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# A Hybrid Differential Evolution Method for the Design of Low Pass Digital Fir Filter

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***Abstract— This paper presents the methodology for the design of Low pass FIR digital filter using hybrid Differential Evolution (DE) method. It is an evolutionary algorithm that exhibits the features of both the basic DE and Hooke-Jeeves exploratory move to attain the optimal solution. The basic DE and exploratory move have the capability to exploit and explore the search space to design the optimal filter parameters. A multivariable objective function is optimized to attain the minimum magnitude error and pass band and stop band ripples. The designed Low pass digital FIR filter authenticates that the proposed hybrid DE algorithm yields more accurate and efficient solution and can be effectively applied for the design of High pass, Band pass and Band stop digital filters.***

***Keywords— Digital Infinite-Impulse Response (IIR) filter, Digital Finite-Impulse Response (FIR) filter, DE, exploratory move***

## I. INTRODUCTION

Optimization plays a substantial role in engineering, science and digital signal and image processing fields. Optimization is the process of determining the conditions that give the minimun and maximum value of a function [1]. Optimization deals with the study of these kinds of problems in which one has to minimize or maximize one or more objectives that are function of some real or integer variables. Optimization can be single objective or multi objective. Single objective optimization results in optimizing only one objective. On the other hand multiobjective optimization deals with the task of simultaneously optimizing two or more objectives with respect to set of certain constraints. The design of optimum digital FIR (Finite Impulse Response) filter using optimization is a very challenging task now- a- days. FIR filters are the digital filters having finite impulse response as it requires only finite number of previous samples and present sample to find the impulse response. FIR filters are non recursive filters having linear phase and always stable [2]. Therefore these are the most suitable for phase sensitive applications such as radar and audio applications. The performance of digital FIR Filters is measured in terms of magnitude error and ripple magnitude of pass band and stop band.

Various optimization algorithms such as simulated annealing, Tabu search, ant colony algorithm, immune algorithm, Genetic algorithm, particle swarm optimization and differential evolution algorithm are available for digital FIR filter design [3]. Kumar et. al. (2013) has implemented window method for the design of band pass FIR filter using various windows such as Blackmann, Hamming, Hanning and Kaiser window [4]. The results of Kaiser window proved to be better among all others. Kaur et. al. (2012) has discussed the design of FIR filters using Genetic algorithm in which the number of operations in the design process are reduced and coefficient calculation is realized [5]. Genetic algorithm may converge towards local optima instead of global optima if the value of fitness function is not properly defined. GA is difficult to operate on dynamics sets. It also requires complex genetic operators of mutation and crossover. The computational cost of GA is also very large. Dai et. al. (2010) designed digital IIR filter using Seeker optimization algorithm [6]. It uses the concept of simulating the task of human search in which the empirical gradient is used to find the search direction using a simple fuzzy rule. Although it is simple to implement and good at local convergence but it requires too many calculations to obtain global minima. Neha et. al. (2014) presented the PSO algorithm for Low pass FIR filter design. This algorithm is used with constriction factor approach to solve the multimodal, highly non linear filter design problems. This method has the property of parameter impedance and converges in very less execution time [7]. Mondal et. al. (2012) implemented noval PSO to design the FIR digital low pass filter [8] and compared it with existing PM, RGA and PSO algorithm. NPSO proved to be better in terms of convergence characteristics, execution time, magnitude response, minimum stop band ripple and maximum stop band attenuation. But the major shortcoming of PSO is that the control parameters affect the convergence behaviour to a great extent. So hybrid differential evolution can be used to overcome the shortcomings of PSO.

Initially DE algorithm was developed by Storn and Price in 1995 which is simple, fast and population based stochastic algorithm. DE has the capability to handle non linear, non differentiable and multimodal cost function. It is easy to implement due to need of less control parameters. It acquires good convergence speed due to its parallelizability. DE has the capability to explore the search space globally. But the basic drawback is that out of ten mutation strategies available, only one particular strategy is suitable for the desired global optimum solution. So to overcome this drawback, basic DE is hybridized with exploratory move to search the local neighbourhood of global solution and find better solution.

The premise of this paper is to implement a hybrid DE algorithm for the design of Low pass FIR digital filter. This algorithm explores and exploits the search space by varying various control parameters such as population size, crossover rate and mutation factor and optimizes the filter coefficients to achieve minimum magnitude error and pass band and stop band ripples.

The remaining paper is arranged in the four sections. The Low pass digital FIR filter design problem is discussed in Section II. The brief algorithm regarding the Hybrid DE is explained in Section III. Section IV depicts the performance and results of the proposed algorithm. Section V successfully negotiates the conclusion.

## II. PROBLEM FORMULATION FOR FIR FILTER DESIGN

Digital filters usually operate on discrete-time signals. FIR Filters have finite impulse response because the filter output is computed as a weighted, finite term sum of past, present, and future values of the filter input. The difference equation of FIR filter is as below:

$$y(n) = \sum_{k=0}^{N-1} b_k \, x(n-k) \quad (1)$$

where x(n) represents the filter input

$b_k$ represents the filter coefficient

y(n) represents the filter output

N is the number of filter coefficients (order of the filter)

The transfer function of FIR filter is given as:

$$H(z) = \sum_{n=0}^{N} h(n) \, z^{-n} \quad (2)$$

where $H(z)$ is the impulse response in frequency domain and $h(n)$ is the impulse response in time domain. N is the filter order. This paper presents the design of Low pass FIR digital filter which is even symmetric and with even order. The total number of filter coefficients is N+1. But due to even symmetric property of FIR filters, we have to calculate only half number of filter coefficients i.e. (N/2 + 1). So the dimension of the problem is reduced to half.

There are many filter types, but the most common are low pass, high pass, band pass and band stop. A low pass digital FIR filter allows only low frequency signals (below some specified cut-off) through to its output, so it can be used to eliminate high frequencies. For a Low pass digital filter the ideal response is defined as:

$$H_i(e^{jw}) = \begin{cases} 1 & for\ w \le w_c \\ 0 & otherwise \end{cases} \quad (3)$$

The performances of digital FIR filter can be calculated by using $L_1$-norm and $L_2$-norm approximation error of magnitude response and ripple magnitude of both pass-band and stop-band. The FIR filter is designed by optimizing some coefficients. For this purpose, we have to minimize the $L_p$-norm approximation error function. $L_p$-norm is expressed as :

$$\mathrm{E}(x) = \left\{\sum_{i=0}^{k} \left|H_d(w_i) - |H(w_i,x)|\right|^p\right\}^{1/p} \quad (4)$$

where $H_d(w_i)$ is the desired magnitude response of the ideal FIR filter and $H(w_i,x)$is the obtained magnitude response of the FIR filter.

For p=1, magnitude error denotes the $L_1$-norm error and for p=2 magnitude error denotes the $L_2$-norm error.

The $L_1$-norm error is given by the expression:

$$e_1(x) = \sum_{i=0}^{k} \left|H_d(w_i) - |H(w_i,x)|\right| \quad (5)$$

The $L_2$-norm error is given by the expression:

$$\mathrm{e}_2(x) = \left\{\sum_{i=0}^{k} \left|H_d(w_i) - |H(w_i,x)|\right|^2\right\}^{1/2} \quad (6)$$

The desired magnitude response $H_d(w_i)$of FIR filter is given as:

$$H_d(w_i) = \begin{cases} 1 & for\ w_i \in pass\ band \\ 0 & for\ w_i \in stop\ band \end{cases} \quad (7)$$

The ripple magnitude of pass band and stop band is expressed as:

$$\delta_p = max\{|H(w_i,x)|\} - \min\{|H(w_i,x)|\} \qquad for\ w_i \in pass\ band \quad (8)$$

$$\delta_s = max\{|H(w_i,x)|\} \qquad for\ w_i \in stop\ band \quad (9)$$

Four objective functions for optimization are:

$Z_1(x) = Minimize\ e_1(x)$

$Z_2(x) = Minimize\ e_2(x)$

$Z_3(x) = Minimize\ \delta_p$

$Z_4(x) = Minimize\ \delta_s$

 

The multi objective function is converted into single objective function:

$$Minimize\ f(x) = w_1 Z_1(x) + w_2 Z_2(x) + w_3 Z_3(x) + w_4 Z_4(x) \quad (10)$$

where $w_1, w_2, w_3, w_4$ are the weighting functions.

## III. HYBRID DIFFERENTIAL EVOLUTION

In this paper the Low pass digital FIR filter is designed by exploring hybrid DE algorithm. DE is a population based evolutionary algorithm. The term evolutionary comes from the fact that it uses biological operators of evolution like mutation, crossover and selection along with the arithmetic operators. DE is a stochastic technique as it gives different results everytime the program is run. In this algorithm first some random population is initialized and then it is evolved to yield better results for the optimization problem.

### *A. Parameter Setup*

The parameters which have been selected initially are population size (S), boundary constraints of population, mutation factor ($f_m$), crossover rate (CR) and maximum number of iterations ($T_{max}$). If there are N number of filter coefficients then each population should be N-dimensional. S number of populations are generated each of length N. So population is represented in the form of matrix $x_{ij}^t$ of size S×N.

### *B. Population Initialization*

For initialization of population, firstly random numbers are generated with uniform probability distribution and these numbers are mapped within the given boundary constraints according to the following equation:

$$x_{ij}^t = x_j^{min} + rand()\left(x_j^{max} - x_j^{min}\right) \quad (11)$$

where $(j = 1,2, \ldots \ldots N; i = 1,2, \ldots \ldots S)$

where $x_{ij}^t$ is the $j^{th}$ element of $i^{th}$ population and t represents the generation.

rand () is the uniform random number with values in the range 0 and 1.

### *C. Mutation*

In mutation, a set of new vectors is generated by combining the randomly chosen vectors from the initial population in different ways. The resultant vector is known as mutant vector. There are ten mutation strategies available for the combination of vectors but first five are studied in this paper which are as below:

$$Z_{ij1}^t = P_{R_1 j}^t + f_m\left(x_{R_2 j}^t - x_{R_3 j}^t\right) \quad (12)$$

$$Z_{ij2}^t = x_{Bj}^t + f_m\left(x_{R_1 j}^t - x_{R_2 j}^t\right) \quad (13)$$

$$Z_{ij3}^t = x_{ij}^t + f_B\left(x_{Bj}^t - x_{ij}^t\right) + f_m\left(x_{R_1 j}^t - x_{R_2 j}^t\right) \quad (14)$$

$$Z_{ij4}^t = x_{Bj}^t + f_m\left(x_{R_1 j}^t + x_{R_2 j}^t - x_{R_3 j}^t - x_{R_4 j}^t\right) \quad (15)$$

$$Z_{ij5}^t = x_{R_5 j}^t + f_m\left(x_{R_1 j}^t + x_{R_2 j}^t - x_{R_3 j}^t - x_{R_4 j}^t\right) \quad (16)$$

where $i = 1,2, \ldots \ldots \ldots, S; j = 1,2 \ldots \ldots \ldots \ldots, N$

$Z_i^t = [Z_{i1}^t, Z_{i2}^t, \ldots \ldots ., Z_{iN}^t]^T$ represents the mutant vector for the position of the $i^{th}$ individual.

$f_m$ is the mutation factor in the range [0.5,1]. $R_1$, $R_2$, $R_3$, $R_4$, $R_5$ are randomly chosen population vector with uniform distribution.

### *D. Recombination*

Recombination is accomplished by exchanging the parameters of mutant vector with that of randomly generated population vector with crossover probability of CR. It is mainly used to increase the population diversity. The resultant vector is called trial vector or crossover vector. Recombination is carried out according to following equation:

$$U_{ij}^{t+1} = \begin{cases} Z_{ij}^t & if\ (r_{and}(j) \le CR)\ or\ (j = j_{rand}) \\ x_{ij}^t & if\ (r_{and}(j) > CR)\ or\ (j \ne j_{rand}) \end{cases} \quad (17)$$

$$(i = 1,2, \ldots \ldots \ldots, S; j = 1,2 \ldots \ldots \ldots \ldots, N)$$

where $U_i^{t+1} = [U_{i1}^{t+1}, U_{i2}^{t+1}, \ldots \ldots ., U_{iN}^{t+1}]^T$ is the trial vector.

rand(j) is the $j^{th}$ evaluation of random number in the range [0,1]. $j_{rand}$ is randomly chosen index within the range [1,N]. CR is the crossover rate in the range [0, 1].

### *E. Selection*

In selection process, the trial vector $U_{ij}^{t+1}$ is checked for survival in the next generation. The values of objective functions of both the population vector and trial vector are compared. If the value of objective function of trial vector is less than that of population vector, then it is selected for evolution in the next generation, otherwise not.

$$x_{ij}^{t+1} = \begin{cases} U_{ij}^{t+1}(j = 1,2, \ldots \ldots . N)\ ; if\ \left(A(U_i^{t+1}) < A(X_i^t)\right) \\ X_{ij}^t\,(j = 1,2, \ldots \ldots . N)\ ; otherwise \end{cases} \quad (18)$$

where A represents the objective function and $x_{ij}^{t+1}$ represents the next generation of population.

 

### *F. Exploratory move*

Hooke and Jeeves pattern search method has two kinds of moves: Pattern move and Exploratory move. The exploratory move explores the global solution with a constant step in all directions. An exploratory move is performed in the vicinity of the current point systematically to find the best point around the current point [9].

Thus if the objective function with positive or negative perturbation is less than the global best objective function, then that positive or negative perturbed location is updated, otherwise the location remains the same. The perturbation of filter coefficient $x_i$ is performed according to following equation:

$$X_i^n = x_i^o \pm \Delta_i u_i^j \quad ; (i = 1,2, \ldots\ldots S; j = 1,2, \ldots\ldots N) \qquad (19)$$

$$where \quad u_i^j = \begin{cases} 1 & if\ i = j \\ 0 & if\ \ i \neq j \end{cases} \qquad (20)$$

N represents number of variables.

The value of the objective function $A(x_i^n)$ is calculated as below:

$$X_i^n = \begin{cases} x_i^o + \Delta_i u_i \ ; A(x_i^o + \Delta_i u_i) < A(x_i^o) \\ x_i^o - \Delta_i u_i \ ; A(x_i^o - \Delta_i u_i) < A(x_i^o) \\ x_i^o \quad ; otherwise \end{cases} \qquad (21)$$

If the value of objective function improves in any attempted perturbations then exploratory move is a success otherwise failure. The whole process is repeated to explore all the filter coefficients until overall minimum is selected for next iteration.

## IV. RESULTS AND DISCUSSIONS

This section presents the work performed to implement the design of low pass digital FIR filter. The hybrid differential evolution algorithm has been applied to design low pass digital FIR filter by using various mutation strategies.

The designing of low pass filter is done by setting 200 equally spaced points within frequency domain [0,π]. The prescribed design conditions for the low pass FIR digital filter are given in Table I.

Table I: Design conditions for Low Pass digital FIR filter

| FILTER TYPE | PASS BAND | STOP BAND | MAX VALUE OF $H(\omega, x)$ |
|---|---|---|---|
| Low Pass | $0 \leq w \leq 0.2\pi$ | $0.3\pi \leq w \leq \pi$ | 1 |

The performance of this algorithm is improved by varying various parameters such as order of the filter, population size, mutation factor and crossover rate. The values of the parameters used and that are optimized are shown in Table II.

Table II: DE algorithm Parameters

| PARAMETER | VARIATION | OPTIMIZED PARAMETERS |
|---|---|---|
| Filter Order | 20 to 50 | 44 |
| Population Size | 60 to 140 | 130 |
| Mutation Factor | 0.5 to 1.0 | 0.8 |
| Crossover Rate | 0.1 to 0.5 | 0.25 |

Table III: Design results of Low pass FIR digital filter for different orders

| FILTER ORDER | OBJECTIVE FUNCTION | MAGNITUDE ERROR 1 | MAGNITUDE ERROR 2 | PASS BAND PERFORMANCE | STOP BAND PERFORMANCE |
|---|---|---|---|---|---|
| 20 | 6.200774 | 2.914095 | 0.339507 | 0.187787 | 0.085787 |
| 22 | 4.433193 | 2.029395 | 0.256602 | 0.130244 | 0.083952 |
| 24 | 4.19703 | 1.813965 | 2.262119 | 0.144386 | 0.065282 |
| 26 | 3.77079 | 1.931759 | 0.246496 | 0.070935 | 0.088361 |
| 28 | 2.669703 | 1.395317 | 0.170318 | 0.060984 | 0.049428 |
| 30 | 2.141216 | 1.075933 | 0.130562 | 0.061327 | 0.032740 |
| 32 | 2.016073 | 0.996107 | 0.134389 | 0.056164 | 0.036414 |
| 34 | 1.828172 | 0.995920 | 0.129214 | 0.028922 | 0.041600 |
| 36 | 1.300113 | 0.687170 | 0.081103 | 0.029087 | 0.024570 |
| 38 | 1.027605 | 0.537584 | 0.065203 | 0.025103 | 0.017586 |
| 40 | 1.078196 | 0.531352 | 0.075378 | 0.056286 | 0.005961 |
| 42 | 0.938382 | 0.496865 | 0.060052 | 0.027294 | 0.012525 |

 

| | | | | | |
|---|---|---|---|---|---|
| 44 | 0.694658 | 0.427289 | 0.045880 | 0.029392 | 0.007026 |
| 46 | 2.299818 | 1.417947 | 0.179437 | 0.077720 | 0.061152 |
| 48 | 3.703213 | 2.281801 | 0.293949 | 0.075482 | 0.105088 |
| 50 | 0.984591 | 0.716939 | 0.069344 | 0.023280 | 0.016233 |

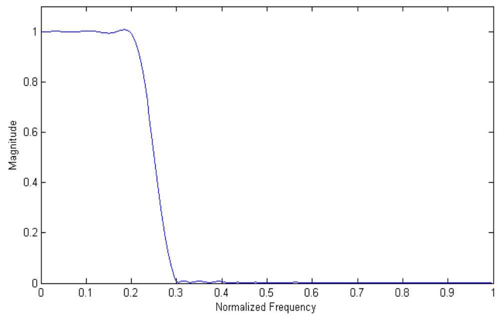


Fig. 1: Plot of Absolute Magnitude Response vs. Normalized Frequency

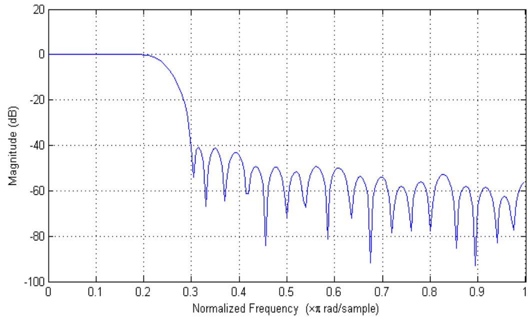


Fig. 2: Plot of Magnitude Response in dB1 vs. Normalized Frequency

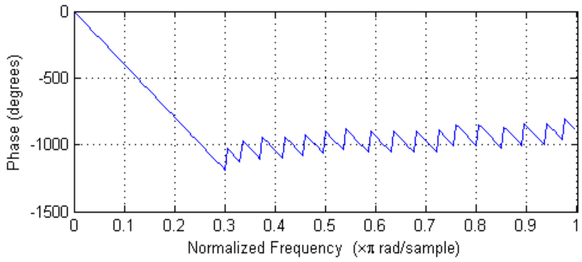


Fig. 3: Plot of Phase Response vs. Normalized Frequency

Table IV: Coefficients obtained for designing Low Pass digital FIR filter

| **h(n)** | **Filter Coefficients** |
|---|---|
| h(0)=h(44) | -0.00208 |
| h(1)=h(43) | -0.00242 |
| h(2)=h(42) | -0.00041 |
| h(3)=h(41) | 0.003571 |
| h(4)=h(40) | 0.006191 |
| h(5)=h(39) | 0.005476 |
| h(6)=h(38) | 0.000228 |
| h(7)=h(37) | -0.00741 |
| h(8)=h(36) | -0.01245 |
| h(9)=h(35) | -0.01065 |
| h(10)=h(34) | -0.00032 |
| h(11)=h(33) | 0.014056 |
| h(12)=h(32) | 0.02357 |
| h(13)=h(31) | 0.019755 |

 

| | |
|---|---|
| h(14)=h(30) | 0.00006 |
| h(15)=h(29) | -0.02745 |
| h(16)=h(28) | -0.04745 |
| h(17)=h(27) | -0.04194 |
| h(18)=h(26) | -0.00017 |
| h(19)=h(25) | 0.072923 |
| h(20)=h(24) | 0.157313 |
| h(21)=h(23) | 0.224448 |
| h(22) | 0.250241 |

In this first five mutation strategies have been applied on various filter orders varying between 20-50. The mutation strategy 2 yields minimum objective function. The mutation strategy 2 has been implemented by varying the filter order from 20 to 50 and the algorithm runs 100 times for each order. The design results are given in Table III for different filter orders. From the Table III it is observed that filter order 44 depicts the minimum value of objective function. As the order increases the objective function decreases. The objective function at order 44 is minimum with value 0.694658, but again at order 46 the objective function increases. Then the population size is varied from value 60 to 140 on mutation strategy 2 and order 44. The best population is obtained at 130.This population is selected to design low pass FIR filter having objective function value as 0.682568. The mutation factor is varied from 0.5 to 1.0 on population size 130, mutation strategy 2 and order 44. The minimum value of objective function is found at 0.8 mutation factor. The crossover rate value is varied from 0.1 to 0.5 at 0.8 mutation factor. The minimum objective function is obtained at 0.25 having value 0.676348 is selected to design Low pass FIR digital filter. The corresponding filter coefficients are shown in Table IV.

The Fig. 1 shows the graph for Absolute Magnitude Response of order 44 for Low pass digital FIR filter. The Fig. 2 shows the graph between Magnitude Response in dB and normalized frequency of order 44 to design Low pass digital FIR filter and Fig. 3 shows the graph between Phase Response and normalized frequency of Low pass digital FIR filter for order 44.

The statistical data including maximum, minimum, average value of objective function and standard deviation is shown in Table V.

Table V: Maximum, Minimum, Average value of objective function and Standard Deviation

| **Filter Type** | **Maximum Value** | **Minimum Value** | **Average Value** | **Standard deviation** |
|---|---|---|---|---|
| Low Pass | 0.919994 | 0.676348 | 0.814785 | 0.051565 |

From the results, it is evident that the proposed filter design approach produces minimum objective function, low pass band attenuation and higher stop band attenuation.

## V. CONCLUSION

The Hybrid DE is used to design the low pass digital FIR Filter by optimizing the various control parameters. The first five mutation strategies have been applied for different filter orders. On the basis of above results obtained for the design of Low pass digital FIR filter, it is concluded that it gives better performance for mutation strategy 2 with filter order 44. Simulation results show better performance of the proposed algorithm in terms of magnitude error and maximum pass band and stop band performance. The achieved value of standard deviation is 0.051565 which is less than 1. Thus the exploration search and global search optimization yields a powerful tool for the design of FIR filters. The hybrid DE technique can also be further used to design high pass, band pass and band stop filters.

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