,0.3,0.1) (C_6,0.7,0.2,0.1)},0.8]

F_α (e_2) = [{(C_1 ,0.3,0.4,0.5) (C_2,0.2,0.1,0.7) (C_3,0.3,0.4,0.4) (C_4,0.8,0.8,0.3) (C_5,0.9,0.7,0.1) (C_6,0.2,0.3,0.1)},0.7]

F_α (e_3) = [{(C_1 ,0.8,0.6,0.2) (C_2,0.2,0.2,0.8) (C_3,0.3,0.4,0.5) (C_4,0.7,0.7,0.3) (C_5,0.8,0.7,0.4) (C_6,0.4,0.2,0.3)},0.6]

F_α (e_5) = [{(C_1 ,0.3,0.3,0.4) (C_2,0.5,0.3,0.4) (C_3,0.7,0.5,0.4) (C_4,0.8,0.5,0.2) (C_5,0.6,0.4,0.5) (C_6,0.7,0.6,0.1)},0.5]

Tabular representation of the Generalized Fuzzy Neutrosophic soft set F_α

COLLEGES	e_5 , $\alpha(e_5) = 0.5$	e_3 , $\alpha(e_3) = 0.6$	e_2 , $\alpha(e_2) = 0.7$	e_1 , $\alpha(e_5) = 0.8$
C_1	(0.3,0.3,0.4)	(0.8,0.6,0.2)	(0.3,0.4,0.5)	(0.2,0.3,0.7)
C_2	(,0.5,0.3,0.4)	(0.2,0.2,0.8)	(0.2,0.1,0.7)	(0.6,0.4,0.1)
C_3	(0.7,0.5,0.4)	(0.3,0.4,0.5)	(0.3,0.4,0.4)	(0.2,0.1,0.0)
C_4	(0.8,0.5,0.2)	(0.7,0.7,0.3)	(0.8,0.8,0.3)	(0.9,0.9,0.4)
C_5	(0.6,0.4,0.5)	(0.8,0.7,0.4)	(0.9,0.7,0.1)	(0.6,0.3,0.1)
C_6	(0.7,0.6,0.1)	(0.4,0.2,0.3)	(0.2,0.3,0.1)	(0.7,0.2,0.1)

COLLEGES	e_5 , $\alpha(e_5)$ = 0.5	e_3 , $\alpha(e_3)$ = 0.6	e_2 , $\alpha(e_2)$ = 0.7	e_1 , $\alpha(e_1)$ = 0.8	α_i
C_1	0	1	0	0	0.6
C_2	0	0	0	0	0
C_3	1	0	0	0	0.5
C_4	1	1	1	1	2.6
C_5	0	1	1	0	1.3
C_6	1	0	0	0	0.5

Maximum score = 2.6.
Decision: the concern x will select the college C_4.

VI. CONCLUSION

A new type of sets called generalized fuzzy neutrosophic soft sets are introduced. Relations on generalized fuzzy neutrosophic soft set are defined and a few of its properties are studied A new in approach in approach in decision making via generalised neutrosophic soft is proposed. It is expected this method will a efficient tool for decision making problems.

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