



Design of 100Hz High-Q IIR Notch Filter

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Abstract— Filter is a well known device used to select the frequency band to pass as per desire. In Digital Signal Processing application there are mainly two types of filters depending upon the impulse response and they are IIR filters and FIR filters. Notch filter is one of the important filter to deselect the desired frequency band and thereby passes all other frequency bands. In this paper, the design of a efficient High Quality Factor notch filter is proposed that will efficiently can stop the desired frequency band without declining any other band of frequency. The proposed design is with high Q and thereby producing sharp magnitude response with accurate pole-zero values and so the high order filter can be achieved by this design.

Keywords— IIR Filter, Notch filter, Analog-to-Digital Mapping, Magnitude response, Coefficients

I. INTRODUCTION

In Signal Processing system, filter is required to process the signal of desired band of frequencies. If there is a requirement of declining a bands of frequency and thereby pass all other frequencies, the band stop filter is essentially required. In this case let we have $f_1, f_2, f_3, \dots, f_{n-1}, f_n$ band of frequencies and the band f_r, \dots, f_{k-1} have to be stopped. So to perform this specific operation, band stop filter will have to be used. But if only one frequency will have to be stopped, the special type of band stop filter that is the Notch filter will be in use.[1][2][3][4][5].

The passive components such as the resistors, capacitors etc are generally used to construct the passive filters. That will be analog in construction type. Actually filter is a device and so it must have a transfer function and after Laplace transform of this, the transfer function in s-domain can be achieved[2][3][4][5]. After getting the s-domain function, the order of the filter can be determined[3][4][5]. The order of the filter is essential to determine the characteristic of the system of filtration technique. A high order filter is nearly behaves like an ideal filter. The so generated filter equation can be observed in frequency domain[4][5][6][8]. If it is required to view the stability and the pole and zeros of the system more accurately, the Z-Transform is required. The generated filter with ZT values and transfer function can be defined as the Digital Filter[3][5][7][8][10][13]. Now the Z-Transform of the analog filter can be obtained either by taking the input and output values in transformed version and thereby generating the transfer function in the digital plane or perform the analog to digital mapping technique to convert the analog transfer function to its equivalent digital transfer function directly. There are a number of techniques are available for analog to digital mapping technique and hence the Bilinear Transformation technique is applied in this paper for the design of a digital filter.[2][4][5][8][7][12].

II. PREVIOUS WORK

There different works and experiments were visualized on behalf of the Notch Filter specially on the emphasis of varying the Q factor and minimizing the noise[1][4][5][6][9][10]. Frey, D.R and Steigerwald, L. works on the Notch for adaptive control on it which requires very high frequencies[6]. The analogous work was proposed by Ye Wu, Yongge Wan and Yingqiu Li for adaptive filter design in data processing[1]. Qiusheng Wang and Deepa Kundur have proposed the design of multiple notch filter[24]. Erwin H. W. Chan and Robert A. Minasian have proposed a design to use the notch filter in RF frequency[25]. So, regarding to the notch filter and notching the required or selected frequency with determination of quality factor and thereby introducing the quality of efficiency have been proposed..

III. ANALOG FILTER DESIGN

The analog filter is essentially constructed from the passive components. That is the input signal is supplied from the input section either by function generator or from other sources. This signal is passed through the filter section, designed by passive components, and produce an output signal[2][5][8]. If the voltage of the input signal is fixed and the frequency of the signal will be varied, a time comes when the filter starts working that is if this is a band stop filter the selected band of frequencies will be stopped. This can be well described by deriving the gain function of the filter[8][11][13][14].

The design of the conventional Notch filter is shown below using Twin-T topology [2][3][4][5][8].

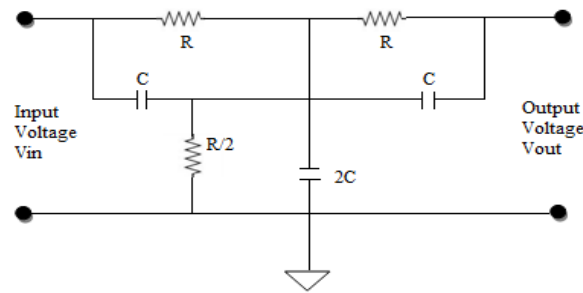


Fig.1 Twin-T Notch Filter

The Fig.1 shows the conventional Twin-T Band Stop filter. The values of the filter can be adjusted well accurately to design the notch frequency and thereby designing the notch filter. The objective is the construction of the notch filter transfer function in frequency domain and transfer it to the digital domain for the design of the digital filter and then by calculating the gain, the magnitude response can be obtained[5][8][10][12][16][22][23].

IV. PROPOSED DESIGN

In the proposed design of the Notch filter, the values of the resistors and capacitors are so calculated that it can efficiently able to cut the desired frequency only and the just former and the later frequency bands can pass and thereby implementing an efficient filter. The design of the Notch filter is done using Multisim v12 by National Instrument..The proposed design is shown below in Fig.2.

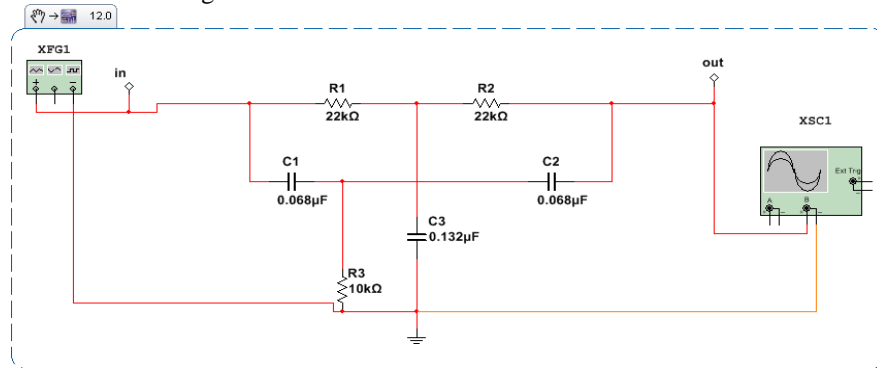


Fig.2 Proposed Design

The design of the Proposed Notch is constructed and simulated in NI Multisim Pro Edition 12.0. The Q factor for proposed design is derived as the ration of the centre frequency to the -3dB bandwidth. The required calculations will be shown on the preceding section.

V. IMPLEMENTATION

After designing the circuit, the simulation have been done. As the filter is analog in type of construction, this is converted into its equivalent digital filter to check the stability and magnitude response as well as the pole-zero positions of the filter. Firstly, the proposed analog filter is converted into digital filter using analog to digital mapping technique with several modification with DTA algorithm[5].

Analog to Digital mapping is very essential for the design of a digital filter. This is essentially required for the calculation of the transfer function of the digital filter in z-plane[2][3][5][13][17][18].

A. Transfer function in digital plane

The transfer function of an analog filter in s-plane can be mapped into z-plane with the application of bilinear transform. Bilinear transform operates on the analog plane to convert it to z-plane using the following equation[2][4],

$$z = \frac{1 + s}{1 - s} \dots\dots\dots(1)$$

where,

$$s = \text{complex variable}$$

$$= s = \sigma + j\Omega$$

Now, putting the value of s into equation (1), we get,

$$z = \frac{1 + (\sigma + j\Omega)}{1 - (\sigma + j\Omega)} \dots\dots\dots(2)$$

$$\Rightarrow z = \frac{(1 + \sigma) + j\Omega}{(1 - \sigma) - j\Omega}$$

$$\Rightarrow |z|^2 = \frac{(1 + \sigma)^2 + \Omega^2}{(1 - \sigma)^2 + \Omega^2} \dots\dots\dots(3)$$

The transfer function of an IIR Digital filter can be described by the following equation[1][2][4][5][8][11][12][15],

$$H(z) = \frac{\sum_{n=0}^M b(n)z^{-n}}{1 + \sum_{n=1}^N a(n)z^{-n}} \dots\dots\dots(4)$$

$$\Rightarrow H(z) = \frac{B(z)}{A(z)}$$

$$\Rightarrow H(z) = \frac{b(0) + b(1)z^{-1} + b(2)z^{-2} + \dots\dots\dots + b(M)z^{-M}}{1 + a(1)z^{-1} + a(2)z^{-2} + \dots\dots\dots + a(N)z^{-N}} \dots\dots\dots(5)$$

Where,

H(z) = Transfer function and Z-transform of impulse response h(n)

b(n) = Numerator coefficient

a(n) = Denominator coefficient

Now, for a realizable filter, h(n) and H(z) can be described by[4][5][8][9][14][15][17],

$$h(n) = 0 \quad \text{for } n \leq 0 \dots\dots\dots(6)$$

$$\sum_{n=0}^{\infty} |h(n)| < \infty \dots\dots\dots(7)$$

$$H(z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n} \dots\dots\dots(8)$$

Equation (7) shows the satisfactory condition for a stable filter. Now, it is necessary to determine the transfer function of a digital filter i.e. of z-domain from the transfer function of s-domain. Next subsection shows the necessary derivation of the transfer function in z-domain.

B. Determination of Transfer Function in z-plane

Let we take the impulse response of a realizable filter in time domain is h(t). If the Laplace Transform is applied on h(t), the transfer function in s-plane can be achieved[1][3][13][16][20] by,

$$H(s) = L\{h(t)\} = \int_0^{\infty} h(t).e^{-st} dt \dots\dots\dots(9)$$

Now, for continuous time signal, the t of h(t) can be replaced by nT, that is,

$$h(t) = h(nT) \dots\dots\dots(10)$$

Where, T=Sampling time

If, T=1, then equation (10) becomes,

$$h(t) = h(n) \dots\dots\dots(11)$$

Now, h(n) is the impulse response in z-plane. So, in this process, the impulse response of z-plane can be obtained from the impulse response in s-plane. If the Z-Transform is applied on h(n), the transfer function in z-plane can be obtained[2][3][5][7][12][15].

$$H(z) = Z\{h(n)\} = \sum_{n=-\infty}^{\infty} h(n)z^{-n} \dots\dots\dots(12)$$

C. Domain Transfer Algorithm

The DTA algorithm is shown in Fig.3 for the design of the analog filter to digital filter for the checking of its stability and implementation[1][4][5][18][19][21][22]. This algorithm actually effectively realizes the digital filter structure by first obtaining the transfer function of analog domain that is in s-plane. When the transfer function is obtained, the coefficients are collected as backup for future use for comparison or in development purposes. After that suitable cutoff frequency, pass band frequency and stop band frequency is calculated with respect to the designed circuit. On the transfer function in s-plane, the domain transformation is applied using s-plane to z-plane mapping technique. Generally it uses Bilinear Transformation where the transfer function in z-plane can be obtained with less errors. Again, the coefficients are observed and on the basis of the result, the required digital filter will be designed.

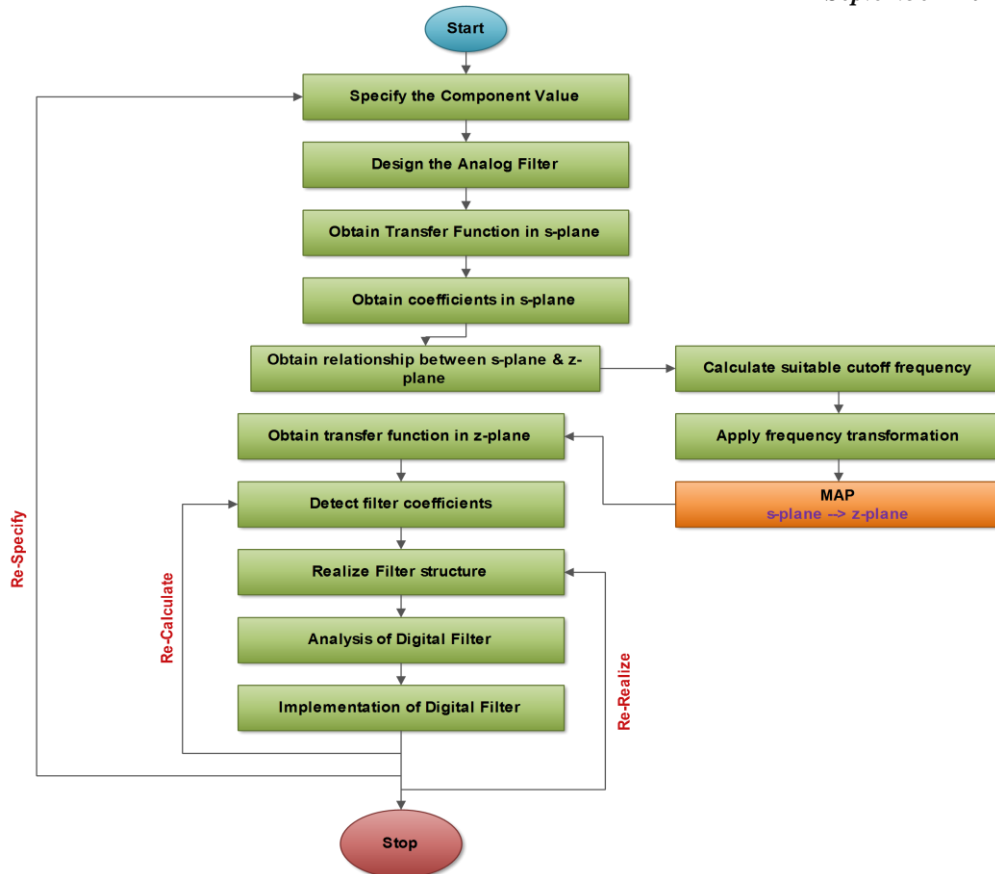


Fig.3 Domain Transfer Algorithm

So, by applying the DTA, the preferred IIR Digital filter can be constructed. The stability of the generated digital filter can be found its pole-zero plot[5][8][21][22][23].

VI. CALCULATION OF DIFFERENT QUANTITIES

The prime target of the design is to achieve high quality factor(Q-Factor). To get the quality factor of the proposed Notch filter, the primary requirement is to obtain the resonant frequency. The peak response of the filter can be obtained at the centre frequency. The peak response is essentially responsible for the resonant frequency which can be obtained by the geometric mean of lower and upper cutoff points i.e. the mean of two -3dB points. The resonant frequency can be expressed by,

$$f_R = \sqrt{f_L \times f_H} \quad \dots\dots\dots(13)$$

Where:

f_R = Resonant or Centre Frequency

f_L = Lower -3dB cut-off frequency point

f_H = Upper -3db cut-off frequency point

From equation 13, we can easily calculate the Notch frequency. Now to calculate the quality factor, it is essential to have knowledge about the bandwidth of the signal or frequency as it is expresses by the ration of Resonant frequency to the bandwidth i.e.,

$$Q = \frac{f_R}{BW} \quad \dots\dots\dots(14)$$

A high Q produces a sharp cut whereas a low Q produces a wide cut. But to design a notch filter, the high Q is desirable. It is worth nothing to mention that Q Factor is a ration and hence has no unit.

Again, the notch frequency for the proposed analog Notch filter can be determined by the values of resistor and capacitor. The expression is given below:

$$f_{Notch} = \frac{1}{2\pi\sqrt{RC}} \quad \dots\dots\dots(15)$$

So, in that case, the value of f_R and f_{Notch} must be same to obtain a perfect filter. Generally, the transfer function in Laplace domain of the Notch filter for second order design can be obtained with help of center frequency and quality factor, given by[19][20][21][22],

$$H(s) = \frac{s^2 + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \dots\dots\dots(16)$$

Where,

ω_0 =Central frequency in Laplace plain

Q= Quality factor

VII. SIMULATION RESULT

The simulation result for the proposed design are discussed in this section. The filter output voltage is simulated and shown in Fig.4 to Fig.10. The values of the parameters are choosen for the design are given in Table1 and Table 2.

Table -1

Input Section (Function Generator)		Output Section (Oscilloscope)	
Parameter	Value	Parameter	Value
Frequency	100Hz	Voltage	~50 mV _{PP}
Voltage	50 mV _{PP}	Duty Cycle	50%
Duty Cycle	50%	Pulse Type	Sinusoidal

Table -2

Component Values	
R1=R2	22 k Ω
R3	10k Ω
C1=C2	0.068μF
C3	0.13μF

A. Simulation Result in Multisim

I. Output Voltage Waveform:

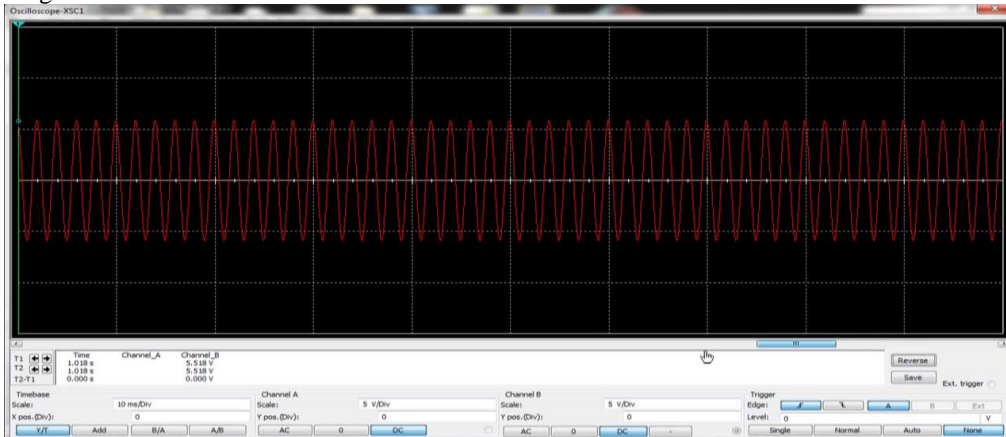


Fig.4 Oscilloscope Output

The input voltage is provided as 4V_{PP} and the output waveform is generated with 5.518V and thus this model amplifies the input signal without using any amplifying module.

II. AC Analysis:

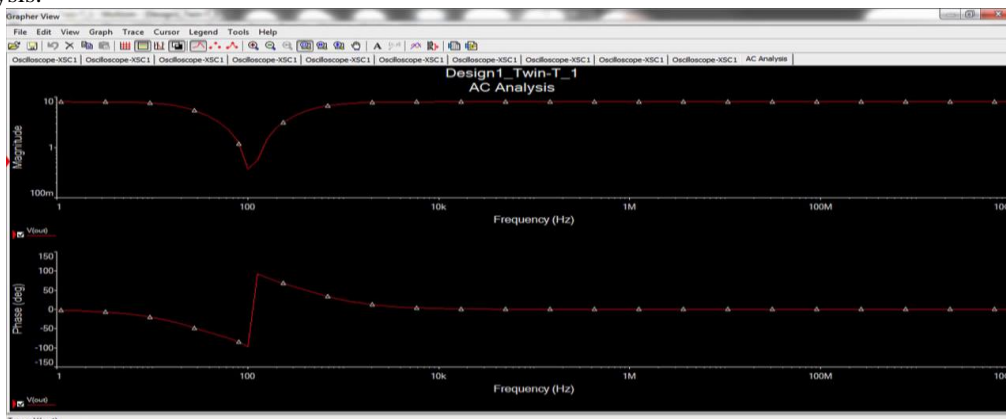


Fig.5 AC Analysis

While analyzing the AC, we are getting the magnitude response as well as the phase response of the proposed diagram. In magnitude response, it cuts sharply 100Hz frequency and so it changes its phase at the same frequency which can be referred as the centre frequency.

III. Fourier Analysis:

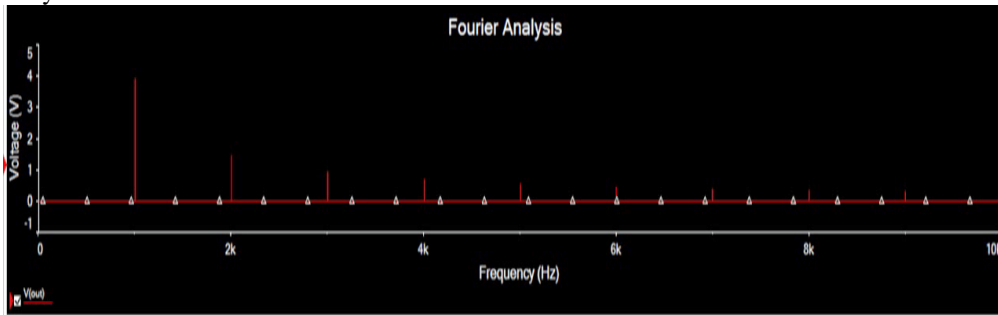


Fig.6 Fourier Analysis

In Fig.6, the Fourier transform of the output pulse is found and literally plotted. Now to find the pole-zero, magnitude response, impulse response we have to look on the Matlab simulation where the values are taken from the proposed circuit and simulated and thereby produced the desired output which is equal to the Multisim simulation. The simulation results for Matlab 7.0 are shown next sub section.

B. Simulation Result in Matlab7.0

I. Magnitude Response

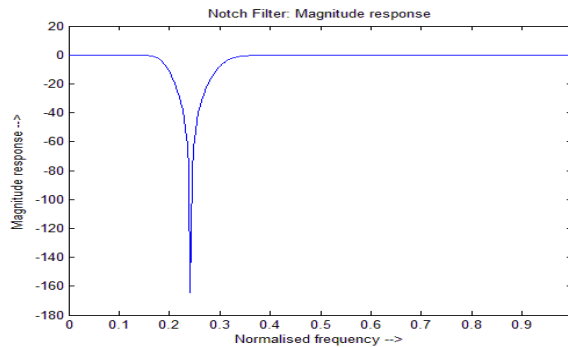


Fig. 7 Magnitude Response

II. Phase Response

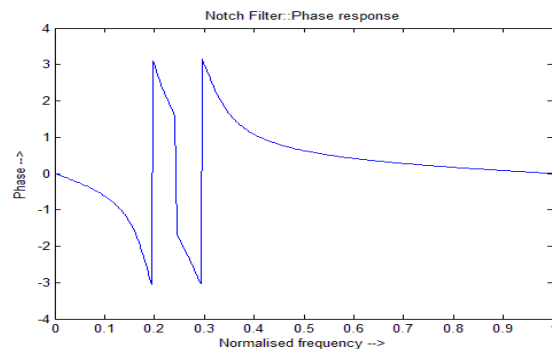


Fig.8 Phase Response

III. Impulse Response

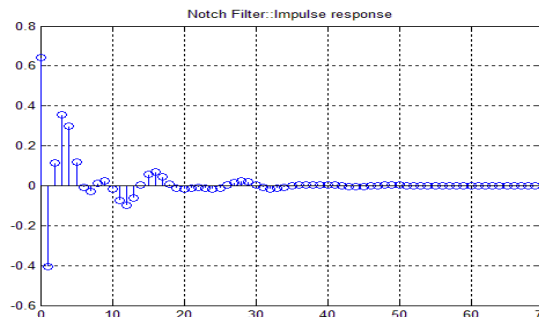


Fig.9 Impulse Response

IV. Pole-Zero Plot

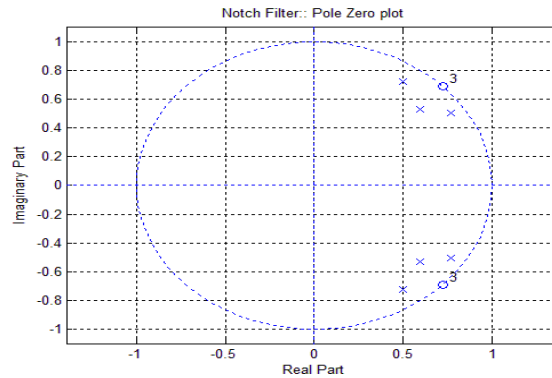


Fig.10 Pole-Zero Plot

The simulation shows the proposed Notch filter is so designed to generate a third order response by minimizing noise as the magnitude response produce a sharp cut without producing a moderate ripple.

VIII. CONCLUSION

The Multisim simulation shows the results where all the values are practical one and not the ideal. But the Matlab simulation reflects the responses with all ideal values of the components and so phase responses are differed from each other. But in case of magnitude responses, both two produces same result thereby designing a 100Hz Notch Filter which can efficiently cut sharply 100Kz signal and passes all the other frequency bands.

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The proposed design of the 100Hz Notch filter is constructed with help of Twin-T topology by minimizing the passband and stopband ripple. The analog circuit design obviously produces a transfer function in s-plane which is described by $H(s)$. This analog transfer function is mapped into digital domain or in z-plane by DTA algorithm. After successful mapping, the simulation results are achieved which are shown in the simulation result section. The idea of the design is taken from different previous works and the references are given in the reference section.

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