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On the Complete (k,2)- Arcs of the Hall Plane of Order 9

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***Abstract*— We investigate the complete (*k*,2)- arcs with the quadrangles which do not generate the Fano planes in the projective plane P_2S over a Hall system S of order 9.**

***Keywords*— Near field, Projective plane, (*k*,2)-arcs**

I. INTRODUCTION

In a finite projective plane π (not necessarily Desarguesian) a set K of k $(k \geq 3)$ points such that no three points of K are collinear (on a line) is called a k-arc. If the plane π has order p then $k \leq p+2$, however the maximum value of k can only be achieved if p is even. In a plane of order p , a (p+1)-arc is called an oval and, if p is even, a (p+2)-arc is called a hyperoval. A general reference for ovals is Hirschfeld [8]. There are known plenty of examples of arcs in projective planes; see [2,3].

There are four known non-isomorphic projective planes of order 9. The known four distinct projective planes of order 9 are extensively studied by Room Kirkpatrick [5]. These are Desarguesian plane, the left nearfield plane, the right nearfield plane and Hughes plane. The last three planes of order 9 are called "miniquaternion planes" because they can be coordinatized by the miniquaternion near field. O. Veblen and J. M. Wedderburn [6] discovered these miniquaternion planes in 1907.

In [1], an algorithm (implemented in $C\#$) to determine and classify Fano subplanes of the projective plane of order 9 coordinatized by elements of a left nearfield of order 9 is given. In [2,3], all complete a (k,2)-arcs containing complete quadrangles which generate the Fano planes in the projective plane whose algebraic structure is the left nearfield of order 9 are examined.

Our present investigation includes (k,2)-complete arcs containing the quadrangles not-constructing the Fano planes in the projective plane P_2S over a Hall system S of order 9 by using completion procedure. We use an algorithm for checking above arcs in this projective plane and apply the algorithm (implemented in $C\#$) to determine and classify (k,2)-complite arcs.

II. THE HALL PLANE P_2S OF ORDER 9

The earliest application of the concept of near-field was in the study of geometries, such as projective geometries. Many projective geometries can be defined in terms of a coordinate system over a division ring, but others can't. It was found that by allowing coordinates from any near-ring the range of geometries which could be coordinatized was extended. For example, Marshall Hall used the near-field of order 9 given above to produce a Hall plane, the first of a sequence of such planes based on Dickson near-fields of order the square of a prime. In 1971, T. G. Room and P.B. Kirkpatrick provided an alternative development.

The original construction of Hall planes was based on a Hall quasifield (also called a Hall system). To build a Hall quasifield, start with a Galois field $F = GF(p^n)$, for p a prime and a quadratic irreducible polynomial $f(t) = t^2 - rt - s$ over F .

We consider an algebraic system $(S, \oplus, \otimes)$ over the Galois field $(F_3, +, .)$ of order 3. The nine elements of S are $a + \lambda b,\ a, b \in F_3, \lambda \notin F_3$. Addition in S is defined by the rule

$$(a+\lambda b)\oplus(c+\lambda d)=(a+c)+\lambda(b+d)$$

and multiplication by

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$$(a+\lambda b)\otimes(c+\lambda d)=\begin{cases} ac+\lambda(ad), & \text{if } \ b=0 \\ ac-b^{-1}df(a)+\lambda(bc-(a-1)d), & \text{if } \ b\neq 0 \end{cases}$$

where, $a,b,c,d\in F_3, \lambda\notin F_3$ and $f(t)=t^2+1$ is a irreducible polynomial on F_3. The system $(S,\oplus,\otimes)$ is a Hall system of order 9. The table of all homogeneous coordinates of the 91 points and lines in the projective plane P_2S defined in terms of a coordinate system over the Hall system $(S,\oplus,\otimes)$ is given , see [3].

III. COMPLETE ARCS CONTAINING THE QUADRANGLES NOT CONSTRUCTING FANO PLANES IN THE PROJECTIVE PLANE P_2S

In [1], the list of the all quadrangles that generate Fano plane in P_2S is given. In this section, we take the set $A=\{O,I,X,P\}$ such that $O=(0,0,1), I=(1,1,1), X=(1,0,0), P\neq(a,b,1)$ with $a\in F_3, b\in F_3-S$. The distinct complete $(k,2)$- arcs can be constructed by adding to A in each time a new point not in spanned by the previous set from the remaining points of P_2S. $(k,2)$-complete arcs containing the quadrangles not constructing the Fano planes in the projective plane P_2S over a Hall system S of order 9 by using this completion procedure are obtained by applying the algorithm (implemented in $C\#$). The obtained complete arcs sets are as follows:

List of (7,2)-arcs is

$\{1,2,11,21,37,68,90\},\{1,2,11,21,46,52,68\},\{1,6,11,21,62,68,76\},$

$\{1,2,11,21,52,58,68\},\{1,2,11,21,52,60,68\},\{1,2,11,21,52,68,82\},$

$\{1,2,11,21,60,68,90\},\{1,2,11,21,62,68,90\},\{1,2,11,21,68,82,90\},$

$\{1,5,11,21,32,48,68\},\{1,5,11,21,34,53,68\},\{1,5,11,21,37,62,68\},$

$\{1,5,11,21,39,62,68\},\{1,5,11,21,39,68,86\},\{1,5,11,21,45,68,85\},$

$\{1,5,11,21,46,58,68\},\{1,5,11,21,32,58,68\},\{1,5,11,21,58,68,77\},$

$\{1,5,11,21,62,68,75\},\{1,6,11,21,29,48,68\},\{1,6,11,21,38,57,68\},$

$\{1,7,11,21,29,50,68\},\{1,7,11,21,29,57,68\},\{1,7,11,21,29,59,68\},$

$\{1,7,11,21,33,46,68\},\{1,7,11,21,46,56,68\},\{1,7,11,21,46,68,83\},$

$\{1,9,11,21,29,59,68\},\{1,9,11,21,38,68,86\},\{1,9,11,21,58,68,80\},$

$\{1,10,11,21,37,42,68\},\{1,10,11,21,37,47,68\},\{1,10,11,21,37,56,68\},$

$\{1,10,11,21,38,57,68\},\{1,10,11,21,38,59,68\},\{1,10,11,21,38,68,84\},$

$\{1,11,21,29,40,57,68\},\{1,11,21,29,42,68,88\},\{1,11,21,29,45,60,68\},$

$\{1,11,21,29,50,68,80\},\{1,11,21,29,52,60,68\},\{1,11,21,29,60,68,80\},$

$\{1,11,21,29,68,80,88\},\{1,11,21,32,42,62,68\},\{1,11,21,32,42,68,76\},$

$\{1,11,21,32,45,68,78\},\{1,11,21,32,46,68,90\},\{1,11,21,32,62,68,90\},$

$\{1,11,21,32,68,75,90\},\{1,11,21,32,68,78,83\},\{1,11,21,33,38,54,68\},$

$\{1,11,21,33,39,58,68\},\{1,11,21,33,39,68,80\},\{1,11,21,33,40,50,68\},$

$\{1,11,21,33,40,58,68\},\{1,11,21,33,42,68,82\},\{1,11,21,33,50,68,80\},$

$\{1,11,21,33,50,68,90\},\{1,11,21,33,53,57,68\},\{1,11,21,33,53,68,78\},$

$\{1,11,21,33,53,68,80\},\{1,11,21,33,54,68,84\},\{1,11,21,33,58,68,83\},$

↑1, 11, 21, 33, 58, 68, 84↓, ↑1, 11, 21, 33, 60, 68, 80↓, ↑1, 11, 21, 33, 68, 78, 84↓,
↑1, 11, 21, 34, 38, 60, 68↓, ↑1, 11, 21, 34, 39, 68, 78↓, ↑1, 11, 21, 34, 45, 47, 68↓,
↑1, 11, 21, 34, 45, 54, 68↓, ↑1, 11, 21, 34, 45, 68, 83↓, ↑1, 11, 21, 34, 45, 68, 88↓,
↑1, 11, 21, 34, 47, 68, 78↓, ↑1, 11, 21, 34, 48, 68, 80↓, ↑1, 11, 21, 34, 52, 58, 68↓,
↑1, 11, 21, 34, 54, 59, 68↓, ↑1, 11, 21, 34, 58, 68, 83↓, ↑1, 11, 21, 34, 59, 68, 78↓,
↑1, 11, 21, 34, 59, 68, 85↓, ↑1, 11, 21, 34, 60, 68, 75↓, ↑1, 11, 21, 34, 60, 68, 78↓,
↑1, 11, 21, 34, 68, 75, 83↓, ↑1, 11, 21, 35, 38, 59, 68↓, ↑1, 11, 21, 35, 42, 62, 68↓
↑1, 11, 21, 35, 42, 68, 84↓, ↑1, 11, 21, 35, 53, 59, 68↓, ↑1, 11, 21, 35, 54, 56, 68↓,
↑1, 11, 21, 35, 54, 68, 75↓, ↑1, 11, 21, 35, 68, 76, 86↓, ↑1, 11, 21, 35, 68, 77, 88↓,
↑1, 11, 21, 37, 39, 52, 68↓, ↑1, 11, 21, 37, 46, 52, 68↓, ↑1, 11, 21, 37, 46, 68, 90↓,
↑1, 11, 21, 37, 52, 68, 77↓, ↑1, 11, 21, 37, 52, 68, 82↓, ↑1, 11, 21, 37, 54, 68, 77↓,
↑1, 11, 21, 37, 54, 68, 90↓, ↑1, 11, 21, 37, 68, 76, 85↓, ↑1, 11, 21, 37, 68, 76, 86↓,
↑1, 11, 21, 37, 68, 82, 90↓, ↑1, 11, 21, 38, 54, 68, 76↓, ↑1, 11, 21, 38, 60, 68, 76↓,
↑1, 11, 21, 38, 60, 68, 90↓, ↑1, 11, 21, 38, 68, 76, 84↓, ↑1, 11, 21, 39, 47, 68, 78↓,
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↑1, 11, 21, 40, 54, 68, 75↓, ↑1, 11, 21, 40, 56, 68, 88↓, ↑1, 11, 21, 40, 57, 68, 85↓,
↑1, 11, 21, 40, 68, 77, 88↓, ↑1, 11, 21, 42, 47, 62, 68↓, ↑1, 11, 21, 42, 50, 62, 68↓,
↑1, 11, 21, 42, 50, 68, 78↓, ↑1, 11, 21, 42, 50, 68, 88↓, ↑1, 11, 21, 42, 59, 68, 85↓,
↑1, 11, 21, 42, 60, 68, 76↓, ↑1, 11, 21, 42, 68, 76, 84↓, ↑1, 11, 21, 42, 68, 76, 85↓,
↑1, 11, 21, 42, 68, 78, 85↓, ↑1, 11, 21, 45, 47, 62, 68↓, ↑1, 11, 21, 45, 47, 68, 77↓,
↑1, 11, 21, 45, 53, 57, 68↓, ↑1, 11, 21, 45, 57, 68, 78↓, ↑1, 11, 21, 45, 57, 68, 88↓,
↑1, 11, 21, 45, 60, 68, 77↓, ↑1, 11, 21, 45, 60, 68, 78↓, ↑1, 11, 21, 45, 62, 68, 90↓,
↑1, 11, 21, 45, 68, 76, 86↓, ↑1, 11, 21, 45, 68, 78, 83↓, ↑1, 11, 21, 47, 58, 68, 90↓,
↑1, 11, 21, 47, 68, 82, 90↓, ↑1, 11, 21, 46, 48, 68, 80↓, ↑1, 11, 21, 46, 52, 68, 82↓,
↑1, 11, 21, 46, 52, 68, 88↓, ↑1, 11, 21, 46, 53, 68, 80↓, ↑1, 11, 21, 46, 68, 75, 88↓,
↑1, 11, 21, 46, 68, 75, 90↓, ↑1, 11, 21, 46, 68, 82, 90↓, ↑1, 11, 21, 48, 60, 68, 80↓,
↑1, 11, 21, 48, 60, 68, 90↓, ↑1, 11, 21, 48, 68, 80, 90↓, ↑1, 11, 21, 50, 62, 68, 90↓,
↑1, 11, 21, 50, 68, 78, 85↓, ↑1, 11, 21, 52, 58, 68, 84↓, ↑1, 11, 21, 52, 60, 68, 77↓,
↑1, 11, 21, 52, 60, 68, 86↓, ↑1, 11, 21, 52, 62, 68, 83↓, ↑1, 11, 21, 52, 62, 68, 88↓,
↑1, 11, 21, 52, 68, 76, 86↓, ↑1, 11, 21, 52, 68, 82, 83↓, ↑1, 11, 21, 53, 57, 68, 82↓,
↑1, 11, 21, 53, 57, 68, 85↓, ↑1, 11, 21, 53, 57, 68, 88↓, ↑1, 11, 21, 53, 59, 68, 78↓,
↑1, 11, 21, 53, 59, 68, 85↓, ↑1, 11, 21, 53, 56, 68, 78↓, ↑1, 11, 21, 53, 68, 78, 84↓,
↑1, 11, 21, 53, 56, 68, 80↓, ↑1, 11, 21, 54, 58, 68, 90↓, ↑1, 11, 21, 54, 59, 68, 85↓,
↑1, 11, 21, 56, 68, 76, 85↓, ↑1, 11, 21, 56, 68, 78, 85↓, ↑1, 11, 21, 57, 68, 78, 85↓,

{1,11,21,59,68,82,85},{1,11,21,60,68,75,90},{1,11,21,60,68,76,86}.

List of (8,2)-arcs is

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↑1, 2, 11, 21, 34, 60, 68, 80↓, ↑1, 2, 11, 21, 35, 42, 68, 88↓, ↑1, 2, 11, 21, 35, 54, 68, 76↓,
↑1, 2, 11, 21, 37, 54, 68, 85↓, ↑1, 2, 11, 21, 40, 68, 80, 88↓, ↑1, 2, 11, 21, 42, 68, 82, 85↓,
↑1, 2, 11, 21, 45, 60, 68, 76↓, ↑1, 2, 11, 21, 45, 62, 68, 88↓, ↑1, 2, 11, 21, 46, 53, 68, 88↓,
↑1, 5, 11, 21, 29, 45, 68, 77↓, ↑1, 5, 11, 21, 29, 48, 68, 82↓, ↑1, 5, 11, 21, 29, 50, 60, 68↓,
↑1, 5, 11, 21, 29, 50, 68, 85↓, ↑1, 5, 11, 21, 29, 60, 68, 75↓, , ↑1, 5, 11, 21, 29, 68, 75, 85↓,
↑1, 5, 11, 21, 32, 45, 53, 68↓, ↑1, 5, 11, 21, 32, 53, 62, 68↓, ↑1, 5, 11, 21, 32, 53, 68, 84↓,
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↑1, 9, 11, 21, 35, 48, 68, 86↓, ↑1, 9, 11, 21, 35, 68, 80, 86↓, ↑1, 9, 11, 21, 35, 68, 80, 88↓,
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↑1, 10, 11, 21, 34, 38, 68, 86↓, ↑1, 10, 11, 21, 34, 47, 68, 75↓, ↑1, 10, 11, 21, 34, 47, 68, 86↓,
↑1, 10, 11, 21, 34, 52, 59, 68↓, ↑1, 10, 11, 21, 34, 52, 60, 68↓, ↑1, 10, 11, 21, 34, 59, 68, 75↓,
↑1, 10, 11, 21, 34, 60, 68, 80↓, ↑1, 10, 11, 21, 34, 68, 75, 88↓, ↑1, 10, 11, 21, 34, 68, 80, 86↓,
↑1, 10, 11, 21, 37, 46, 68, 75↓, ↑1, 10, 11, 21, 37, 54, 68, 75↓, ↑1, 10, 11, 21, 40, 47, 68, 75↓,
↑1, 10, 11, 21, 40, 47, 68, 80↓, ↑1, 10, 11, 21, 40, 47, 68, 86↓, ↑1, 10, 11, 21, 40, 56, 68, 86↓,
↑1, 10, 11, 21, 40, 57, 68, 88↓, ↑1, 10, 11, 21, 42, 52, 60, 68↓, ↑1, 10, 11, 21, 42, 52, 68, 88↓,
↑1, 10, 11, 21, 42, 59, 68, 78↓, ↑1, 10, 11, 21, 46, 52, 59, 68↓, ↑1, 10, 11, 21, 46, 52, 68, 84↓,

$\{1,10,11,21,46,59,68,75\},\{1,10,11,21,46,59,68,83\},\{1,10,11,21,46,68,75,83\}$

$\{1,10,11,21,47,57,68,78\},\{1,10,11,21,47,68,80,86\},\{1,10,11,21,52,59,68,83\}$

$\{1,10,11,21,54,56,68,84\},\{1,10,11,21,60,68,78,84\},\{1,10,11,21,60,68,80,86\}$

$\{1,11,21,29,40,48,59,68\},\{1,11,21,29,40,58,68,77\},\{1,11,21,29,40,58,68,80\}$

$\{1,11,21,29,40,59,68,85\},\{1,11,21,29,42,48,60,68\},\{1,11,21,29,52,68,77,84\}$

$\{1,11,21,29,59,68,75,85\},\{1,11,21,32,40,47,58,68\},\{1,11,21,32,40,47,68,75\}$

$\{1,11,21,32,45,48,68,90\},\{1,11,21,32,46,48,68,83\},\{1,11,21,32,46,53,56,68\}$

$\{1,11,21,32,46,56,68,84\},\{1,11,21,32,46,58,68,84\},\{1,11,21,32,53,56,68,84\}$

$\{1,11,21,32,56,68,76,84\},\{1,11,21,32,58,68,76,84\},\{1,11,21,33,38,60,68,86\}$

$\{1,11,21,33,42,50,68,84\},\{1,11,21,33,46,68,82,83\},\{1,11,21,34,39,52,68,86\}$

$\{1,11,21,34,48,59,68,83\},\{1,11,21,34,53,68,80,88\},\{1,11,21,35,38,53,57,68\}$

$\{1,11,21,35,38,57,68,86\},\{1,11,21,35,38,62,68,75\},\{1,11,21,35,38,62,68,76\}$

$\{1,11,21,35,39,62,68,83\},\{1,11,21,35,39,68,77,83\},\{1,11,21,35,48,56,68,80\}$

$\{1,11,21,35,48,68,80,86\},\{1,11,21,35,53,56,68,84\},\{1,11,21,35,53,56,68,88\}$

$\{1,11,21,35,53,57,68,77\},\{1,11,21,35,56,68,76,84\},\{1,11,21,35,62,68,75,88\}$

$\{1,11,21,37,39,47,68,86\},\{1,11,21,37,39,50,56,68\},\{1,11,21,37,39,50,62,68\}$

$\{1,11,21,37,39,56,68,85\},\{1,11,21,37,42,47,68,82\},\{1,11,21,37,47,57,68,82\}$

$\{1,11,21,37,56,68,82,86\},\{1,11,21,37,57,68,82,85\},\{1,11,21,38,50,68,75,90\},$

$\{1,11,21,38,53,57,68,77\},\{1,11,21,39,50,56,68,80\},\{1,11,21,39,50,56,68,85\}$

$\{1,11,21,39,50,62,68,80\},\{1,11,21,40,48,68,76,86\},\{1,11,21,40,50,56,68,80\}$

$\{1,11,21,40,50,56,68,85\},\{1,11,21,40,54,56,68,85\},\{1,11,21,40,54,58,68,77\}$

$\{1,11,21,40,56,68,76,86\},\{1,11,21,40,59,68,75,85\},\{1,11,21,45,47,57,68,86\},$

$\{1,11,21,45,54,68,76,85\},\{1,11,21,47,58,68,77,84\},\{1,11,21,46,48,56,68,82\}$

$\{1,11,21,46,53,59,68,82\},\{1,11,21,46,59,68,82,83\},\{1,11,21,48,56,68,76,86\}$

$\{1,11,21,48,56,68,80,86\},\{1,11,21,48,56,68,82,86\},\{1,11,21,50,62,68,75,83\}$

$\{1,11,21,50,62,68,80,88\},\{1,11,21,35,38,53,57,67\},\{1,11,21,54,58,68,76,84\}.$

List of (9,2)-arcs is

$\{1,11,21,29,42,50,60,68,84\},\{1,11,21,29,52,57,68,77,88\},\{1,11,21,32,45,47,57,68,90\}$

$\{1,11,21,33,38,50,60,68,84\},\{1,11,21,34,39,52,59,68,83\},\{1,11,21,38,54,59,68,75,90\}$

List of (10,2)-arcs is
$\{1,2,11,21,34,45,53,68,78,85\},\{1,2,11,21,33,46,58,68,80,90\},\{1,2,11,21,37,42,52,62,68,76\}.$

IV. CONCLUSIONS

we take the set $A=\{O,I,X,P\}$ such that $O=(0,0,1), I=(1,1,1), X=(1,0,0)$, $P\neq(a,b,1)$ with $a\in F_3$, $b\in F_3-S$.

We described procedure for searching all complete (k,2)- arcs with $6 < k \leq 10$ containing the quadrangles $A = \{O, I, X, P\}$ such that $O = (0,0,1), I = (1,1,1), X = (1,0,0), P \neq (a,b,1)$ with $a \in F_3, b \in F_3 - S$ not constructing the Fano subplanes in the left nearfield plane of order 9. By applying the method and algorithm (implemented $C\#$) in [2], a full listing the complete (k,2)-arcs were determined and classified . These can be represented by four groups. There are 168 classes of the complete (7,2)- arcs, 276 classes of the complete (8,2)-arcs, 6 classes of the complete (9,2)-arcs, 3 classes of the complete (10,2)-arcs.

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