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Unbalanced Assignment Problem by Using Modified Approach

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Abstract*— The assignment problem is one of the main problems while assigning task to the worker. It is an important problem in mathematics and is also discuss in real physical world. It is a combinatorial optimization problem in the field of operational research. In a normal case of transportation problem where the objective is to assign the available resources to the activity going on so as to get the minimum cost or maximize total benefits of allocation. In this paper we proposed modified assignment model for the solution of assignment problem. Here in this paper with the help of numerical examples or problem is solved to show its efficiency and also its comparison with Hungarian method is shown. A new cost is achieved by using unbalanced assignment problem.

Keywords*— Unbalanced Assignment problem, Optimization, Hungarian method, proposed method.

I. INTRODUCTION

An ideal way to put straight forward immediately after the transportation problem in the assignment problem [2]. The assignment problem come under the type of linear programming problem in which our main focus is to assign n number of tasks to m number of workers at a minimum cost/maximum profit. Assignment problem may be any type of problem like person to jobs, teacher to classroom, operators to lathe machine, driver to bus, bus to delivery routes etc. Balanced assignment problem is a assign problem where n==m. Otherwise that problem is known to be unbalanced assignment problem where n≠m [2]. There are various optimization method to solve the assignment problem like genetic algorithm, simulated annealing etc. Over the 5 decades many variations of assignment problem are proposed e.g generalized assignment problem, quadratic assignment problem, bottleneck assignment problem etc.[3]Here while solving the assignment problem by using Hungarian method it takes space complexity is $O(n^2)$[3]. So to solve an unbalanced assignment problem we propose a new method with space complexity O(nm). But it does not always provide a minimal total cost. This paper mainly focus to solve an unbalanced assignment problem by proposing a new method to improve the existing assignment cost.

II. MATHEMATICAL NOTATIONS AND MODEL OF ASSIGNMENT PROBLEM

Assume
n: number of task.
M: number of worker.
C_{ij} :Be the cost of assigning the ith task to j^{th} worker.
T: total assigment cost.
Y_{ij} : 1 if task i is assigned to worker j .
Y_{ij}: 0 if task I is not assigned to task j.

Minimum cost can be achieved by

$$\text{Min}(T) = \sum_{i=1}^{n} = \sum_{i=1}^{m} C_{ij} Y_{ij}.$$

Subject to

$$\sum_{i=1}^{n} Y_{ij} = 1 \quad i=1,2,3\ldots.n$$
$$\sum_{i=1}^{n} Y_{ij} = 1 \quad j=1,2,3\ldots.m$$

III. PROPOSED METHOD

E.g: Let us consider a problem with 8 worker and 4 tasks. Its worker-task assignment cost matrix is a shown in table 1[3].

i/j	1	2	3	4
A	53	62	42	89
B	18	35	39	55
C	93	80	91	83
D	79	23	96	56
E	43	16	12	20
F	87	70	87	31

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G	35	79	25	59
H	27	16	12	20

Step 1: Create an unbalanced matrix problem for worker task assignment cost matrix.
Step 2: Obtain table 2 by adding duplicate or one dummy task with all 1 assignment cost to table 1.

i/j	1	2	3	4	5
A	53	62	42	89	1
B	18	35	39	55	1
C	93	80	91	83	1
D	79	23	96	56	1
E	43	16	12	20	1
F	87	70	87	31	1
G	35	79	25	59	1
H	27	16	12	20	1

Step 3:Calculate a reduced cost matrix by dividing each row by minimum cost of its column. Reduced cost matrix obtained from table 2 is shown in table 3.

i/j	1	2	3	4	5
A	2.94	3.87	3.5	4.45	1
B	1	2.18	3.25	2.75	1
C	5.16	5	7.58	4.15	1
D	4.38	1.43	8	2.8	1
E	2.38	1	1	1	1
F	4.83	4037	7.25	1.55	1
G	1.94	4.93	2.083	2.95	1
H	1.5	1	1	1	1

Step 4: Look for the row with one 1 reduced cost excluding 1 from dummy column. Assign worker to task according to position of the chosen 1 and mark entire row to avoid later redundant assignment as shown in table 4.

i/j	1	2	3	4	5
A	2.94	3.87	3.5	4.45	1
B	~~1~~	~~2.18~~	~~3.25~~	~~2.75~~	~~1~~
C	5.16	5	7.58	4.15	1
D	4.38	1.43	8	2.8	1
E	2.38	1	1	1	1
F	4.83	4037	7.25	1.55	1
G	1.94	4.93	2.083	2.95	1
H	1.5	1	1	1	1

Step 5: Look for the column with one 1 reduced cost. Assign worker to task according to position of the chosen 1 and mark entire column to avoid later redundant assignment[4]. There is no column with one 1 in table 4. Therefore, step 5 gives the same matrix as table 4.
Step 6:Choose one of remaining 1 in the reduced cost matrix as a position to assign worker to task. Mark row and column of chosen 1 to avoid later redundant assignment. After applying step 6 to the matrix in table 4, we get the new matrix as shown in table 5.

i/j	1	2	3	4	5
A	2.94	3.87	3.5	4.45	1
B	~~1~~	~~2.18~~	~~3.25~~	~~2.75~~	~~1~~
C	5.16	5	7.58	4.15	1
D	4.38	1.43	8	2.8	1
E	~~2.38~~	~~1~~	~~1~~	~~1~~	~~1~~
F	4.83	4037	7.25	1.55	1
G	1.94	4.93	2.083	2.95	1
H	~~1.5~~	~~1~~	~~1~~	~~1~~	~~1~~

Step 7: Assign dummy task to the first n-m agent from table 5, we assign dummy task to 8-4 worker as shown in table 6.

i/j	1	2	3	4	5
A	2.94	3.87	3.5	4.45	1
B	~~1~~	~~2.18~~	~~3.25~~	~~2.75~~	~~1~~
C	5.16	5	7.58	4.15	1
D	4.38	1.43	8	2.8	1

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E	2.38	1	1	1	1
F	4.83	4037	7.25	1.55	1
G	1.94	4.93	2.083	2.95	1
H	1.5	1	1	1	1

Step 8: Count the number of assigned task excluding dummy task. If it is equal to the number of tasks then go to step11[5]. Otherwise unmark all marked rows/columns and go to the next step.
Step 9: Mark row with assignment s excluding dummy task assignment and mark dummy column. The marked rows/column from our examples are as shown in table 7.

i/j	1	2	3	4	5
A	2.94	3.87	3.5	4.45	1
B	1	2.18	3.25	2.75	1
C	5.16	5	7.58	4.15	1
D	4.38	1.43	8	2.8	1
E	2.38	1	1	1	1
F	4.83	4037	7.25	1.55	1
G	1.94	4.93	2.083	2.95	1
H	1.5	1	1	1	1

Step 10: Look for the minimum cost among remaining(unmarked) costs[6]. Recalulate the reduced cost table by dividing each remaining cost by the minimum cost and replace 0 at intersection with the minimum cost. And go to step 4. Table 8 shows the modified reduced cost matrix.

i/j	1	2	3	4	5
A	1.51	2.44	2.07	3.02	1
B	1	2.18	3.25	2.75	1.43
C	3.73	3.57	6.15	2.72	1
D	2.95	0	6.57	1.37	1
E	2.38	1	1	1	1.43
F	3.4	2.94	5.82	0.12	1
G	0.51	3.5	0.65	1.52	1
H	1.5	1	1	1	1.43

Step 11: Calculate total assignment cost. Suppose B->1,D->2,E->3 and H->4. The total cost for this assignment is calculated as follows.
$Min(T) = \sum^{n}_{i=1} = \sum^{m}_{i=1} C_{ij}\ Y_{ij}$.=(53*0) + (62*0)+(42*0)+(89*0)+(18*1)+(35*0)+(39*0)+(55*0)+(93*0)+(80*0)+(91*0)+(83*0)+(79*0)+(23*1)+(96*0)+(56*0)+(43*0)+(16*0)+(12*1)+(20*0)+(87*0)+(70*0)+(87*0)+(31*0)+(35*0)+(79*0)+(25*0)+(59*0)+(27*0)+(16*0)+(12*0)+(20*1) = 73

IV. NUMERICAL EXAMPLE SOLVED BY HUNGARIAN METHOD:

Consider the following cost minimizing Assignment Problem with 10 workers and 4 jobs solved by Hungarian method.

Worker	Tasks			
	1	**2**	**3**	**4**
A	11	8	9	8
B	4	5	29	33
C	10	5	29	33
D	1	18	25	31
E	23	22	33	30
F	3	9	13	19
G	6	8	27	32
H	32	30	39	38
I	36	35	31	21
J	15	11	10	28

Step 1: Subtract minimum value of each row from every element of the row
Step 2: And then subtract the minimum value of each column from every column. The result is shown in table 2.

Worker	**Tasks**			
	1	**2**	**3**	**4**
A	3	0	1	0

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B	0	3	22	16
C	5	0	24	28
D	0	17	24	30
E	2	0	11	8
F	0	6	10	16
G	0	2	21	26
H	2	0	9	8
I	15	14	10	0
J	5	1	0	18

Step 3: Draw the minimum number of lines to cover all the zeros of the matrix.Here the matrix is unbalanced. If the number of drawn lines ls < n, then the assignment is not complete, and if the number of line is equal to n, then the assignment is complete.

Worker	**Tasks**			
	1	**2**	**3**	**4**
A	3	0	1	0
B	0	3	22	16
C	5	0	24	28
D	0	17	24	30
E	2	0	11	8
F	0	6	10	16
G	0	2	21	26
H	2	0	9	8
I	15	14	10	0
J	5	1	0	18

It means that for A→2, B→1 , J→3 and I→4 so the following assignment is done. The optimal assignment policy is
Worker A should be assigned to task 2.
Worker B should be assigned to task 1.
Worker J should be assigned to task 3.
Worker I should be assigned to task 4.
The total cost associated with these assignments is **Rs. 43.**

V. NUMERICAL EXAMPLE SOLVED BY PROPOSED METHOD:

Consider the following cost minimizing Assignment Problem with 10 workers and 4 jobs solved by proposed method.

Worker	Tasks			
	1	**2**	**3**	**4**
A	11	8	9	8
B	4	5	29	33
C	10	5	29	33
D	1	18	25	31
E	23	22	33	30
F	3	9	13	19
G	6	8	27	32
H	32	30	39	38
I	36	35	31	21
J	15	11	10	28

Step 1: Add one dummy task column by value 1 to all worker.

Worker	Tasks				
	1	**2**	**3**	**4**	**5**
A	11	8	9	8	1
B	4	5	29	33	1
C	10	5	29	33	1
D	1	18	25	31	1
E	23	22	33	30	1
F	3	9	13	19	1
G	6	8	27	32	1
H	32	30	39	38	1
I	36	35	31	21	1
J	15	11	10	28	1

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Step 2: Calculate a reduced cost matrix by dividing each row by minimum cost of its column. The matrix is as:

Worker	Tasks				
	1	**2**	**3**	**4**	**5**
A	11	1.6	1	1	1
B	4	1.4	2.88	2.5	1
C	10	1	3.22	4.12	1
D	1	3.6	2.77	3.87	1
E	23	4.4	3.66	3.75	1
F	3	1.8	1.44	2.37	1
G	6	1.6	3	4	1
H	32	6	4.33	4.75	1
I	36	7	3.44	2.62	1
J	15	2.2	1.11	3.5	1

Step 3:Look for the row with one 1 reduced cost excluding 1 from dummy column. Assign worker to task according to position of the chosen 1 and mark entire row to avoid later redundant assignment.

Worker	Tasks				
	1	**2**	**3**	**4**	**5**
A	11	1.6	1	1	1
B	4	1.4	2.88	2.5	1
C	~~10~~	~~1~~	~~3.22~~	~~4.12~~	~~1~~
D	~~1~~	~~3.6~~	~~2.77~~	~~3.87~~	~~1~~
E	23	4.4	3.66	3.75	1
F	3	1.8	1.44	2.37	1
G	6	1.6	3	4	1
H	32	6	4.33	4.75	1
I	36	7	3.44	2.62	1
J	15	2.2	1.11	3.5	1

Step 4: Look for the column with one 1 reduced cost. Assign worker to task according to position of the chosen 1 and mark entire column to avoid later redundant assignment[6]. Column 4 is having only 1.

Worker	Tasks				
	1	**2**	**3**	**4**	**5**
A	11	1.6	1	1	1
B	4	1.4	2.88	2.5	1
C	~~10~~	~~1~~	~~3.22~~	~~4.12~~	~~1~~
D	~~1~~	~~3.6~~	~~2.77~~	~~3.87~~	~~1~~
E	23	4.4	3.66	3.75	1
F	3	1.8	1.44	2.37	1
G	6	1.6	3	4	1
H	32	6	4.33	4.75	1
I	36	7	3.44	2.62	1
J	15	2.2	1.11	3.5	1

Step 5: Here the worker A is assigned to task 4 A→4,C→2,D→1. So task 1,2 and 4 is assigned but for last task 3 check for the minimum value in column.Here the minimum value is 1.11 for worker J.So assign J→3.The matrix looks like.

Worker	Tasks				
	1	**2**	**3**	**4**	**5**
A	11	1.6	1	1	1
B	4	1.4	2.88	2.5	1
C	~~10~~	~~1~~	~~3.22~~	~~4.12~~	~~1~~
D	~~1~~	~~3.6~~	~~2.77~~	~~3.87~~	~~1~~
E	23	4.4	3.66	3.75	1
F	3	1.8	1.44	2.37	1
G	6	1.6	3	4	1
H	32	6	4.33	4.75	1
I	36	7	3.44	2.62	1
J	15	2.2	1.11	3.5	1

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It means that for A→4, C→2 , D→1 and J→3 so the following assignment is done. The optimal assignment policy is
Worker A should be assigned to task 2.
Worker B should be assigned to task 1.
Worker J should be assigned to task 3.
Worker I should be assigned to task 4.
The total cost associated with these assignments is **Rs. 24**

VI. CONCLUSION

In this paper, the assignment costs are considered not an exact number described by Hungarian method for unbalanced problem. The space require for Hungarian to solve the problem is O(n2). The main goal is to find out the minimum optimal assignment by assigning all tasks to the worker. But here when we are solving the unbalanced assignment problem, the Hungarian algorithm requires more time to solve the problem as well as space. Therefore, a simple and more efficient method is determined to solve the unbalanced problem and it is presented in this paper. The proposed method uses a new space solution arranged in or extending along a straight line. Based on the n number of cost matrices, our proposed method provides lower feasible and optimal cost than of the previous method. In our new method for the problem we are getting the cost Rs.24 compare to Hungarian method which is Rs.43. So the proposed method is giving the better result.

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