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Graph Theoretic Approach for Enhancing the Lifetime of Wireless Sensor Networks

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Abstract— In Wireless Sensor Networks, the energy consumption is a critical issue. Since each node has a limited energy and uses it in sensing, processing, gathering and transmitting the data, therefore the main issue in WSN is to utilize the energy of sensor nodes carefully to enhance the lifetime of the network. In this paper, a graph theoretic approach for energy efficiency and power management is given. An algorithm is constructed for the disjoint dominating sets in the network, which may be used cyclically for sensing, processing, data gathering and data transmitting. So that the energy of the sensor nodes is uniformly consumed throughout the network and all the sensor nodes may actively participate in above functions.

Key Words— Unit Disk graph, dominating set, connected dominating set.

I. INTRODUCTION

Ad-hoc Networks are formed by mobile devices. A mobile device unit consists of a processor, some memory, a radio communication unit and a power source (battery or solar cell). Sensor Networks can be considered as special form of ad-hoc networks in which nodes are equipped with sensors measuring certain physical quantity such as humidity, brightness, temperature, pressure, velocity, acceleration and vibration. A Wireless Sensor Networks is composed of a set of battery powered sensors. These sensors can communicate with one another through wireless links if they are within their transmission range, otherwise they can communicate via other sensors between them. WSNs has a wide variety of applications such as battle field surveillance, target tracking, fire detection, security, oil or gas pumping, habitat monitoring and environment control.

Sensor nodes are generally assumed to be autonomous and operate for a considerable time period may be for several years. Energy is a scarce resource for sensor nodes which run on battery as energy sources. Each node has limited energy for the data transmission. The improvement of energy efficiency and power management is a critical issue in the WSNs. The energy consumption in WSNs is central issue for the researchers. The WSNs should be designed so that the system can maintain itself for several months or year, without paying attention and is required to collect and send the accurate information for a long period of time.

Energy efficiency and power management is usually characterized before deployment using simulators. Usually the targeted area is very large, so we use robotic helicopter to deploy sensors to monitor outer environment. Such rapid and broad deployment may have a chance of some errors. Sensors may not be placed in exactly their desired locations due to wind or inaccurate localization. Sensor may fail due to physical damage from impact of deployment of sensors, fire or extreme heat, animal or vehicular accident, extended use and lack of sufficient energy. These reasons may effect our network and we won't get the whole desired information. Graph theory based concept may be used to overcome this problem approximately.

A graph G = (V, E), where V is the set of vertices and E is the set of edges. Each edge i.e. element of E is an unordered pair of element of V. Unit Disk graph is an important class of graphs, which finds application in modeling a WSNs. The radio coverage range of sensors is based on Euclidean distance between the nodes and we utilize this concept of Euclidean distance in graph theory. This has given rise to a new branch termed as 'geometric graph theory'. WSNs can be modeled as Unit Disk Graph. In this modeling sensors are denoted as vertices. The sensing coverage area of a sensor is represented by a unit disk centered at the corresponding vertex. The connectivity between two sensor nodes is determined if the first sensor is within the sensing coverage area of the second sensor. Thus there is an edge between u and v iff $d(u,v) \leq R$, where $d(u,v)$ is the Euclidean distance between u and v and R is the sensing range. Thus Unit Disk (UD) graph is a suitable model for a Wireless Sensor Networks.

Initially sensor nodes are deployed with a limited amount of energy. They spend their energy in sensing, processing, gathering and transmitting the data. Therefore, the main issue in WSNs is to utilize the energy of sensor nodes carefully to enhance the life time of the network. This problem can be solved by constructing dominating sets in the network graph. The main concept behind this approach is to keep only a few number of sensor node busy in sensing, processing, gathering and transmitting the data and to save the energy of remaining nodes by putting them in sleeping mode. Dominating set in a graph G = (V, E) is a subset D of V such that every node $u \in V$ is in D or adjacent to some node $v \in D$. A dominating set D is a Connected Dominating Set (CDS) if the induced subgraph of G by D is connected. A Minimum Connected Dominating Set (MCDS) is a connected dominating set with smallest possible cardinality among all the CDSs of G. CDSs are popularly used for constructing virtual backbones for broadcasting operation in WSNs. Virtual backbone is basically a subset of sensor nodes which can transmit and receive messages throughout the network. Establishing a virtual backbone in WSNs is an important issue because it reduces unnecessary message transmission or flooding in the network. It helps in reducing interference and energy consumption because a limited number of sensors are engaged in message transmission and thus it helps in improving the Quality of Service (QoS) in the network.

However, the problem of unbalanced energy distribution between sensor nodes may arise in this approach. The sensor nodes which belong to the dominating set will quickly consume their energy and the sensor nodes which are in sleeping mode will save their energy for long time until they are not in active mode. It will generate the problem of non uniform consumption of energy of sensor nodes. It creates problem in some applications where the network's life time is based on the functioning of the individual sensors. This drawback can be removed by finding a number of disjoint dominating sets in network graph and by activating them cyclically one after another. So that the energy of the sensor node is uniformly consume throughout the network and all the sensor node may actively perform the assigned function in the network. On the other hand, if any sensor node is not working due to pre defined reasons, then we may activate the other disjoint dominating set for sensing, processing, gathering and transmitting the data.

In this Paper, we have introduced an algorithm on the construction of disjoint dominating sets in the network graph. It is based on id, degree and neighbor information of every node. Dominating set is constructed iteratively in this algorithm. Let G = (V,E) be the network graph. Firstly we have given distinct id to every node. It may be ascending order in numbers or may be lexicographic order in alphabets or may be some other predefined rule. To construct first independent set, we select the highest degree node u_1 in G (in case of tie, we select the minimum id node). Put it in the first dominating set D_1 and then select the next highest u_2 degree node except the nodes in $N[u_1]$. Put it in D_1 and so on until we have no node to select. We repeat this process with the set $V \setminus D_1$ to find next dominating set and iteratively we can find all the disjoint dominating sets.

The paper is organized as follows: In Section II, we recall the works that have been carried out in this field. In Section III, we have introduced the basic definitions and terminologies related to the work. Model Definition is given is Section IV. The main algorithm is described in section V with an example. The last Section VI concludes the work.

II. RELATED WORK

K. Islam et. al. [3] have determined the Minimum connected dominating sets using convex hull of the sensors. K. Islam et. al. [4] have studied the problem of computing a family of connected dominating sets in Wireless Sensor Networks (WSN) in a distributed manner. A distributed algorithm is described that computes a family non-trivial connected dominating sets (CDS) as many CDSs as possible while minimizing the number of frequencies of each node in these sets. An interesting and important relationship has been given between minimum vertex-cuts and CDSs where the cardinality of a minimum vertex-cut limits the number of disjoint CDSs. They show that this algorithm achieves a constant approximation factor of the optimal solution for a subclass of unit disk graphs.

K. Islam et. al. [5] investigate the problem of finding the maximum number of disjoint dominating sets called the domatic partition problem in unit disk graphs. Although the domatic partition problem is NP hard in general graphs, it is unknown whether the same is true for unit disk graphs. An algorithm towards solving this problem (approximately) together with experimental results and give a conjecture based on our results about the maximum number of disjoint dominating sets in unit disk graphs are presented. They have provided extensive simulation results to assess the performance of our algorithm where the experiments provide good results. However, one intriguing open question is to come up with an algorithm that gives an approximation factor (preferably 'small') for unit disk graphs.

In [7], the k-domatic partition problem is considered. A k-dominating set is a subset D of nodes such that every node in the network is at distance at most k from D. The k-domatic partition problem seeks to partition the network into maximum number of k-dominating sets. Three deterministic, distributed algorithms for finding large k-domatic partitions are given there.

A distributed algorithm for constructing the CDP using our maximal independent set (MlS)-based proximity heuristics is developed in [6], which depends only on connectivity information and does not rely on geographic or geometric information

III. AUXILIARY DEFINITIONS

In order to develop the algorithm, we state some definition and introduce some terminology relevant to the paper.

A. Dominating Set

Dominating Set for a graph G = (V, E) is a subset D of the Vertex Set V such that each vertex $u \in V$ is either in D or adjacent to some vertex v in D. The elements of dominating set are called dominators and the remaining vertices are called dominatees. Examples of dominating set in a graph G are given below:

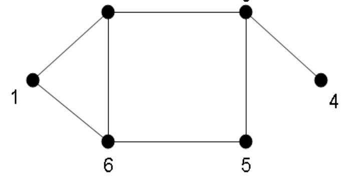

Fig. 1 {1, 3}, {2, 3, 5} and {1, 2, 3, 4} are Dominating Sets

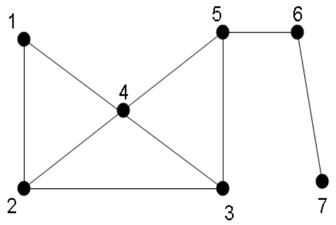

Fig. 2 {4, 6}, {1, 5, 7} and {4, 5, 6} are Dominating Sets

B. Connected Dominating Set

A Connected Dominating Set (CDS) D of a graph G = (V, E) is a set of vertices with two properties:

1) D is a dominating set in G.
2) D induces a connected subgraph of G.

In Fig.1, {2, 3, 5} and {1, 2, 3, 4} are Connected Dominating Sets. Similarly in Fig. 2, {4, 5, 6} is a Connected Dominating Set.

C. Minimum Connected Dominating Set

A minimum Connected Dominating Set (MCDS) is a connected dominating set with smallest possible cardinality among all the CDSs of G. As in Figs. 1 and 2, {2, 3, 5} and {4, 5, 6} are Minimum Connected Dominating Sets respectively.

D. Neighborhood of a vertex

Neighborhood of a vertex u in a graph G = (V,E) is a set of vertices which are adjacent to u in G.

1) *Open Neighborhood*: If neighborhood does not include u itself, then it is called open neighborhood of u and denoted by N(u). As in figure 1, N(1) is {2, 6}, N(2) is {1, 3, 6}, N(3) is {2, 4, 5} and so on.
2) *Closed neighborhood*: If neighborhood includes u itself, then it is called closed neighborhood of u and denoted by N[u]. For example in Fig. 2, N[1] is {1, 2, 4} and N[2] is {1, 2, 3, 4}and so on.

E. Neighborhood of a vertex set

The neighborhood of a vertex set D ⊆ V in a graph G = (V, E) is the union of neighborhoods of all the vertices of D in G.

1) *Open Neighborhood of a vertex set:* The neighborhood of a vertex set D⊆V in a graph G = (V, E) is called the open neighborhood of D if it is the union of open neighborhoods of all the vertices of D in G and not included the vertices of D. It is denoted by N(D). In Fig. 2, Let D = {1, 2} then N(D) = N(1) ∪ N(2) = {3, 4}.
2) *Closed Neighborhood of a vertex set:* The neighborhood of a vertex set D⊆V in a graph G = (V, E) is called the closed neighborhood of D if it is the union of closed neighborhoods of all the vertices of D in G. It is denoted by N[D]. In Fig. 2, Let D = {1, 2} then N[D] = N[1] ∪ N[2] = {1, 2, 3, 4}.

F. Unit Disk Graph

A graph G is a Unit Disk graph if there is an assignment of unit disks centered at its vertices such that two vertices are adjacent if and only if one vertex is within the unit disk centered at the other vertex. We denote a unit disk graph by G_{UD}.

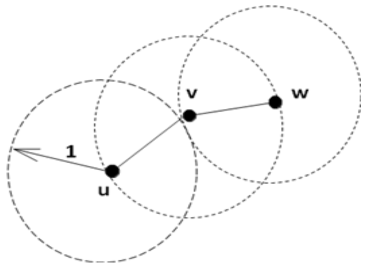

Fig. 3 Unit Disk Graph

IV. MODEL DEFINITION

A. Modeling of a Wireless Sensor Network as Unit Disk (UD) Graph

A Wireless Sensor Network can be modeled as a UD Graph since the transmission range of sensor nodes is based on Euclidean distance. In this modeling, sensors are denoted as vertices. The sensing coverage area of a sensor is represented by a unit disk centered at corresponding vertex. The connectivity between two sensor nodes is determined if the first sensor is within the sensing coverage range of the second sensor. Thus there is an edge between two vertices *u* and *v* iff $d(u,v) \leq 1$, where $d(u,v)$ is the Euclidean distance between *u* and *v*. In this way UD Graph is most suitable model for a wireless sensor network. The modeled graph is called network graph.

B. Domatic Partition problem in a graph

By Domatic partition problem in a graph G = (V, E) is meant to find the maximum number of mutually disjoint dominating sets $D_i \subseteq V$ such that:

(i) $\bigcup_i D_i \subseteq V$, where $\{D_i : i \in N\}$ is the collection of dominating sets.

(ii) $D_i \cap D_j = \emptyset, \forall\, i \neq j$

V. ALGORITHM TO FIND DOMATIC PARTITION OF THE NETWORK GRAPH

In this section we present a description of our algorithm. We find disjoint dominating set D_i such that $\bigcup_i D_i \subseteq V$. Firstly we model the given network as a unit disk graph to find network graph. Let it be G = (V, E). Then we assign a unique id to each vertex according to some predefined order. It may be lexicographic order in alphabets or may be ascending order in numbers or some other method. We select the maximum degree vertex in V (in case of tie, select the vertex with minimum id). Let it be v_1. Put this vertex in first dominating set D_1. In the next step we find the $N[D_1]$ and $V\backslash N[D_1]$. Then again select the maximum degree vertex in $V\backslash N[D_1]$ and put it in D_1. Repeat the above process until $V\backslash N[D_1] = \emptyset$. If $V\backslash N[D_1] = \emptyset$, then we go for the next dominating set D_2. In order to find D_2, we assume $V\backslash D_1$ as V_1. Repeat the above process for V_1 until $V_1\backslash N[D_2] = \emptyset$. Similarly to find D_3, we assume $V_1\backslash D_2$ as V_2. Repeat the above process for V_2 until $V_2\backslash N[D_3] = \emptyset$ and so on. We will stop on the case when $V_n\backslash D_{n+1} = \emptyset$ for some n. We will find at the most δ+1 disjoint dominating sets of the network graph G by applying this algorithm, where δ is the minimum degree of the graph G.

Algorithm for Domatic partition

Input: The Network graph G (V, E)
Output: The Disjoint Dominating Sets $D_1, D_2, \ldots\ldots\ldots$

```
1.  V' = V
2.  Begin For (i=1; i ≤ δ+1; i++)
3.  Di = ∅
4.  Select the maximum degree vertex v in V'
5.  Di = Di ∪{v}
6.  N[Di] = ∪vr N[vr] ∀ vr ∈ Di
7.  V' = V'\ N[Di]
8.  If (V' ≠ ∅)
9.  Then go to step 4.
10. Else if (V' = ∅)
        {
11.     If (V\N[Di] = ∅)
12.     Then Di is a Dominating Set.
13.     Else if (V\N[Di]≠ ∅)
14.     Then
                {
15.             Di is not a Dominating Set.
15.             i = i-1
                }
16.     STOP
        }
17. V'= V'\ Di
18. End for
```

To explain the algorithm we illustrate an example as follows:

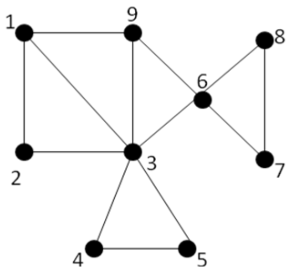

Fig. 4 A Network Graph

Consider the above graph G = (V, E) is the network graph for a given WSN. We assign the unique id (numbers) to each vertex. Then we find the degree of each vertex.

Step 1: Select the maximum degree vertex in V i.e. 3. Put it in D_1. i.e. $D_1 = \{3\}$.

Step 2: Find $N[D_1]$.

$N[D_1] = N[3] = \{1,2,3,4,5,6,9\}$.

Step 3: Find $V\backslash N[D_1]$.

$V \setminus N[D_1] = \{7,8\}$.

Step 4: Select the maximum degree vertex in $V \setminus N[D_1]$ i.e. 7 (since it has minimum id). Put it in D_1.

$D_1 = \{3,7\}$.

Step 5: Find $N[D_1]$.

$N[D_1] = N[3] \cup N[7] = \{1,2,3,4,5,6,7,8,9\}$.

Step 6: Find $V \setminus N[D_1]$.

$V \setminus N[D_1] = \emptyset$.

Here we stop and our first dominating set is constructed. Now we go for second dominating set and repeat the above process for $V_1 = V \setminus D_1$. In this way we find $D_2 = \{1,4,6\}$ and $D_3 = \{2,3,5,8,9\}$. These dominating set can be used cyclically for the broadcasting operation in the network so that uniform consumption of the energy could happen throughout the network.

We can consider another example in which all nodes are not included in disjoint dominating sets. We have obtained following dominating sets in the following network graph: {2} and {1,3} by applying above algorithm and vertex 4 is not included in the dominating sets.

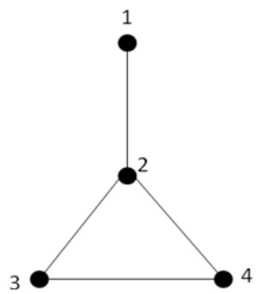

Fig. 5 Network Graph

Now we prove some results related to dominating sets.

Theorem 4.2: Two distinct dominating sets in a network graph G obtained by the above algorithm are disjoint. i.e. If D_i and D_j be two distinct dominating sets, then $D_i \cap D_j = \emptyset, \forall\, i \neq j$.

Proof: Let D_i and D_j be two dominating sets obtained by the above algorithm and assume that D_i is obtained before D_j in the algorithm. In each step of the algorithm, we exclude all the nodes, which are included in the dominating set for the next step to find the next dominating set. Therefore a node which has been included in D_i, cannot be included in D_j. *i.e.* D_i and D_j have no common node. Hence $D_i \cap D_j = \emptyset, \forall\, i \neq j$.

Theorem 4.3: The number of disjoint dominating sets constructed by the above algorithm is at the most $\delta+1$, where δ is the minimum degree of the graph.

Proof: The proof of this theorem is given by contradiction method. If possible suppose that more than $\delta+1$ disjoint dominating sets are obtained by the algorithm. *i.e.* $D_1, D_2, \ldots\ldots D_k$ are disjoint dominating sets obtained by the above algorithm and $k > \delta+1$. Without loss of generality let $k = \delta+2$. *i.e.* $D_1, D_2, \ldots\ldots D_\delta, D_{\delta+1}, D_{\delta+2}$ are disjoint dominating sets obtained by the algorithm.

Let u be the vertex with minimum degree δ and let $u_1, u_2, \ldots\ldots u_\delta$ are the neighbors of u. At the most possibility is that all the nodes are included in some dominating set. We consider that u and its neighbors $u_1, u_2, \ldots\ldots u_\delta$ are included in distinct dominating set. Let $u \in D_1$, $u_1 \in D_2$, $u_2 \in D_3, \ldots\ldots \ldots\ldots u_\delta \in D_{\delta+1}$. Since all D_i's are disjoint Dominating Sets, therefore u, $u_1, u_2, \ldots\ldots u_\delta$ cannot be included in $D_{\delta+2}$ and hence u is not dominated by any node of $D_{\delta+2}$. $D_{\delta+2}$ is not a dominating set, which is a contradiction. Therefore our assumption is wrong. Hence, the number of disjoint dominating sets is at the most $\delta+1$, where δ is the minimum degree of the graph.

VI. CONCLUSIONS

In this paper we have introduce an algorithm to construct disjoint dominating sets in the given network graph. It is based on id, degree and neighbor information of every node. Dominating set is constructed iteratively in this algorithm. These dominating set are the virtual backbone of the network and are used for the broadcasting operation in the network. We may activate only one of these dominating set for broadcasting so that energy of the remaining sensor nodes in the network may be saved. It will also increase the lifetime of the network. Cyclic activation of these dominating set will ensure the uniform energy consumption throughout the network.

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