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Stability and Link Reduction Methodology of Multi-agent System

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Abstract— This paper investigates about the structural stability and link reduction methodology for high order integrated dynamic multi-agent systems. Recently, the majority of multi-agent system's problems have been focused on structural controllability aspects. Here, we consider that a group of agents in leader-follower structure are under a fixed topological arrangement and the agent's interconnection is a weighted graph with freely chosen weights such that the corresponding system is structurally controllable. Under this organization, it is shown that structural stability guarantees structural controllability. Moreover, a methodology namely-Link reduction is introduced to minimize the numbers of links of an intricate graph. Finally, a numerical example and simulation results are presented to illustrate the approach.

Keywords— Graph Theory, Multi-agent system, Pole-Placement method, Link Reduction, Complex Network

I. INTRODUCTION

Distributed coordination of any network's dynamics agents has attracted massive attention as a hot research area in the last few years. This is due to its huge application base considering areas like cooperative control of unmanned aerial vehicles, smart grid, infrastructure less networks like wireless sensor networks (WSN), self-organizing networks, swarm robot control and many others [1]-[9]. These application domains can always be described with the key term, namely "Multi-Agent System".

The primary goal of a multi-agent system that consists of a group of simple dynamic agents is to achieve a complicated task through the employment of multiple agents. The motivation behind this can be understood intuitively. So, the behaviour of individual agent is not only depended on its dynamics but on the behaviour of its neighbours [11]. In this field, the initial criterion is to design an interconnected topology amongst the agents such that whole system is well-organized. But here the major difficulty is that, how an interconnected system could be steered to some specific destination by controlling the motion of a single agent which plays a vital role of this group (leader) [12]. Structural controllability satisfies this criterion. If all the agents of a group can be controlled by their interconnection topology then the overall system is treated as structurally controllable system [11]. The idea of controllability was first introduced by Lin [10]. Many researchers were involved in studying the problem. In [11], the authors considered the system as a single integrator dynamic agent. Based on this work, double integrator [14] and high order integrator [13], [14] were taken into account. First, the structural controllability problem was proposed from graph theoretic point of view for multi-agent system in [11], and [12].

In contrast to the existing literature, we will consider weighted graph under a fixed topological feature. The link weights of the graph can be freely chosen such that it is structurally controllable. Controllability does not guarantee stability. From a structural stable system with bounded input, bounded output can be expected. But controllability does not fulfil this criterion. So, in this paper our main focus is to assign the link weights among agents in such a way that the overall system structurally stable as well as controllable. Based on that, we propose another methodology called link reduction which minimizes the number of links of a complicated graph to make it stable and controllable.

The rest of the paper is organized as follows: in section II, we have discussed some mathematical basics of graph theory and introduced the necessary and sufficient condition for a graph to be structurally controllable one. In section III, we have formulated our problem. Section IV has presented the Pole-Placement methodology [15] which is applied to our problem for stability purpose. In section V, we have discussed about link reduction technique and discussed how it is integrated into the system. Section VI, concludes the present work.

II. MATHEMATICAL BASICS OF GRAPH THEORY

A. *Basic Graph Theory*

A weighted graph is a depiction for the communication links among agents because it establishes the strength of these links among agents. A weighted graph [16] G with N vertices consists of a vertex set $\nu = \{v_1, v_2, \ldots, v_N\}$ and an edge set $e = \{e_1, e_2, \ldots, e_N\}$, which is the set of interconnection links among the vertices. Each agent is represented by a node and each pair $e_{ij}(v_i, v_j)$ implies the agents v_i and v_j share information among themselves. The links present in the graph, have some specific weight w_{ij} associated with them and it is assumed that $w_{ij} \neq w_{ji}$. Two vertices i and j are called as neighbours if $(i, j) \in e$. Vertices x and y are adjacent if x is a neighbour of y. A path is an alternating sequence of the distinct edges in e such that all consecutive agents are adjacent. The graph G is said to be connected if there exist a path between any two different vertices [16].

The weighted adjacency matrix [16], ‘A’ can be defined as

$$A_{(i,j)} = \begin{cases} w(i,j) & (i,j)\in e \\ 0 & otherwise \end{cases} \quad (1)$$

The Laplacian matrix of a graph G [16] can be denoted by ‘L’,

$$L_{(i,j)} = \begin{cases} \sum_{i\neq j} w(i,j) & i=j \\ -w(i,j) & (i,j)\in e \\ 0 & otherwise \end{cases} \quad (2)$$

B. Structural Controllability of Multi-agent System

According to the graph theoretical point of view [17], a leader-follower oriented graph can be written in the state-space form;

$$\dot{x} = Ax + Bu \quad (3)$$

Where, A (state matrix) depicts the interconnection between the agents, B (input matrix) represents the interconnection between the followers and leaders and u the control input is given to the leader. The condition of controllability (Kalman’s full rank condition [15]) is,

$$Q = [B \quad AB \quad A^2B \quad .... \quad A^{n-1}B] \quad \text{is of rank 'n'}.$$

To make a system structurally controllable, we have to find the state and input matrices in such a manner that the system obeys the classical rank condition (Kalman’s full rank condition).Therefore, the problem could be posed in a manner so that it dictates us to obtain a set of values for the link weight matrix such that it is possible to drive the agents to any configuration by properly designed control signal.

For high order multi-agent system, each agent’s dynamics [13] can be given by an n^{th} order differential equation,

$$\dot{x}_i^{(1)} = x_i^{(2)}, \ldots, \dot{x}_i^{(n-1)} = x_i^{(n)}$$
$$\dot{x}_i^{(n)} = u_i \qquad i=1 \ldots N+1. \quad (4)$$

Where n is a positive integer, representing the order of the equation. $\dot{x}_i^{(m)} \in R$ is the m^{th} order derivative of the i^{th} agent state and $u_i \in R$ is the single input to control the overall system dynamics. It might be assumed that the graph has a single input (leader) with N follower agents.

The control input (neighbor-based law [12]) can be represented by;

$$u_i = -\sum_{j\in N} \sum_{m=0}^{n-1} K_m (x_i^{(m+1)} - x_j^{(m+1)}) \quad (5)$$

Where $K_0, K_1, \ldots, K_{n-1}$ are the non-zero feedback gains of the relative states.

Example 1: Consider a multi-agent system with six agents having one leader and five followers are shown in the figure 1 [11]. The matrices A and B can be written as,

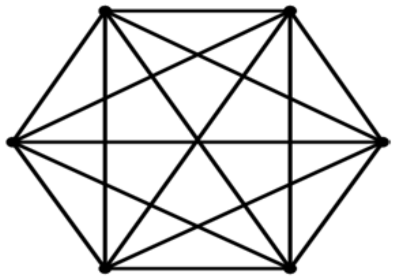

Fig. 1 A graph of 1 leader and 5 followers

$$A = \begin{bmatrix} 7 & -2 & -2 & -2 & -1 \\ -2 & 9 & -3 & -2 & -2 \\ -2 & -3 & 13 & -5 & -3 \\ -2 & -2 & -5 & 11 & -2 \\ -1 & -2 & -3 & -2 & 8 \end{bmatrix} \quad (6)$$

$$B = \begin{bmatrix} -1 \\ -2 \\ -5 \\ -3 \\ -1 \end{bmatrix} \quad (7)$$

From the above pair (A, B), it is clearly seen that the above system is controllable in classical sense. But, if it is checked very carefully, then it would be seen that the system is unstable.

Now, it is our objective to stabilize the system i.e. allocate the link weight matrices of the system in such a manner that the system is stable but still controllable. For doing this, we use pole-placement technique.

III. PROBLEM STATEMENT

The problem is whether we can find a weighting scheme, i.e., a set of values for link weight matrix, such that the overall system would be formally stable and structurally controllable. If the link weights are selected freely and properly, the system (equation 3) is reduced to an LTI system. Its controllability and stability can be directly answered by well-developed linear system theory [15]. In paper [11], a weighting scheme was proposed to make a graph structurally controllable. But that was not much efficient to satisfy the stability criterion. In this paper, we have introduced a state-feedback gain matrix (Pole-placement methodology [15]) into the system which makes the system stable as well as controllable. Figure 2 represents the corresponding block diagram.

IV. POLE PLACEMENT METHODOLOGY

If all state variables are measurable and are available for feedback. It can be shown that if a system is completely state controllable, poles of the closed loop system can be placed at any desired locations by the use of state feedback mechanism. This methodology is termed as "pole-placement" [15].

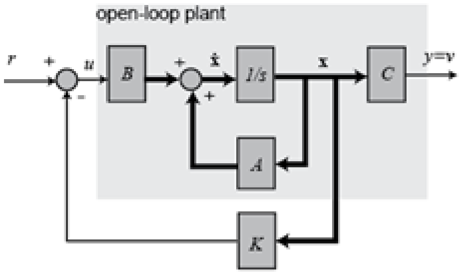

Fig. 2 Block Diagram of Pole-Placement Method

Let assume that the desired closed loop are located at s=m_1, m_2, ..., m_n. Then by choosing an appropriate gain matrix for state feedback, it is possible to place the close loop pole at desired locations but the original system should be completely state controllable.

The control signal is,

$$u = -K * x \tag{8}$$

This means that the control signal u is determined by an instantaneous state. Such scheme is known as state feedback and 'K' is called the state feedback gain matrix [15].

Then the state-space equation for such system is;

$$\dot{x} = (A - BK) * x \tag{9}$$

If the K matrix is properly chosen, then the matrix (A - BK) can be made asymptotically stable. The eigenvalues of matrix (A - BK) are called regulator poles. If this regulator poles are placed in the left-half of S-plane, then X(t) approaches to 0 as t approaches to infinity.

For the system shown in the figure 1, we place the poles of the system in the locations $\begin{bmatrix} -1 & -2 & -3 & -4 & -5 \end{bmatrix}$. Then the corresponding gain matrix for the system would be

$$K = \begin{bmatrix} 263.5 & -2571 & 113.7 & 1026.6 & 1167.2 \end{bmatrix}$$

The pair (A_{new}, B_{new}) can be written as;

$$A_{new} = \begin{bmatrix} 270 & -2573 & 112 & 1025 & 1166 \\ 525 & -5133 & 224 & 2051 & 2332 \\ 1315 & -12858 & 582 & 5128 & 5833 \\ 788 & -7715 & 336 & 3091 & 3500 \\ 262 & -2573 & 111 & 1025 & 1175 \end{bmatrix} \tag{10}$$

$$B_{new} = \begin{bmatrix} -1 \\ -2 \\ -5 \\ -3 \\ -1 \end{bmatrix} \tag{11}$$

From the above pair, it is clearly seen that the multi-agent system shown in the figure 1, is completely stable and controllable. But one thing should be noted is that structural stability guarantees structural controllability.

Then from the above argument we can say that a multi agent system is said to be structurally controllable and stable if and only if there exists $w_{ij} \neq 0$ which can make the system stable.
In the above example, there are twenty five (25) links (consider bi-directional links) among the agents (same as that of original one). Now we are trying to minimize the number of links without hampering the system dynamics. For doing so, we have proposed a technique called Link reduction.

V. LINK REDUCTION METHODOLOGY

Suppose, a system is defined by;

$$y^{(n)} + a_1 y^{(n-1)} + \cdots + a_{n-1}\dot{y} + a_n y = bu \tag{12}$$

Where u is the input and y is the output. The transfer function obtained from eqn. (12) under the assumption of zero initial conditions is;

$$T(s) = \frac{Y(s)}{U(s)} = \frac{b}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n} \tag{13}$$

This is the simple case, where the transfer function does not have zeros.
The above equation results in the following state equation;

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & \dots & -a_{n-1} & a_n \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} b \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} u \tag{14}$$

Or, $\dot{x} = Ax + Bu$

This form of matrix **A** is known as the bush form or companion form [15]. Now, if closer look is given into the above equation, it would be seen that, the state where the input command is given, is comprised of the negative of the coefficients of the original differential equation and all other elements are zero. B matrix also obeys the rule.

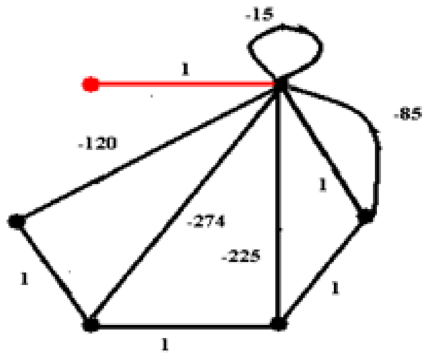

FIG. 3 MODIFIED SYSTEM AFTER LINK REDUCTION

Now, for the system shown in the figure 1, we use the above stated methodology. The corresponding state and input matrices would become

$$A_{Reduced} = \begin{bmatrix} -15 & -85 & -225 & -274 & -120 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{15}$$

$$B_{Reduced} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{16}$$

From the above matrices it is clearly seen that, the corresponding graph has nine (9) links which is the minimum links that would require for a 5^{th} order graph to be stable and controllable.

The system after link reduction methodology is given in the figure 3.From the above discussion, it might be come to mind is that what is the minimum number of links would be needed to make a graph formally stable and structurally controllable. The following theorem answers this;

Theorem 1: For an n^{th} order graph, the minimum number of links required to make it stable and controllable is (2n-1).

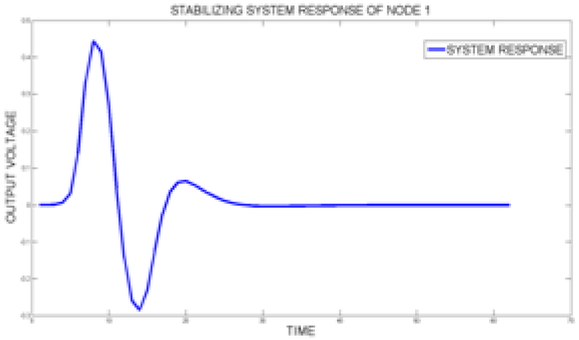

FIG. 4: RESPONSE OF THE NODE 1

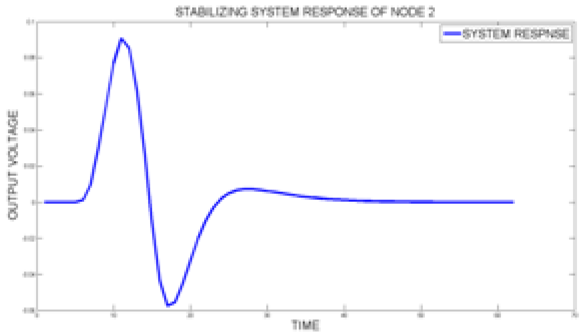

FIG. 5: RESPONSE OF THE NODE 2

The figures 4-8 represent the node's response of the graph shown in figure 3. For optimize its performance, we have used a well-known classical optimization technique namely linear quadratic regulator (LQR) into the system. But for lower order graph it was unable to optimize the performance, furthermore for higher order graph it was underperforming than the original one.

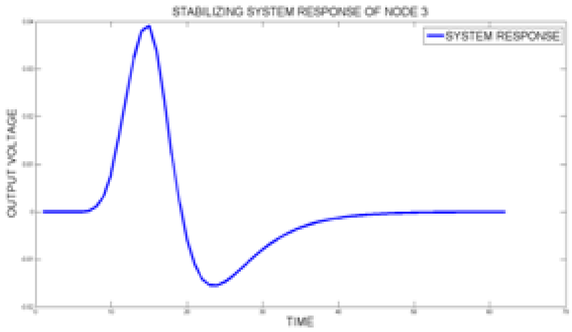

Fig.6: Response of the node 3

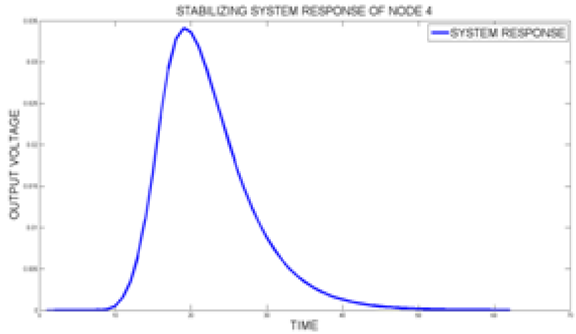

Fig. 7: Response of the node 4

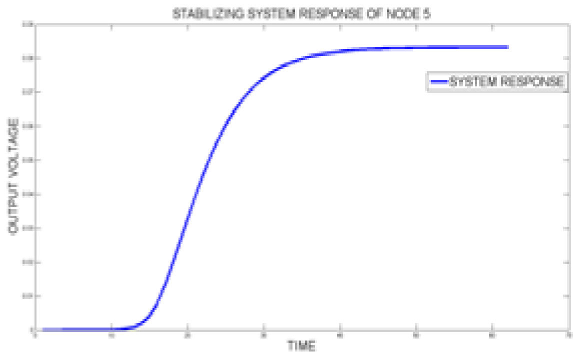

Fig. 8: Response of the node 5

VI. CONCLUSION

In this paper, controllability as well as stability problem for multi-agent system's interconnection via a fixed weighted topology is investigated. It is shown that structural controllability does not provide stability but vice versa is true. Link reduction methodology is introduced and it is shown that using the technique, we can easily minimize the number of links of a graph without disturbing its dynamics.

Further research is directed on developing some algorithms to optimize different node's response. In addition, assuming more than one leader in a group and high order dynamics realization for each agent are future challenge.

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