



Spasmodic Regression of Images Using Decimated Wavelet Transform

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Abstract— Visual information can be most efficiently represented by images. They are repeatedly deprived by noise. Wavelet shrinkage denoising is used to assess the visual information from noisy context. A novel thresholding function for regression of images is proposed in this paper. This proposed thresholding function is applied on the images corrupted with additive white gaussian noise using visu, false discovery rate and translation invariant shrinkage rules. This novel method performance is compared with existing hard, soft and scad thresholding functions using root mean square error (RMSE) and peak signal to noise ratio (PSNR) as testing parameters. From the assessment, the new thresholding function attains superior performance than all other existing thresholding functions in visu, false discovery rate and translation invariant methods.

Keywords— Wavelet shrinkage denoising, additive white gaussian noise, novel thresholding function, root mean square error (RMSE) and peak signal to noise ratio (PSNR).

I. INTRODUCTION

The most recent accomplishments in multimedia technology lead to immense study in the data and image processing. The major disadvantage in digital images is due to inheritance of noise introduced in the datasets collected by the image sensors, transmission errors, compression, storage and reproduction. Noise reduction is the initial action to be taken before performing supplementary analysis on images before performing the visual quality. The main objective of denoising algorithms is to reduce the noise content by preserving the edges and it is possible only in nonlinear or spasmodic regression techniques as in [1]. Each denoising algorithm has its own expectations, advantages and constraints.

The process of noise clipping in undeniable amount of wavelet coefficients is referred to as wavelet shrinkage as in [2]. In this paper, a newly designed thresholding function is proposed for removing noise in images. This composite function is evaluated by using visu, false discovery rate and translation invariant methods and the results are compared with existing hard, soft and SCAD using various quality measure parameters.

II. WAVELET SHRINKAGE DENOISING

In the process of wavelet shrinkage denoising as in [3], firstly discrete wavelet transform is enforced on the noise contaminated image to get the noisy wavelet coefficients. The resulting noisy coefficients are modified using novel thresholding function subjecting to the threshold value calculated using visu, FDR and translation invariant thresholding rules. Afterwards reverse wavelet transform is enforced on the modified noisy coefficients to get the denoised image as in [4].

III. THRESHOLDING TECHNIQUES

The notion of thresholding as in [5] in wavelet transform domain tries to take out noise in the degraded image due to excellent property of energy compaction provided in the wavelet transform. By virtue of this property majority of the signal power concentrates in the top LL band of wavelet decomposition, while the noise power is scattered all over the other detail subbands. Thus we make use of the wavelet decomposition to segregate the content of actual signal from the noise content. This type of segregation process facilitates as long as the signal power is much greater than the noise power.

The most active thresholding techniques are

A. Visu Method

Visu shrink thresholding was first anticipated by Donoho and Johnstone as in [6] to facilitate the rule of hard threshold. In this method, the threshold value is computed from high pass detailed wavelet coefficients at the initial level of wavelet transform. In this method, the threshold value t is directly proportional to the noise variance. In denoising of images, visu shrink effectively dealt with additive white gaussian noise to produce a large threshold value than other supplementary methods to yield an excessively smooth reconstructed estimate of the original image. The key drawback with this method is that it is unable to remove salt & pepper noise and the minimum mean square error will be very high. Visu Shrink

greatly depends on size of the image but not on content within it. In Visu Shrink method the threshold can be estimated as

$$T_V = \hat{\sigma} \sqrt{2 \log M}$$

$$\hat{\sigma}^2 = \left[\frac{\text{median}(|w_{j-1,k} - \text{median}(w_{j-1,k})|)}{0.6475} \right]^2$$

Where $\hat{\sigma}$ is estimation of noise variance based on the median of absolute deviation (MAD) which can be treated as a robust estimator and M is the number of pixels in the original image.

B. FDR Method

The minimizing false discovery rate (min FDR) method was proposed by B. Vidakovic as in [7] for 1-D data. It is the accepted fraction of false discoveries among the discarded null hypotheses. FDR preserves the identical threshold value for all the thresholding functions by enduring the likely value of the tiny proportion of detailed coefficients imperfectly incorporated in the reconstruction of coefficients below a given fraction α . Given the L detailed coefficients (e_n , $n = 1, 2, \dots, L$), first it computes p-values.

$$p_n = 2[1 - \delta(|e_n|/\sigma)]$$

Where $\delta(\cdot)$ is the cumulative distribution function and σ is an estimation of the standard deviation. Then p_n values are ordered as $p_{(1)} \leq p_{(2)} \leq p_{(3)} \dots \leq p_{(L)}$. From $n = 1$, let k be the major index value then

$$p_{(k)} \leq \frac{k}{L} \alpha$$

The threshold value is obtained as

$$\lambda = \sigma \delta^{-1}(1 - (p_{(k)}/2))$$

C. Translation Invariant Method

Donoho and Coifman have introduced the translation invariant rule. It consists of carrying out shrinkage on each basis and taking the average of the obtained de-noised signals. There are two challenging effects present for improved detection of singularities due to taking into account all the shifts in the analysis and an effect of more powerful smoothing due to averaging of the de-noised signals on each basis. This method can get better compensation of the edges and it is also used for estimation of images as in [8].

IV. THRESHOLDING FILTERS

After the calculation of threshold value for each subband (LL, LH, HL and HH) except the low pass or approximation subband, apply thresholding filter to each noisy wavelet coefficient given below, by substituting the calculated threshold value.

Thresholding filters can be used for applying the calculated threshold value on the noisy image. In this paper existing Hard, Soft and Scad filters are considered along with hybrid thresholding filter.

Donoho and Johnstone proposed a widely recognised filter known as hard thresholding filter. Hard thresholding can also be treated as remain or destroy method or gating. The transfer function of the same is shown in figure 1. Hard thresholding is very sensitive and it has very large variance. In this method, threshold value is fixed by adopting some existing threshold rules. Hard thresholding set the value of wavelet coefficients less than or equal to the threshold to zero. The coefficients greater than this threshold value are remained same as in [9].

The hard thresholding hard (W, λ) is denoted as

$$\text{hard}(W, \lambda) = W \quad \text{for } |W| > \lambda$$

$$= 0 \quad \text{otherwise}$$

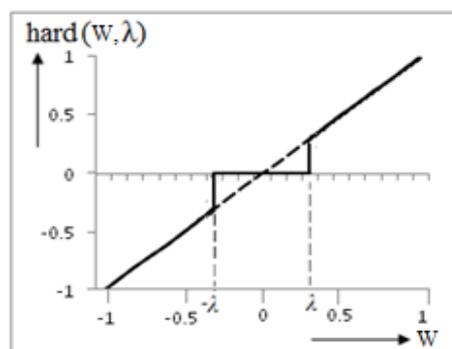


Fig. 1 Hard Thresholding Filter

The soft threshold function as in [10] takes the argument and shrinks it toward zero by the threshold T. Soft thresholding filter is used to analyse the performance of denoising procedure for different levels of DWT decomposition, because the soft thresholding filter achieves near-optimal minimax rate over a large range of Besov spaces and for the generalized Gaussian noise, the soft-thresholding filter yields a minor risk than the hard-thresholding estimator. In practice, the soft-thresholding method reclaims more regretted visually pleasing images over hard-thresholding since the latter one is sporadic and yields abrupt artifacts in the improved images, especially when the noise energy is more significant. Soft filter sets the coefficient value as the difference between coefficient value and threshold value if it is

greater than the threshold value, otherwise the coefficient value is zero. The transfer function of soft thresholding soft (W, λ) is given below and the same is shown in the figure 2.

$$\text{soft}(W, \lambda) = \begin{cases} [\text{sign}(W)] (|W| - \lambda) & \text{for } |W| > \lambda \\ 0 & \text{otherwise} \end{cases}$$

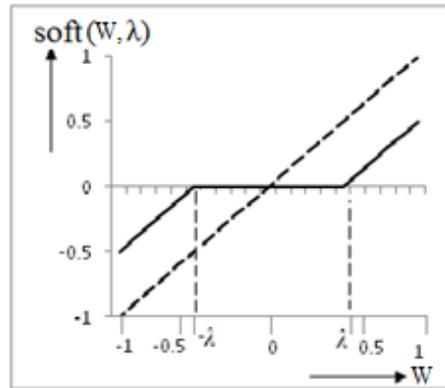


Fig. 2 Soft Thresholding Filter

SCAD, smoothly clipped absolute deviation, is a piecewise linear function and is continuously differentiable in $(-\infty, 0) \cup (0, \infty)$. The SCAD derivatives are zero outside the $[-\alpha\lambda, \alpha\lambda]$. This leads to large coefficients retain as it is, while small coefficients are made zero and some other coefficients have been made shrunk towards zero. It does not create excessive bias when the wavelet coefficients are large and it produces a set of sparse solutions. The transfer function for the SCAD thresholding function is shown in figure 3.

The scad function is denoted as

$$\text{SCAD}(W, \lambda) = \begin{cases} \text{sign}(W) \max(0, |W| - \lambda) & \text{if } |W| \leq 2\lambda \\ (\alpha - 1)W - \alpha\lambda \text{sign}(W) & \text{if } 2\lambda \leq |W| < \alpha\lambda \\ W & \text{if } |W| > \alpha\lambda \end{cases}$$

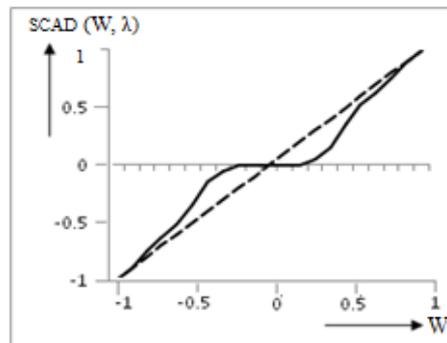


Fig. 3 SCAD Thresholding Filter

V. NOVEL THRESHOLDING FILTER

In this paper a novel thresholding filter function is proposed for modifying the noisy wavelet coefficients. This filter function is designed by subtracting the fraction of the noisy coefficient from the hard thresholding filter for more than threshold value. 25% of detailed coefficient value is considered for less than the threshold value. This is given as

$$N(W, \lambda) = \begin{cases} W - \left(\frac{1}{1+e^{(W/\lambda)}}\right) & \text{for all } |W| > \lambda \\ 0.25 * W & \text{otherwise} \end{cases}$$

Here, W represents wavelet coefficients and λ represents threshold value.

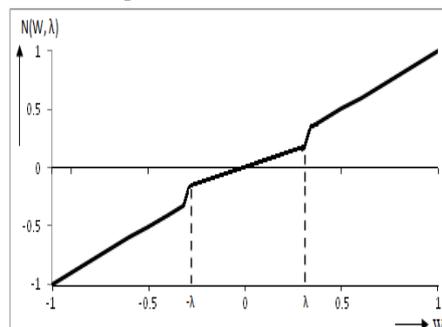


Fig. 4 Novel Thresholding Filter

VI. SIMULATION RESULTS AND DISCUSSION

The results obtained with existing hard, soft, SCAD and novel thresholding functions on noise contaminated image are cited in this section. The Lena image of size 256×256 is taken and it is contaminated with additive white gaussian noise of various standard deviation values. The wavelet used to decompose the noisy image into corresponding wavelet coefficients is coiflet wavelet. These noisy wavelet coefficients are modified using the appropriate thresholding filter function. In thresholding filter the threshold value is fixed using visu, FDR and translation invariant thresholding methods. The inverse wavelet transform is used to reconstruct the modified noisy wavelet coefficients into denoised image. Quality measure parameters like RMSE and PSNR are used to compare the results.

$$RMSE = \frac{1}{n} \sum_{i=1}^n (S(i) - \hat{S}(i))^2$$

$$PSNR = 10 \log_{10} \frac{\sum_{i=1}^n S(i)^2}{\sum_{i=1}^n (S(i) - \hat{S}(i))^2}$$

Where, i represents number of samples, S (i) is original image component and $\hat{S}(i)$ is denoised image component. The simulating process in MATLAB is continuously repeated for about 100 times and then the averages of RMSE and PSNR values are taken.

The same process is implemented on different images then similar type of results will be obtained. The results of image for $\sigma=5, 10$ and 15 using hard, soft, SCAD and novel thresholding filters with visu, FDR and translation invariant method are shown in Table 1-3. The original and denoised image using hybrid thresholding filter with visu, FDR and translation invariant method are shown in Figures 5-9. Figures 10-15 shows the comparison of the results in visu, FDR and translation invariant methods.

For a noise standard deviation of $\sigma =5$, RMSE will be 4.8523 and PSNR will be 34.4228 are obtained on denoising the noisy Lena image with novel thresholding filter using visu method, the results of RMSE and PSNR obtained for all other thresholding filters are presented in Table 1. The comparison indicates that the novel thresholding filter better denoises the Lena image than hard, soft and scad thresholding filters. The same denoising behaviour can be found for $\sigma =10$ and 15 .

Similarly, from the results of RMSE and PSNR for different noise deviations of $\sigma =5, 10$ and 15 in Table 2 and Table 3, the newly designed novel filter performs much better than hard, soft and scad thresholding filters in denoising the Lena image using visu method, FDR method and translation invariant method.

TABLE I DENOISING RESULTS OF LENA IMAGE USING HARD, SOFT, SCAD AND NOVEL THRESHOLDING FILTERS: VISU METHOD

	Noisy Image		Hard Filter		Soft Filter		Scad Filter		Novel Filter	
	RMSE	PSNR	RMSE	PSNR	RMSE	PSNR	RMSE	PSNR	RMSE	PSNR
$\sigma =5$	5.002	34.148	6.19	32.309 3	9.2139	28.863 2	8.7998	29.252 4	4.8523	34.422 8
$\sigma =10$	10.012 5	28.12	9.7737	28.333 2	13.012 1	25.853 4	12.973 6	25.879 8	7.7141	30.389 9
$\sigma =15$	14.985 2	24.617 6	11.601 5	26.848 9	15.366 3	24.406 2	15.160 5	24.530 6	9.7414	28.364 4

TABLE II DENOISING RESULTS OF LENA IMAGE USING HARD, SOFT, SCAD AND NOVEL THRESHOLDING FILTERS: FDR METHOD

	Noisy Image		Hard filter		Soft filter		Scad filter		Novel filter	
	RMSE	PSNR	RMSE	PSNR	RMSE	PSNR	RMSE	PSNR	RMSE	PSNR
$\sigma =5$	4.9973	34.156 1	4.4306	35.204	6.6794	31.658 3	6.3779	32.045 2	4.169	35.740 2
$\sigma =10$	9.9937	28.136 3	7.5972	30.520 6	9.9299	28.226 2	9.6523	28.461 3	6.9501	31.291 7
$\sigma =15$	14.999 2	24.609 5	9.8741	28.241 4	12.796 7	26.027 8	12.446 6	26.244	9.1568	28.898 2

TABLE III DENOISING RESULTS OF LENA IMAGE USING HARD, SOFT, SCAD AND NOVEL THRESHOLDING FILTERS: TI METHOD

	Noisy Image		Hard filter		Soft filter		Scad filter		Novel filter	
	RMSE	PSNR	RMSE	PSNR	RMSE	PSNR	RMSE	PSNR	RMSE	PSNR
$\sigma =5$	4.9988	34.153 5	4.349	35.380 9	8.2423	29.819 3	5.8714	32.785 9	3.6185	36.963 7
$\sigma =10$	10.006	28.125 6	6.8915	31.391 9	12.133 4	26.467 7	9.6922	28.418 7	6.0108	32.555 6
$\sigma =15$	14.999 9	24.609 1	9.3742	28.712 4	15.052 3	24.597 4	11.815 9	26.702	8.1755	29.885 7



Fig. 5 Original Lena Image



Fig. 6 Noisy Lena Image



Fig. 7 Denoised Lena Image using novel thresholding filter in visu method



Fig. 8 Denoised Lena Image using novel thresholding filter in FDR method



Fig. 9 Denoised Lena Image using novel thresholding filter in TI method

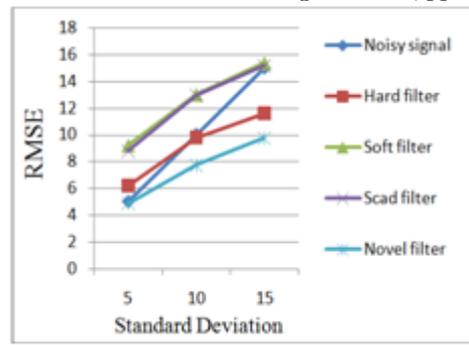


Fig. 10 RMSE values of different filters in visu method.

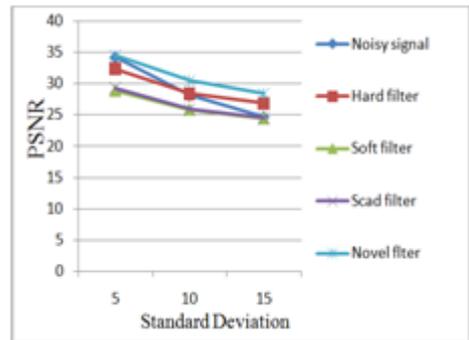


Fig. 11 PSNR values of different filters in visu method.

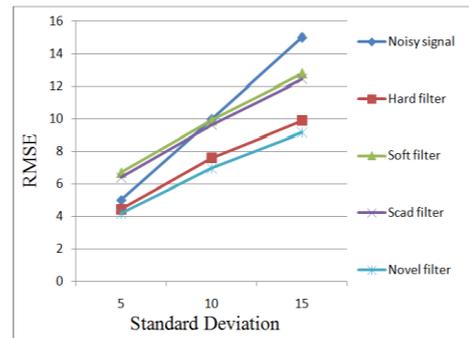


Fig. 12 RMSE values of different filters in FDR method.

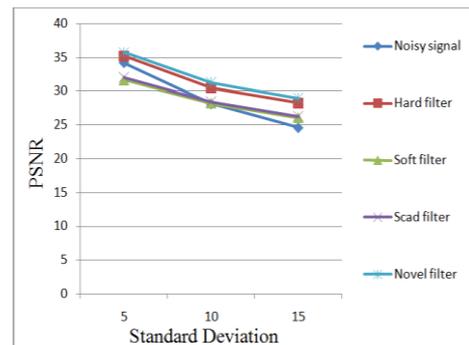


Fig. 13: PSNR values of different filters in FDR method.

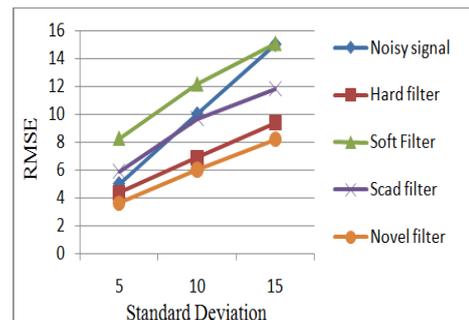


Fig. 14 RMSE values of different filters in TI method.

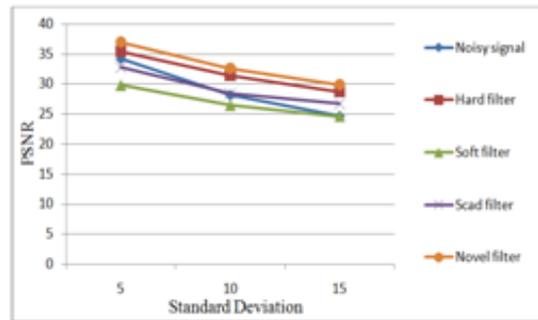


Fig. 15 PSNR values of different filters in TI method.

VII. CONCLUSION

In this paper, a novel thresholding filter for discrete wavelet shrinkage denoising of images is proposed. The performance of this newly designed filter is evaluated by using different images. The results obtained are compared with existing hard, soft and SCAD filters. It is found from the results that the novel thresholding filter performs much comfortable than hard, soft and SCAD filters using visu, FDR and translation invariant method.

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