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Characterization of Fuzzy b-open Set in Fuzzy Topological Spaces

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***Abstract:** The aim of this paper is to create and estimate the theorems which exhibit the characterization of fuzzy b-open set in fuzzy topological space by using fuzzy b-open set introduce by Benchalli and Karnel [4] and We establish the relation between f b-closure & f b-interior along with their representations in terms of fscl, fpcl, fsint & fpint.*

***Keywords:** Fuzzy b-open set, fuzzy α-open set, fuzzy topological space. AMS Classification No. 54C05; 54C08; 54C10.*

I. INTRODUCTION

The mathematical paper [1] introduce and investigate fuzzy pre-open set and fuzzy semi-pre-open sets which are some of the weak forms of fuzzy open sets and the complements of these sets are obviously the same type of fuzzy closed set.

In the recent years a number of generalization of fuzzy open sets have been considered in the literature.

II. PRELIMINARIES

Throughout this paper X means fuzzy topological spaces (fts in short).The notation *f b*-open and *f* α -open will denote respectively fuzzy *b*-open and fuzzy α-open in fts *X*. Let *A* be a fuzzy subset of *X*. We denote the fuzzy closure and fuzzy interior of a fuzzy set *A* by *cl(A)* and *int(A)* respectively.

A family $\delta \leq I^X$ of fuzzy sets is called a fuzzy topology for X if it satisfies the following three axioms;

1. $0_X, 1_X \in \delta$
2. $\forall A, B \in \delta \Rightarrow A \wedge B \in \delta$
3. $\forall (A_j)_{j\in J} \in \delta \Rightarrow \vee_{j\in J} A_j \in \delta$

The pair (X, δ) is called a **Fuzzy topological space or fts.**

Fuzzy operations: The closer and interior of a fuzzy set *A* of *X* are define as

cl(A) = inf $\{K: A \leq K, K^c \in \delta\}$

int(A)= sup$\{O: O \leq A, O \in \delta\}$

Definition 2.1[1]: A fuzzy set S of a fts X is called fuzzy pre-open (fuzzy pre-closed) if S ≤int cl S (cl int S≤S)

Definition 2.2 [1]: A fuzzy set S of a fts X is called fuzzy semi-open (fuzzy semi- closed) if S≤ cl int S (int cl S≤S)

Definition 2.3[1]: A fuzzy set S of a fts X is called fuzzy semi-pre-open (fuzzy semi-pre closed) if S≤cl int cl S(int cl int S≤ S)

Definition 2.4[1]: A fuzzy set S of a fts X is called fuzzy α-open (fuzzy α-closed) if S≤int cl int S(cl int cl S≤ S)

Fuzzy b-open set and fuzzy b-closed sets

Definition 2.5[4]: A fuzzy set *S* in a fts *X* is called fuzzy *b*-open (*f b*-open) set
if and only if *S≤int (cl S)* ∨ *cl (int S)*

Definition 2.6[4]: A fuzzy set S in fts X is called fuzzy b-closed (fb-closed)set
if and only if *int (cl S)* ∨ *cl (int S) ≤S*

All the above given definitions are different and independent as illustrated by the following example:

Example 2.1: Let *X*= $\{a, b\}$ and consider the fuzzy sets on *X* are α=$(a_{0.3}, b_{0.4})$, β= $(a_{0.7}, b_{0.8})$, $\gamma = (a_{0.6}, b_{0.5})$. Let τ= $\{0_X, 1_X$,α$\}$ fuzzy topology on X .Then γ is f fb-open set but not fp-open set.

Example 2.2: Let *X*= $\{a, b\}$ and consider the fuzzy sets on *X* are α=$(a_{0.4}, b_{0.4})$, β= $(a_{0.7}, b_{0.8})$, $\gamma = (a_{0.6}, b_{0.5})$. Let τ= $\{0_X, 1_X$,α$\}$ fuzzy topology on X. Then γ is fsp - open set but not f b-open set.

Lemma 2.1: Let (X, δ) be a Fuzzy topological space, then

(i) An arbitrary union of fb-open set is fb-open set.

(ii) An arbitrary intersection of fb-closed set is fb-closed.

Proof (i) Let $\{A_\alpha\}$ be the collection of fb-open sets. Then for each α , $A_\alpha \leq$ (cl int A_α) ∨ (int cl A_α). Now ∨ A_α ≤∨((cl int , A_α)∨(int cl , A_α)) ≤(((cl int(∨A_α))∨(cl int (∨, A_α))). Thus ∨A_α is fuzzy b-open set.

(ii) Similar by taking complement.

Lemma 2.2: Let (X, δ) be a Fuzzy topological space and A≤X, then

(i) $(\delta\text{- fbcl}(A))^C = \delta\text{- fbint}(A^C)$

(ii) $(\delta\text{-fbint}(A))^C = \delta\text{- fbcl}(A^C)$

Proof: Let A≤X where (X, δ) is a fts

(i) Now, δ-fbcl(A)=∩ {F:A<F and F is δ –fb-closed set}

$(\delta\text{-fbcl}(A))^C = [\cap \{F{:}A<F \text{ and } F \text{ is } \delta\text{–fb-closed set}\}]^C$
$= \cup \{F^C{:}\ F^C<A^C \text{ and } F^C \text{ is a } \delta\text{-fb-open set}\}$
$= \delta\text{- fbint}(A^C)$

(ii) Similarly, $(\delta\text{-fbint}(A))^C = \delta\text{- fbcl}(A^C)$

III. CHARACTERIZATION OF FB-OPEN SETS

We now prove the following theorem which characterizes a fb-open set:

Theorem (3.1):

If (X, δ) is a Fuzzy topological space then, the intersection of an fα–open set and a fb-open set is a fb-open set.

Proof: Suppose that (X, δ) is a Fuzzy topological space.

Let A be an fα-open set and B a fb-open set.

Now, S=A∩B
=fαint (A) ∩ fbint (B)
≤ fbint(A)∩ fbint (B)
=fbint (A∩B)

i.e. S≤fbint(S)

But, fbint(S) <S

Hence, S=fbint(S) i.e. S= A∩B is a fb-open set.

Theorem 3.2:

If S be a subset of a fts (X, δ), then

(i) fbcl S=fscl S ∩ fpcl S

(ii) fbint S=fsint S∪ fpint S

Proof:

Let S ≤ X where (X, δ) is a fts.

(i) Since fbcl S is a b- closed set.

Hence, int(cl(fbcl S)) ∩ cl(int(fbcl S)) ≤ fbcl S.

Again, int(cl S) ∩ cl(int S) ≤ int(cl(fbcl S) ∩ cl(int(fbcl S)).

i.e. int(cl S) ∩ cl(int S) ≤ fbcl S.

i.e. S∪{int(cl S) ∩ cl(int S)} ≤ S∪fbcl S.

i.e. [S∪int(cl S)] ∩ [S ∪ cl(int S)] ≤ fbcl S.

i.e. fscl S ∩ fpcl S ≤ bcl S ……………..(1)

Next, fbcl S ≤ fscl S and fbcl S ≤ fpcl S

i.e. fbcl S ≤ fscl S ∩ fpcl S………………(2)

from (1) & (2) it follows that

fbcl S= fscl S∩ fpcl S

(ii) Since fbcl S is a fb-open set, we have

cl(int(fbint S)) ∪ int(cl(fbint S))≥fbint S

Again, cl(int(fbint S)) ∪ int(cl(fbint S)) ≤ cl(int S) ∪int(cl S)

i.e. fbint S≤ cl(int S) ∪int(cl S)

i.e. S∩fbint S≤{S∩cl(int S)} ∪ {S ∩ cl(int S)}

i.e. fbint S≤ fsint S ∪ fpint S…………………..(1)

Next, fsint S ≤ fbint S & fpint S≤ fbint S.

i.e. fsint S ∪ fpint S≤ fbint S…………………(2)

From (1) & (2), it follows that

fbint S=fsint S ∪fpint S.

Hence theorem.

IV. CONCLUSIONS

Characterization ensures that separation axiom can be framed in terms of b-open sets. Spaces profounded by b-open set as b-connected and b-compact space are in existence. Also different types of continuity evolve in the form of b-open set.

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