Volume 6, Issue 1, January 2016 ISSN: 2277 128X
International Journal of Advanced Research in Computer Science and Software Engineering
Research Paper
Available online at: www.ijarcsse.com

Connected Double Geodetic Domination Number and Strong Split Double Geodetic Domination Number of A Graph

[1]A. Nellai Murugan*, [2]A. Esakkimuthu, [3]G. Mahadevan
[1, 2] Dr.S.S.Pillai Centre for Research in Mathematics, V. O. Chidambaram College, Thoothukudi, Tamilnadu, India
[3] Department of Mathematics, Gandhigram Rural Institute Deemed University, Gandhigram,
Dindigul District, Tamilnadu, India

***Abstract** − The Concept of Double Geodetic Dominating Set of a Graph was introduced in [9]. A subset S of vertices in a graph G is a called a double geodetic dominating set if S is both a double geodetic set and a dominating set. The double geodetic domination number $\gamma_{DG}(G)$ is the minimum cardinality of a double geodetic dominating set. Any double geodetic domination of cardinality $\gamma_{DG}(G)$ is called $\gamma_{DG} - set$ of G. In this paper we introduce the new concept of ConnectedDouble Geodetic Dominating Set and Strong Split Double Geodetic Dominating set of Graphs.A double geodetic dominating set S of G is said to be a Connected Double Geodetic Dominating Setof G if the sub graphs of G induced by S is connected. The minimum cardinality of all such connected double geodetic dominating set of G is called the Connected Double Geodetic Domination Number of G and is denoted by$\gamma_{DG}^{c}(G)$. A connected double geodetic dominating set of cardinality $\gamma_{DG}^{c}(G)$ is called a γ_{DG}^{c} set of G.A Double Geodetic Dominating set S of a graph G is said to be a Strong Split Double Geodetic Dominating set of G if the subgraph induced by $V - S$ is totally disconnected. That is, the subgraph induced by $V - S$ is independent. The minimum cardinality of all such strong split double geodetic dominating set of G is called the Strong Split Double Geodetic Domination Number of G and is denoted by $\gamma_{DG}^{ss}(G)$. A strong split double geodetic dominating set of cardinality $\gamma_{DG}^{ss}(G)$is called a γ_{DG}^{ss} −set of G. In this paper, we study Connected Double Geodetic Domination and Strong Split Double Geodetic Domination on Graphs*

***Keywords**− Domination, Geodetic, Double Geodetic, Double Geodetic Dominating Set, Double Geodetic Domination Number, Connected Double Geodetic Dominating Set, Connected Double Geodetic Domination Number. Strong Split Double Geodetic Dominating Set, Strong Split Double Geodetic Domination Number 2000 Mathematical subject classification: 05C12, 05C69*

I. INTRODUCTION

We consider finite graphs without loops and multiple edges. For any graph G the set of vertices is denoted by *V(G)* and the edge set by *E(G)*. We define the order of *G* by $n = n(G) = |V(G)|$ and the size by $m = m(G) = |E(G)|$. For a vertex$v \in V(G)$, the *open neighborhood N(v)* is the set of all vertices adjacent to *v*, and *N[v]* = *N(v)*∪{*v*} is the *closed neighborhood* of *v*. The *degree d(v)* of a vertex *v* is defined by *d(v)* = |*N(v)*|. The *minimum* and *maximum degrees of a graph G are* denoted by $\delta = \delta(G)$ and $\Delta = \Delta(G)$, respectively.

For $X \subseteq V(G)$let *G[X]* the sub graph of *G* induced by *X*, *N(X)* = $\cup_{x\in X}N(x)$ and *N[X]* =$\cup_{x\in X}N(x)$. If *G* is a connected graph, then the distance *d(x, y)* is the length of a shortest $x - y$ path in *G*. The diameter $diam(G)$ of a connected graph is defined by $diam(G) = max_{x,y\in V(G)}d(x,y)$. An $x - y$ path of length *d(x, y)* is called an $x - y$ *geodesic*. A vertex *v* is said to lie on an $x - y$ geodesic *P* if *v* is an internal vertex of *P*. The closed interval *I[x, y]* consists of *x, y* and all vertices lying on some $x - y$ geodesic of *G*, while for $S \subseteq V(G)$, $I[S] = \cup_{x,y\in S} I[x,y]$. If *G* is a connected graph, then a set *S* of vertices is a *geodetic set* if *I[S]* = *V(G)*. The minimum cardinality of a geodetic set is the *geodetic number* of *G*, and is denoted by *g(G)*. The geodetic number of a disconnected graph is the sum of the geodetic numbers of its components. A geodetic set of cardinality $g(G)$ is called a $g(G)$ −*set*. For references on geodetic sets see [1, 2, 3, 4, 6].

Let *G* be a connected graph with at least two vertices. A set *S* of vertices of *G* is called a *double geodetic set* of *G* if for each pair of vertices *x, y* in *G* there exist vertices *u, v* ∈*S* such that x, y ∈I[u, v]. The *double geodeticnumber* $dg(G)$ of G is the minimum cardinality of a double geodetic set. Any double geodetic set of cardinality $dg(G)$ is called $dg(G)$-*set* of G. [8]

A vertex in a graphdominates itself and its neighbors. A set of vertices *S* in a graph *G* is a *dominating set* if *N*[*S*] = *V* (*G*). The *domination number* $\gamma(G)$ of *G* is the minimum cardinality of a dominating set of *G*. The domination number was introduced in [6].

Let $G = (V, E)$ be any connected graph with at least two vertices. A*double geodetic dominating set* of G is a subset S of $V(G)$ which is both dominating and double geodetic set of G. A double geodetic dominating set S is said to be a *minimal double geodetic dominating set* of G if no proper subset of S is a double geodetic dominating set of G. A double geodetic dominating set S is a said to be a minimum double geodetic dominating set of G if there exists no double geodetic dominating set S^1 such that | S^1 | < | S |. The cardinality of a minimum double geodetic dominating set of G is called the *double geodetic domination number* of G. It is denoted by $\gamma_{DG}(G)$. Any double geodetic dominating set S of G of cardinality γ_{DG} is called a $\gamma_{DG} -$ set of G.

It is easily seen that a dominating set is not in general a double geodetic set in a graph G. Also the converse is not valid in general. This has motivated us to study the new domination conception of double geodetic domination. We investigate those subsets of vertices of a graph that are both a double geodetic set and a dominating set. Adouble geodetic dominating set S of G is said to be a *connected double geodetic dominating set* of G if the sub graphs of G induced by S is connected. We call these sets connected double geodetic dominating sets. We call the minimum cardinality of a connected double geodetic dominating set of G, the connected double geodetic domination number of G.A Double Geodetic Dominating set S of a graph G is said to be a *Strong Split Double Geodetic Dominating set* of G if the subgraph induced by $V - S$ is totally disconnected. That is, the subgraph induced by $V - S$ is independent. We call these sets strong split double geodetic dominating sets.We call the minimum cardinality of a strong split double geodetic dominating set of G, the *Strong Split Double Geodetic Domination Number* of G.

The following definition is used in the sequel **:**

1.1 Definition: A dominating set S of a graph G is said to be a *connected dominating set* of G if the sub graph induced by S is a connected. The minimum cardinality among all connected dominating sets is called the *connected domination number* of G and is denoted as$\gamma_c(G)$

1.2 Definition: A connected double geodetic set of G is a double geodetic set S of G such that the sub graph induced by S is connected. The minimum cardinality of all connected double geodetic sets of G is called its *connected double geodetic number* and it is denoted as $dg_c(G)$.

1.3 Theorem: Each extreme vertex of a connected graph G belongs to every double geodetic set of G. In particular, if the set of all end vertices of G is a double geodetic set, then it is the unique dg-set of G.

1.4 Definition: A dominating set S of a graph G is said to be a split dominating set of G if the subgraph induced by $V - S$ is disconnected. The minimum cardinality among all split dominating sets is called the split domination number of G and is denoted as $\gamma_s(G)$.

1.5 Definition: A dominating set S of a graph G is said to be a strong split dominating set of G if the subgraph induced by $V - S$ is totally disconnected or independent. The minimum cardinality among all strong split dominating sets is called the strong split domination number of G and is denoted as $\gamma_{ss}(G)$.

II. CONNECTED DOUBLE GEODETIC DOMINATION NUMBER

Definition 2.1: A double geodetic dominating set S of G is said to be a connected double geodetic dominating set of G if the sub graphs of G induced by S is connected. The minimum cardinality of all such connected double geodetic dominating set of G is called the connected double geodetic dominating set of G and is denoted by $\gamma_{DG}^c(G)$. A connected double geodetic dominating set of cardinality $\gamma_{DG}^c(G)$ is called a γ_{DG}^c set of G.

Observation 2.2: Let G be a connected graph with at least two vertices. Then, the following are observed.

1. $2 \leq \gamma_{DG}(G) \leq \gamma_{DG}^c(G) \leq p$.
2. $\gamma_{DG}^c(K_p) = p$.
3. $\gamma_{DG}^c(G) \geq \max\{\gamma_c(G), dg_c(G)\}$
4. As the extreme vertices of G belong to every double geodetic dominating set of G, every connected double geodetic dominating set of G contains all the extreme vertices of G.

Theorem 2.3: Let G be a graph on $p(\geq 3)$ vertices. Then, $\gamma_{DG}^c(G) \geq 3$.

Proof: Suppose $\gamma_{DG}^c(G) = 2$. Then, the two vertices of a minimum double geodetic dominating set are adjacent and so there is no double geodesic joining them contains any other vertex of G. As$p \geq 3$, this is a contradiction to the definition of double geodetic dominating set. Hence, $\gamma_{DG}(G) \neq 2$. By Observation 2.2 (1) $\gamma_{DG}^c(G) \geq 3$.

Corollary 2.4: $\gamma_{DG}^c(G) = 2$ if and only if $G \cong K_2$.

Example 2.5: considering the graph G in figure 2.5(a) $\gamma_{DG}(G) = 2$ since $D = \{v_1, v_3\}$ is the unique minimum double geodetic dominating set of G. But, the sub graph induced by D is not connected. Let $S = \{v_1, v_2, v_3\}$. S is a connected double geodetic dominating set of G. By theorem 2.3, $\gamma_{DG}^c(G) = 3$.

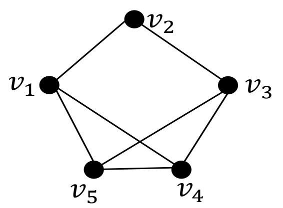

Figure 2.5 (a)

Remark 2.6: In the above example, $\gamma_{DG}^c(G) \neq \gamma_{DG}(G)$. But, $\gamma_{DG}^c(K_2) = \gamma_{DG}(K_2) = p$.

Theorem 2.7: Let S be a connected double geodetic dominating set of a graph G. Then, the sub graph induced by S is complete if and only if G is complete.

Proof: Suppose the sub graph induced by S is complete. Then, $d(u, v) = 1$ for every $u, v \in S$. Since S is a double geodetic set of G, $S = V(G)$. Hence, G is complete. The converse is straight forward.

Proposition 2.8: Let S be a minimal connected double geodetic set of a graph G. If S is a dominating set of G, then S is a minimal connected double geodetic dominating set of G.

Proof: SupposeS is not a minimal connected double geodetic dominating set of G. Then, there exists a proper subset $S'$of S such that S' is a connected double geodetic dominating set of G. Therefore, S' is a connected double geodetic set of G. This is a contradiction. Hence, S is a connected double geodetic dominating set of G.

Remark 2.9: Similarly, the following results are true.

1. Let S be a minimal connected dominating set of a graph G. If S is also a double geodetic set of G, then S is a minimal connected double geodetic dominating set of G.
2. Let S be a minimum connected double geodetic set of G. If S is also a dominating set of G, then S is a minimum connected double geodetic dominating set of G.
3. Let S be a minimum connected dominating set of G. If S is also a double geodetic set of G, then S is a minimum connected double geodetic dominating set of G.

Lemma 2.10: Let G be a connected graph with p vertices. If $dg_c(G) = p, then\ \gamma_{DG}^c(G) = p.$

Proof: As $dg_c(G) = p$, by Observation 2.2, $\gamma_{DG}^c(G) \geq p$. Further, $\gamma_{DG}^c(G) \leq p$. Hence,$\gamma_{DG}^c(G) = p$.

Lemma 2.11: Let G be a non-complete connected graph with p vertices. If $dg_c(G) = p - 1$, then $\gamma_{DG}^c(G) = p - 1$.

Proof: Suppose $dg_c(G) = p - 1$. Let S be a minimum connected double geodetic set of G. Then,$|V - S| = 1$ and let $V - S = \{x\}$. As S is a double geodetic set of G, x lies in au-v double geodetic of length 2 for some $u, v \in S$. Then, S is also a dominating set of G. By Remark 2.9, S is a minimum connected double geodetic dominating set of G. Hence $\gamma_{DG}^c(G) = p - 1$.

Lemma 2.12: Let G be a non-complete connected graph with p vertices. Then, $dg_c(G) \leq p - 2$, if and only if $\gamma_{DG}^c(G) \leq p - 2$.

Proof: Let S be a minimum connected double geodetic set of G. If every vertex of $V - S$ lies in a double geodetic of length≤ 3, then S is also a dominating set of G. Therefore, by Remark 2.9, S is a $\gamma_{DG}^c - set$ of G and so $\gamma_{DG}^c \leq p - 2$. Suppose $v \in V - S$ such that every double geodtiec connected two vertices of S containing v is of length> 3. Choose $P: (v_1, v_2, \dots\dots, v_n)$ as a double geodetic containing v such that length(P) is minimum. Then $v_2, v_3, \dots\dots, v_{n-1} \in V - S$ and $v = v_i$ for some i, $2 \leq i \leq n - 1$. Clearly, $V - \{v_i, v_{i+1}\}$ if $2 \leq i \leq n - 1\ orV - \{v_{i-1}, v_i\}$ if $i = n - 1$ is a connected double geodetic dominating set of G. Therefore $\gamma_{DG}^c(G) \leq p - 2$.

Converse follows from Lemma 2.10 and Lemma 2.11.

Corollary 2.13: Let G be a connected graph with p vertices. Then, $\gamma_{DG}^c(G) = p$ if and only if $dg_c(G) = p$.

Proof: By lemma 2.10, $dg_c(G) = p$ implies $\gamma_{DG}^c(G) = p$. Converse follows lemma 2.11 and form lemma 2.12.

Corollary 2.14: Let G be a non-complete connected graph with p vertices. Then, $dg_c(G) = p - 1$ if and only if $\gamma_{DG}^c(G) = p - 1$.

Proof: The result follows form 2.11, 2.12 and 2.13.

Corollary 2.15: Let G be a connected graph. Then,$\gamma_{DG}^c(G) = p$ if and only if every vertex of G is an extreme vertex of G.

Proof: The result follows from theorem 1.3 and corollary 2.13.

Corollary 2.16: If T is a tree, then $\gamma_{DG}^c(G) = p$.

Proof: Each vertex of a tree is either an extreme vertex or a cut vertex. Hence the result follows form corollary 2.15.

Corollary 2.17: If P_p is a path on p vertices, then $\gamma_{DG}^c(P_p) = p$.

Theorem 2.18: Let G be a non-complete connected graph with $p \geq 3$ vertices. If $\gamma_{DG} = 2$, then $\gamma_{DG}^c(G) \leq 4$.

Proof: Suppose $\gamma_{DG} = 2$. Then, by Theorem, $daim\ G < 4$and G contains two antipodal vertices u and v such that every vertex of $V - \{u, v\}$ lies in some $u - v$ double geodetic in G. Then, the length of $P = daim\ G < 4$. $S = V(P)$ is a connected double geodetic dominating set of G and so $\gamma_{DG}^c(G) \leq |S| = diam\ G + 1 < 5$. In other words, $\gamma_{DG}^c(G) \leq 4$. The above bound is sharp. For, $\gamma_{DG}(P_4) = 2\ \ and\ \gamma_{DG}^c(P_4) = 4$.

III. STRONG SPLIT DOUBLE GEODETIC DOMINATION NUMBER

Definition 3.1: A Double Geodetic Dominating set S of a graph G is said to be a *Strong Split Double Geodetic Dominating set* of G if the subgraph induced by $V - S$ is totally disconnected. That is, the subgraph induced by $V - S$ is independent. The minimum cardinality of all such strong split double geodetic dominating set of G is called the *Strong Split Double Geodetic Domination Number* of G and is denoted by $\gamma_{DG}^{ss}(G)$. A strong split double geodetic dominating set of cardinality $\gamma_{DG}^{ss}(G)$is called a γ_{DG}^{ss} $-$set of G.

Example 3.2: Consider the graph G as in Figure 3.2 (a).

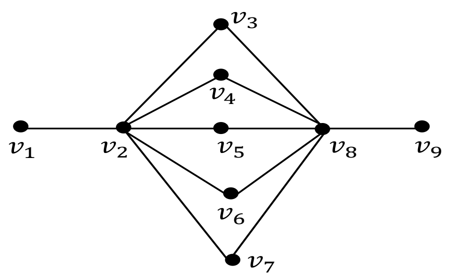

Figure 3.2(a)

Clearly, $\{v_1, v_2, v_8, v_9\}$ is a minimum strong split double geodetic dominating set of G and so $\gamma_{DG}^{ss}(G) = 4$.

Observation 3.3:

1. A complete graph has no strong split double geodetic dominating set and
2. All graphs need not have strong split double geodetic dominating sets.

Problem 3.4: Characterize graphs with strong split double geodetic dominating sets.

Let ζ''denote the collection of all graphs having atleast one strong split double geodetic dominating set.

Definition 3.5: Let $G \in \zeta''$. Then, the minimum cardinality of all strong split double geodetic dominating sets of G is called the strong split double geodetic domination number of G. It is denoted by $\gamma_{DG}^{ss}(G)$. A strong split double geodetic dominating set of cardinality $\gamma_{DG}^{ss}(G)$ is called a γ_{DG}^{ss} −set of G.

Observation 3.6: Let $G \in \zeta''$. The following are observed.

1. Every strong split double geodetic dominating set is a split double geodetic dominating set of G and further a double geodetic dominating set of G. Therefore, $\gamma_{DG}^{ss}(G) \geq \gamma_{DG}^{s}(G) \geq \gamma_{DG}(G) \geq \max\{\gamma(G), dg(G)\}$.
2. Every strong split double geodetic dominating set is a strong split dominating set of G and further a split dominating set of G. Therefore, $\gamma_{DG}^{ss}(G) \geq \gamma_{ss}(G) \geq \gamma_s(G)$. [By definition 1.4 and 1.5]
3. Every extreme vertex of G belongs to all strong split double geodetic dominating sets of G.
4. $2 \leq \gamma_{DG}(G) \leq \gamma_{DG}^{s}(G) \leq \gamma_{DG}^{ss}(G) \leq p$.

Proposition 3.7: Let $G \in \zeta''$. Then, $\gamma_{DG}^{ss}(G) \geq \alpha_0(G)$.

Proof. Let S be a $\gamma_{DG}^{ss}(G)$ −set of G. Then, $V - S$ is independent. Therefore, $|V - S| \leq \beta_0(G)$. That is, $p - \gamma_{DG}^{ss}(G) \leq \beta_0(G) = p - \alpha_0(G)$. Hence, $\gamma_{DG}^{ss}(G) \geq \alpha_0(G)$.

Proposition 3.8: Let $G \in \zeta''$ and let S be a strong split double geodetic dominating set of G. Then, every $w \in V - S$ lies in a $u - v$ geodesic of length 2 for some $u, v \in S$.

Proof. Let S be a strong split double geodetic dominating set of G and $w \in V - S$. Since w is adjacent to no vertex of $V - S$ and S is a double geodetic set of G, w lies in a $u - v$ geodesic P for some $u, v \in S$.

Proposition 3.9: Let $G \in \zeta''$. Then, a double geodetic dominating set S of G is a strong split double geodetic dominating set of G if and only if for $w_1, w_2 \in V - S$, every $w_1 - w_2$ path contains a vertex of S.

Proof. Suppose S is a strong split double geodetic dominating set of G. Then, $V - S$ is an independent set. Also, every path joining two vertices of $V - S$ contains a vertex of S. Conversely, let S be a double geodetic dominating set of G. If two vertices $u, v \in V - S$ are adjacent, then the edge uv is $u - v$ path in the subgraph induced by $V - S$ and it contains no vertex of S. This is a contradiction to our assumption and so $V - S$ is totally disconnected. Hence, S is a strong split double geodetic dominating set of G.

Proposition 3.10: For $n > 3$.

$$\gamma_{DG}^{ss}(P_n) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil & if\ n\ is\ odd \\ \frac{n}{2} + 1 & if\ \ n\ is\ even \end{cases}$$

Proof. Let $n > 3$ and let $P_n = (v_1, v_2, v_3, \dots\dots, v_n)$

Case 1: n is odd

Let $S = \{v_1, v_3, \dots\dots, v_n\}$. Since $v_1, v_n \in S$, if $w = v_i \in V - S$, then $v_{i-1}, v_{i+1} \in S$ and w lies in the $v_{i-1}v_iv_{i+1}$ geodesic joining v_{i-1} and v_{i+1}. Further, it is dominated by both v_{i-1} and v_{i+1}. Therefore, S is a double geodetic dominating set of P_n. By construction of S, no two vertices of $V - S$ are adjacent. Therefore, S is a strong split double geodetic dominating set of P_n and so

$\gamma_{DG}^{ss}(P_n) \leq |S| = \left\lceil \frac{n}{2} \right\rceil. ------- (1)$.

Further, as every double geodetic dominating set contains the end vertices of the path, the maximum cardinality for $V - S$, where S is a strong split double geodetic dominating set of P_n is $\left\lfloor \frac{n}{2} \right\rfloor$.

Therefore, $|V - S| \leq \left\lfloor \frac{n}{2} \right\rfloor$.

That is, $|S| \geq |V| - \left\lfloor \frac{n}{2} \right\rfloor = \left\lceil \frac{n}{2} \right\rceil$, as n is odd.

Therefore,$\gamma_{DG}^{ss}(P_n) \geq \left\lceil \frac{n}{2} \right\rceil -------- (2)$.

From (1) and (2), it is proved that if $n > 3$ is odd, then$\gamma_{DG}^{ss}(P_n) = \left\lceil \frac{n}{2} \right\rceil$.

Case 2: n is even.

Let $S = \{v_1, v_3, \dots\dots, v_{n-1}, v_n\}$. Since $v_1, v_n \in S$, if $w = v_i \in V - S$, then $v_{i-1}, v_{i+1} \in S$ and w lies in the $v_{i-1}v_iv_{i+1}$ geodesic joining v_{i-1} and v_{i+1}. Further, it is dominated by both v_{i-1} and v_{i+1}. Therefore, S is a double geodetic dominating set of P_n. By the construction of S, no two vertices of $V - S$ are adjacent. Therefore, S is a strong split double geodetic dominating set of P_n and so $\gamma_{DG}^{ss}(P_n) \leq |S| = n + 1. --------- (3)$.

Further, as every double geodetic dominating set contains the end vertices of the path, the maximum cardinality for $V-S$, where S is a strong split double geodetic dominating set of P_n, is $\frac{n}{2}-1$.

Therefore, $|V-S| \leq \frac{n}{2}-1$.

That is $|S| \geq \frac{n}{2}+1$.

Then, $\gamma_{DG}^{ss}(P_n) \geq \frac{n}{2}+1$. ------(4).

Hence, from (3) and (4), if $n>3$ is even, then $\gamma_{DG}^{ss}(P_n) = \left\lceil\frac{n}{2}\right\rceil+1$.

Proposition 3.11. For $n>3$ is even, then $\gamma_{DG}^{ss}(C_n) = \frac{n}{2}$.

Proof. Clearly, $S = \{v_1, v_3, \dots\dots, v_{n-1}\}$ is a strong split double geodetic dominating set of C_n and so $\gamma_{DG}^{ss}(C_n) \leq |S| = \frac{n}{2}$. --------(1).

Further, if S is a strong split double geodetic dominating set of C_n. $then$, $V-S$ is independent and so the maximum cardinality for $V-S$ is $\frac{n}{2}$.

Therefore, $\gamma_{DG}^{ss}(C_n) \geq \frac{n}{2}$. ----------(2).

In other words, from (1) and (2), $\gamma_{DG}^{ss}(C_n) = \frac{n}{2}$.

Proposition 3.12: For $m,n \geq 2$, $\gamma_{DG}^{ss}(K_{m,n}) = \min\{m,n\}$.

Proof. Let U,W be the partition of $V(K_{m,n})$ with $|U| = m$ and $|W| = n$. Obviously, U and W are strong split double geodetic dominating sets of $K_{m,n}$. Let S be a strong split double geodetic dominating set of $K_{m,n}$.$Then$, as $V-S$ is independent, it should not contain part of U as well as part of W. If U or W is a proper subset of S, then S is not a minimal strong split double geodetic dominating set of G. Therefore, U and W are the only minimal strong split double geodetic dominating set of G. Hence, $\gamma_{DG}^{ss}(K_{m,n}) = \min\{|U|,|W|\} = \min\{m,n\}$.

Proposition 3.13: If $\gamma_{DG}^{ss}(G) = 2$, then every minimum strong split double geodetic dominating set of G is a minimum independent double geodetic dominating set of G and $i_{DG}(G) = 2$.

Proof. Suppose $\gamma_{DG}^{ss}(G) = 2$. Let $S = \{u,v\}$ be a minimum strong split double geodetic dominating set of G. Then, $V-S$ is totally disconnected and every vertex $w \in V-S$ lies in a uwv geodesic joining u and v. Therefore, u and v are non-adjacent and so S is an independent double geodetic dominating set of G. S is a minimum double independent geodetic dominating set of G and hence $i_{DG}(G) = 2$.

Converse of the above Proposition is not true. For example 3.13(a),

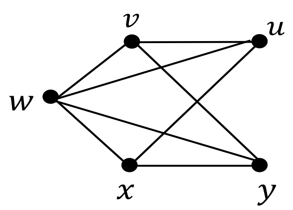

Figure 3.13(a)

Considering the graph in figure 3.13(a), $\{u,y\}$ is a minimum independent double geodetic dominating set of G and so $i_{DG}(G) = 2$. But, $\{u,y\}$ not a strong split double geodetic dominating set of G. Further, form the figure, it is clear that G has no strong split double geodetic dominating set of cardinality 2 and$\{u,y,w\}$ is a minimum strong split double geodetic dominating set of G. Therefore, $\gamma_{DG}^{ss}(G) = 3$.

Corollary 3.14: Let G be a connected graph with $p \geq 3$ vertices. Then, $\gamma_{DG}^{ss}(G) = 2$ if and only if G is isomorphic to $K_{2,p-2}$.

Proof. Suppose $\gamma_{DG}^{ss}(G) = 2$ and let $S = \{u,v\}$ be a $\gamma_{DG}^{ss}-set$ of G. Then, by Proposition 3.13, S is an independent set and every edge of G has one end in S and the other end in $V-S$. Clearly, G is isomorphic to a bipartite graph with vertex partition U,W such that $U=S$ and $W=V-S$. Further, as S is geodesic, every vertex in $V-S$ is adjacent to both u and v in S. Further $V-S$ is an independent set of G. Therefore, $G \cong K_{2,p-2}$.

IV. CONCLUSION

Domination not only in Graph Theory but also in real life Problems plays a vital role. It helps to solve many real life situations.

REFERENCES

[1] Haynes T. W, Hedetniemi S. T and Slater P. J, *Fundamentals of Domination In Graphs*, 208, Marcel Dekker, Inc, New York, 1998.

[2] Buckley F, Harary F, And Quintas L.V, *External Results on The Geodetic Number Of A Graph*, Scientia 2a (1988), 17-26.

[3] Chartrand G, Harary F, Swart H. C and Zhang P, *Geodomination in Graphs*, Bull. Ica 31 (2001), 51-59.

[4] Chartrand G, Harary F and Zhang P, *Geodetic Sets in Graphs,* Discussiones Mathematicae Graph Theory 20 (2000), 129-138.
[5] Chartrand G, Harary F and Zhang P, *On The Geodetic Number of a Graph,* Networks 39 (2002), 1-6.
[6] Kulli V.R and Janakiram B, *The split domination number of a graph*, Graph Theory Notes of New York, New York Academy of Sciences, XXXII, (1997) 16-19.
[7] Kulli V.R and Janakiram B, *The strong split domination number of a graph*, Acta Ciencia Indica, 32 M(2006) 715-720.
[8] Muntean R and Zhang P, *On Geodomination In Graphs,* Congressus Numerantium 143 (2000), 161-174.
[9] Nellai Murugan A and Esakkimuthu A., *Double geodetic Domination Number of a Graph,* Communicated
[10] Robinson Chellathurai S And Padma Vijaya S, *The Geodetic Domination Number For The Product Of Graphs*, Transactions On Combinatorics, (2014 University Of Isfahan), Issn (On-Line): 2251-8665 Vol. 3 No. 4 (2014), Pp. 19-30.
[11] Santhakumaran A.P And Jebaraj T, *Double Geodetic Number of A Graph,* Discussiones Mathematicae ,Graph Theory 32 (2012) 109–119.
[12] Sampathkumar E And Walikar H.B, *The Connected Domination Number of A Graph*, J.Math.Phys.Sci.,13(1979) 607-613.

