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Design of Digital Low Pass Fir Fiter Using Hybrid Particle Swarm Optimization

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Abstract- This paper presents an optimal design of linear phase digital low pass finite impulse response (FIR) filter using hybrid particle swarm optimization (HPSO) technique where PSO has been hybridized with exploratory search technique. PSO is a simple, population based robust global search algorithm capable of handling large search space. Exploratory move is a gradient free deterministic algorithm, exploited as a local search technique. So the proposed Hybrid method calculates the optimal filter coefficients such that error function is minimized by exploring and exploiting search space globally as well as locally by employing exploratory move on global best particle. The simulation results show that the proposed method is superior to its counterparts at higher orders.

Keywords – FIR filter, HPSO, PSO, Exploratory search, Deterministic algorithm, Optimization.

I. INTRODUCTION

Signal is any fundamental quantity representing some information. It is a formal description of any phenomenon evolving around time and space. On the basis of time, it can be classified as continuous time and discrete time signals. An enabling technology used for generation, transformation and interpretation of information from the signals is termed as signal processing. It plays an extremely important and continually developing role in engineering systems. It encompasses the fundamental theory, algorithms, and applications for processing, understanding, learning, regeneration, mining and extraction of information from signals. Method of extraction of information can be of different types depending on the type and nature of information being carried out by the signal. The signal processing operations involved in many applications like communication systems, control systems, instrumentation, biomedical signal processing, geophysical signal processing etc. can be basically implemented in two different ways:

1. Analog or continuous time method
2. Digital or discrete time method

Analog signal processing is achieved by analog components such as resistors, capacitors and inductors. However the analog circuitry can be dramatically affected by the inherent tolerance associated with these components, voltage, temperature changes and mechanical vibrations. Hence digital signal processing is preferred over analog signal processing as it is so much powerful that sometimes it is extremely difficult for analog signal processing to achieve similar performance.

Digital signal processing (DSP) is the mathematics, the algorithms and the techniques for analyzing and modifying a signal to optimize or improve its performance or efficiency. It is a type of signal processing performed by digital means with digital signal processor or a similar device that can execute DSP specific processing algorithms [11]. It involves applying a number of algorithms (mathematical, computational) to both analog and digital signals to generate a new signal, which has better characteristics than the original signal. Typically DSP first converts signal into a digital signal with an analog to digital convertor (ADC) by sampling, quantization. However if the output required is analog then a digital to analog convertor (DAC) is used. DSP is one of the most powerful tools that will shape the science and engineering in this century as it has a major and increasing impact in many key areas of technology. Revolutionary changes have been already made in different range of fields including communications, medical imaging, radar and sonar, reproduction, oil prospecting etc. It has a number of advantages especially in reliability, repeatability, versatility, upgradability and flexibility, high noise immunity, low sensitivity to temperature changes and component tolerances. It also reduces noise susceptibility, development time, chip count, cost, power consumption. Also integrated chips have life time more than 15 years. DSP is not an expensive technology, it is a commonplace in various systems and devices these days.

Filtering is one of the most widely used complex signal processing operations used to boost or attenuate regions of signal spectrum. The systems performing this function are known as filters. So filtering is done to select or to suppress certain frequency components in a frequency band and the range of frequencies those blocked by filter is called Stop band. On the basis of their operation these filters can be classified as (1**)** Low pass filter (2) High pass filter (3) Band pass filter (4) Band stop filter .Filters can also be classified as analog filters and digital filters. Digital filters are programmable, highly flexible, versatile and portable filters which uses a digital processor to perform numerical calculations on sampled values of the signal. The processor may be a general-purpose computer such as a PC, or a specialized DSP (Digital Signal Processor**)** chip. Digital filters are easily designed, tested and implemented on a general-purpose computer or workstation**.** Although speed limitation, finite word length effect long design and development time are some limitations, but digital filters do not vary with environmental parameters like temperature, humidity unlike analog filters .So digital filters are more stable filters than analog filters. Also, there is no need to calibrate digital filters periodically.

Depending on their impulse responses digital filters are generally available as FIR (finite impulse response) filters and IIR (infinite impulse response) filters. Finite impulse response filters have a number of desirable features such as guaranteed stability, the possibility of exact linear phase characteristic at all frequencies, simpler implementation, no feedback requirement, digital implementation as non-recursive structures and great flexibility in shaping and magnitude response [4,12]. One of the major advantages of designing FIR filter is that it is symmetrical, due to which the coefficients are symmetrical. An FIR filter is based on a feed-forward filter which means there is no feedback of past or future outputs to form the present output, just input related terms are there. The filter is known as a Finite-Duration Impulse Response (FIR) filter. Other names used for a non-recursive filter include all-zero filter, feed-forward filter or moving average (MA) filter. FIR filters can be easily realized for both recursive as well as non recursive structures [8] or filters having an arbitrary magnitude response. However these filters have requirement of large storage, complex computational techniques. Also, these cannot simulate prototype analog filters.

Infinite impulse response (IIR) digital filters offer improved selectivity, computational efficiency, and reduced system delay as compared to finite impulse response (FIR) digital filters with comparable approximation accuracy [10]. IIR filters are more compact, more efficient in memory and computational requirements than FIR filters. Output of IIR filter depends upon previous inputs, but also on the previous outputs with impulse response continuing for infinite time. However its design is more difficult than FIR digital filters because IIR digital filters have a rational transfer function. Also it requires feedback and therefore feedback can result in the filter becoming unstable if the coefficients deviate from their true values and have non-linear phase characteristics. Hence FIR filters are preferred over IIR filters due to number of reasons as stated above.

There are many well known traditional methods for filter design such as window method, frequency sampling method etc. Analytic or simple iterative methods usually lead to suboptimal designs [3]. Consequently, design of digital filters requires optimization based methods which can satisfy prescribed specifications. Ideally these methods should lead to a global optimum solution with a minimum computations. There are a number of algorithms proposed for global optimization including stochastic and heuristic algorithms such as genetic algorithms (GAs), evolutionary strategies, ant colony optimization and particle swarm optimization (PSO) [4,7]. PSO is a biological inspired robust population based flexible optimization technique based on social psychological principles. It has advantages of simple implementation, control of convergence speed by controlling few parameters. However PSO gets trapped in local minima. Premature convergence and stagnation problems are two main limitations of this method. To overcome these limitations a new Hybrid Particle Swarm Optimization (HPSO) method is proposed by employing exploratory move on the global best particle of swarm. HPSO method creates search directions iteratively to completely cover the search space [1].

The rest of the paper is organized as follows: section 2 describes the FIR filter design problem statement; PSO, Exploratory move technique and HPSO algorithms are discussed in section 3; Section 4 consists of simulation results obtained for low pass filter; Finally section 5 concludes the paper.

II. FIR FILTER DESIGN PROBLEM

Digital filters are basically classified in two broad categories namely finite impulse response (FIR) or infinite impulse response (IIR) filter depending upon whether the response of the filter is dependent on only the present and past inputs or on the present and past inputs as well as previous outputs, respectively [9]. A finite-duration impulse response filter has a system function of the form given in equation (1).

$$H(z) = h(0) + h(1)z^{-1}+.....+ h(N)z^{-N} \tag{1}$$

$$\text{Or, } H(z)= \sum_{n=0}^{N} h(n)z^{-n} \qquad n=0,1,....N \tag{2}$$

where h(n) is called impulse response. The difference equation representation is

$$y(n)= h(0)x(n) + h(1)\, x(n-1)+....+h(N)x(n-N) \tag{3}$$

where, N is the order of the filter while the length of the filter is (N+1) i.e. number of filter's impulse response coefficients h(n).The FIR filter structures are always stable and can be designed to have linear phase response. The impulse responses h(n) are to be determined in the design process and will determine the type of the filter, e.g., low pass, high pass, band pass and band stop. The choice of the filters is based on three broad criteria , the filters should provide zero distortion to the signal, flat pass band and exhibit highest attenuation characteristics in the stop band. Short filter length, short frequency transition beyond the cutoff point, and the ability to manipulate the attenuation in the stop band are some other desirable characteristics. One of the major advantages of designing linear phase FIR filter is that it is symmetrical, due to which the coefficients are symmetrical. Only half of the coefficients are updated by any algorithm and then they are concatenated to form the other half due to the symmetrical nature of FIR filter. Therefore, the dimension of the problem is halved [5]. There are two possible cases of FIR filters. These can be

1) Symmetric FIR filters: A symmetric FIR filter has linear phase if its unit sample response satisfies following condition: $h(n) = h(M-1-n)$, $n = 0,1,\ldots\ldots.M-1$

2) Asymmetric FIR filters: An asymmetric FIR filter has linear phase if its unit sample response satisfies following condition:

$$h(n) = -h(M-1-n), \qquad n = 0,1,\ldots\ldots M-1$$

The frequency response of the FIR digital filter can be calculated as:

$$H(e^{jw_k}) = H(w_k) = \sum_{n=0}^{N} h(n)e^{-jw_k n} \tag{4}$$

where $W_k = 2\pi k/N$ and $H(e^{jw_k})$ or $H(W_k)$ is the Fourier transform complex vector. The frequency is sampled with N points in $[0, \pi]$ in the FIR filter frequency response. Magnitude response can be given as:

$$H_d(w) = [H_d(w_1), H_d(w_2), H_d(w_3), \ldots. H_d(w_N)]^T \tag{5}$$

$$H_i(w) = [H_i(w_1), H_i(w_2), H_i(w_3), \ldots\ldots\ldots H_i(w_N)]^T \tag{6}$$

Where, H_d represents the approximate magnitude response of the designed filter and H_i represents the magnitude response of the ideal filter for low pass, high pass, band pass and band stop. It is given, respectively as:

$$H_i = \begin{cases} 1 \text{ for } 0 \le w \le w_c; \\ 0 \qquad \text{otherwise} \end{cases} \tag{7.a}$$

$$H_i = \begin{cases} 0 \text{ for } 0 \le w \le w_c; \\ 1 \qquad \text{otherwise} \end{cases} \tag{7.b}$$

$$H_i = \begin{cases} 1 \text{ for } w_{pl} \le w \le w_{ph}; \\ 0 \qquad \text{otherwise} \end{cases} \tag{7.c}$$

$$H_i = \begin{cases} 0 \text{ for } w_{pl} \le w \le w_{ph}; \\ 1 \qquad \text{otherwise} \end{cases} \tag{7.d}$$

Where, w_c is the cut-off frequency of filter. w_{pl} and w_{ph} are lower and upper cut-off frequency respectively. In FIR filter, the coefficients are optimized so that approximation error function for magnitude is to be minimized. Absolute error is given as:

$$E_m^1 = \min\left\{\sum_{k=1}^{N}[\,||H_i(w_k)| - |H_d(w_k, x)||]\right\} \tag{8}$$

$$E(w) = G(w)[H_i(w_k) - H_d(w)_{k,x})] \tag{9}$$

Mean error is given as:

$$E_m^2 = \min\left\{\sum_{k=1}^{N}[||H_i(w_k)| - |H_d(w_k, x)||^2]\right\} \tag{10}$$

The G(w) is the weighting function used to provide different weights for the approximate errors in different frequency bands and N is the number of samples.

$H_d(e^{jw})$ is the frequency response of the approximate filter.

$H_i(e^{jw})$ is the frequency response of ideal filter.

The ripple magnitudes of pass-band and stop-band are to be minimized, which are denoted by $\delta p(x)$ and $\delta s(x)$ respectively. Ripple magnitudes are defined as:

$$\delta p(x) = \max_{w_k}\{|H_d(w_k, x)|\} - \min_{w_k}\{|H_d(w_k, x|\} \qquad \text{for } w_k \in \text{passband} \tag{11}$$

$$\text{And, } \delta s(x) = \max_{w_k}\{|H_d(w_k, x)|\} \qquad \text{for } w_k \in \text{stopband} \tag{12}$$

III. OPTIMIZATION TECHNIQUES

A. PARTICLE SWARM OPTIMIZATION (PSO)

Particle swarm optimization (PSO) is a flexible, biologically inspired, computationally efficient, robust population based algorithm capable of handling larger search space and non differential objective function [10]. The concept of PSO is similar to the behavior of swarm of birds to simulate random movements of bird flocking in multidimensional space. In

PSO each member of the swarm is called 'Particle', which is associated with a position and a velocity. These particles are randomly initialized over the whole search space. The best solution is found in this technique by simply adjusting the trajectory of each individual particle towards its own best location and towards the best particle of the entire swarm. so each particle tries to move toward the optimum solution by accelerating towards the two best values: (1) the best value achieved by the particle so far i.e. pbest, which provides the cognitive information (2) the best value achieved so far among the entire group i.e. gbest, which gives social information[4].
Mathematically, velocity and position of particle is modified in each iteration as following equation:

$$v[\ ] = v[\ ] + c_1 * rand(\ ) * (pbest[\ ] - present[\ ]) + c_2 * rand(\ ) * (gbest[\ ] - present[\tag{13}$$

$$present[\ ] = present[\ ] + v[\ ] \tag{14}$$

Where, v [ ] is the particle velocity, present [ ] is the current particle (solution), pbest [ ] and gbest [ ] are defined as stated before, rand ( ) is a random number between [0, 1], c_1, c_2 are cognitive and social acceleration constants respectively, usually $c_1 = c_2 = 2$.

The population of particles tends to cluster together from different directions. The algorithm runs through above process iteratively until a predefined criterion is reached. PSO has been successfully employed in many areas including function optimization, artificial neural network training, fuzzy system control, and other areas where GA can be applied. PSO is a very simple nature inspired, easy to implement and computationally efficient technique having two main advantages fast convergence, only few parameters are to be controlled. However performance of PSO depends on its control parameters and may be influenced by premature convergence and stagnation problem [4].

B. EXPLORATORY MOVE SEARCH TECHNIQUE

To overcome problems associated with PSO, the PSO algorithm has been hybridized with Hook Jeeves exploratory move on global best particle of the swarm. Hook Jeeves is a gradient free, deterministic local optimization iterative technique consisting of sequence of exploratory moves in all directions. PSO is a global search technique, may converge slowly nearby solutions, when it occurs we can introduce the exploratory move on global best particle [1,6].

In the exploratory move, the current point is perturbed in positive and negative directions along each variable one at a time and the best point is recorded. The current point is the best point at the end of each variable perturbation. If the point found at the end of all variable perturbations is different from the original point, the exploratory move is a success otherwise; the exploratory move is a failure. So, this technique creates set of search directions iteratively in such a way so that the search directions completely cover the search space. As a result of this, global search capability increases for its long jump ability. Also, this technique shows the fast convergence speed greatly overcome the tendency of trapping in a local minima. The filter coefficient x_i is perturbated as follows :

$$x_i^n = x_i^0 \pm \Delta_i u_i^j \tag{15.a}$$

Where, $$u_i^j = \begin{cases} 1 & if\ i = j \\ 0 & if\ i \neq j \end{cases} \tag{15.b}$$

S denotes number of variables. The objective function denoted by $A(x_i^n)$ is calculated as follows:

$$x_i^n = \begin{cases} x_i^0 + \Delta_i u_i & ; A(x_i^0 + \Delta_i u_i) < A(x_i^0) \\ x_i^0 - \Delta_i u_i & ; A(x_i^0 - \Delta_i u_i) < A(x_i^0) \\ x_i^0 & ; otherwise \end{cases} \tag{16}$$

Where (i=1,2,….,S) and Δ_i is random for global search and fxed for local search. The Process is repeated till all the coefficients and overall minimum is selected as new starting point for next iteration [2].

C. HPSO ALGORITHM

1. PSO is initialized with a group of random particles (solutions) ,where particle represent a potential solution (better condition).Then it searches for optima by updating generations.
2. In every iteration , each particle is updated by following two "best" values.
   (a) The first one is the best solution (fitness) it has achieved so far. (The fitness value is also stored) This value is called pbest.
   (b) The second "best" value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the population. This best value is a global best and called gbest.
3. When a particle takes part of the population as its topological neighbours, the best value is a local best and is called local best.
4. After finding the two best values, the particle updates its velocity and positions using the equations (13) and (14) and calculate fitness function.
5. Call Exploratory Move on the global best and then find the new global best .
6. If the new global best is better than the global best, then replace the global best.

IV. SIMULATION RESULTS AND DISCUSSIONS

A Particle Swarm Optimization and Hybrid Particle Swarm Optimization (HPSO) algorithms have been applied to design the digital FIR low pass filter, yielding optimal filter coefficients. The optimized coefficients of this filter are taken and the magnitude and phase response of FIR filter are drawn using MATLAB. The order of the filter has been taken 20 which results in number of coefficients as 21.To design a digital low pass FIR filter 200 equally spaced samples are set within the range [0,π]. The range of pass-band and stop-band has been taken as $0 \le \omega \le 0.2\pi$ and $0.3\pi \le \omega \le \pi$ .The algorithms are run for 100 times and 100 iterations have been taken to obtain best results at different orders. Initially the population size (IPOP) is taken as 100, accelerating constants c_1 and c_2 as 2.0, maximum weight (w_{max}) as 0.45.

A. PSO

1) Selection of order: Order of the filter has been varied from 20 to 50 for the PSO algorithm and objective function has been observed. The Fig. 1 shows objective function variations with respect to order of the filter.

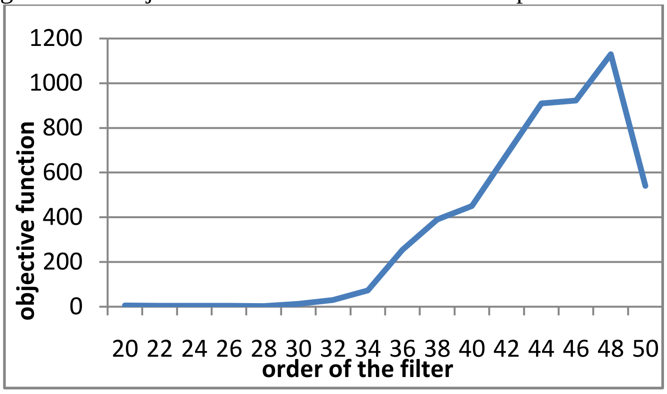

Figure 1: Order v/s Objective Function v/s order of the filter

Fig. 1 shows that with the increase of filter order objective function decreases continuously and we get the minimum value of objective function at order 28 and then after this value of objective function starts to increase with increase in filter order. So order 28 is having optimum value of objective function while at higher orders the value is deteriorated.

B. HPSO

1) Selection of order: PSO algorithm is hybridized using Hooke Jeeves Exploratory Move algorithm and the results have been observed by varying order of the filter. The values of ALF and MIT parameters of this algorithm have been taken as 2.0 and 20 respectively. Order of filter has been varied from 20 to 50 for the HPSO algorithm and objective function is observed. The Fig. 2 shows objective function variations with respect to order of the filter

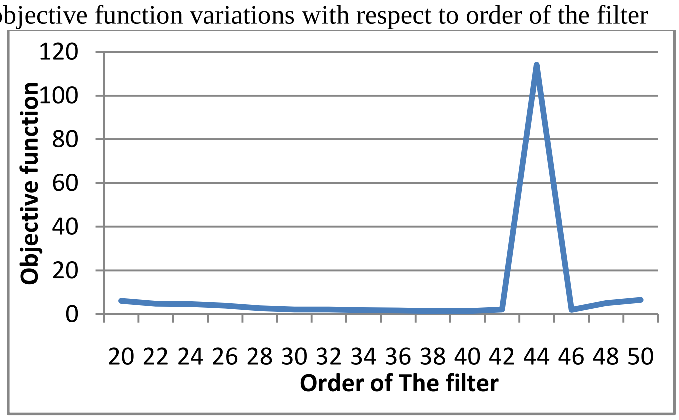

Figure 2: Order v/s Objective Function

Fig. 2 shows that with the increase of filter order objective function decreases continuously and we get the optimum value of objective function at order 40 and then after this value of objective function starts to increase with increase in filter order. As order 40 is having minimum value of objective function so it has been selected for the design of digital low pass FIR filter.

C. Comparison of PSO and HPSO algorithms

In PSO, minimum objective function has been obtained at the order 28, but this value is deteriorated at the higher orders whereas in case of HPSO, its objective function value decreases up to the order 40 i.e. minimum value of objective function is observed at order 40. So HPSO algorithm is capable of giving enhanced performance than PSO at higher orders. Due to superior performance result in terms of objective function, magnitude error HPSO algorithm has been selected for further tuning of parameters like c_1, c_2, w_{max}, IPOP.

1) ***Selection of population:*** Population has been varied from 60 to 120 in steps of 10 for FIR filter order 40 using the HPSO algorithm and objective function is observed. The Fig. 3 shows objective function variations with respect to population size.

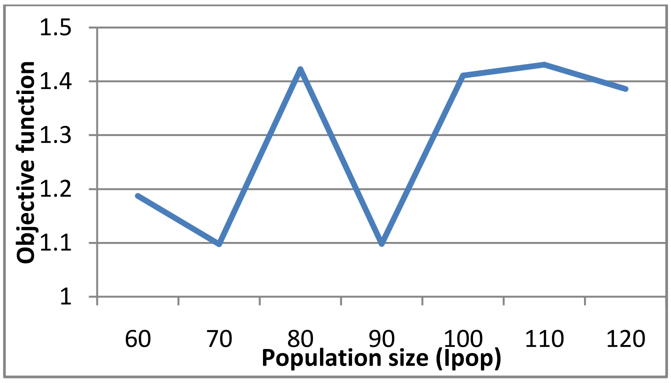

Figure 3: Population size v/s Objective Function

Fig. 3 shows that with the increase of population objective function varies continuously. It initially decreases and then it increases after population is equal to 70. Again it decreases at population size of 80 and then starts increasing up to population size of 110.It further decreases at population size of 120.So we get the minimum value of objective function at population 70. Table 1 shows the values of objective function corresponding to different sizes of population.

Table 1: Population v/s Objective Function

Sr. No.	Population Size (Ipop)	Objective Function
1	60	1.18746
2	70	1.09789
3	80	1.4226
4	90	1.09799
5	100	1.41084
6	110	1.43101
7	120	1.38567

2) ***Selection of constants:*** Constants c_1 & c_2 have been varied from 0.5to 2.5 in steps of 0.5 for FIR Filter order 40 using the HPSO algorithm and objective function is observed. The Fig. 4 shows objective function variations with respect to constants.

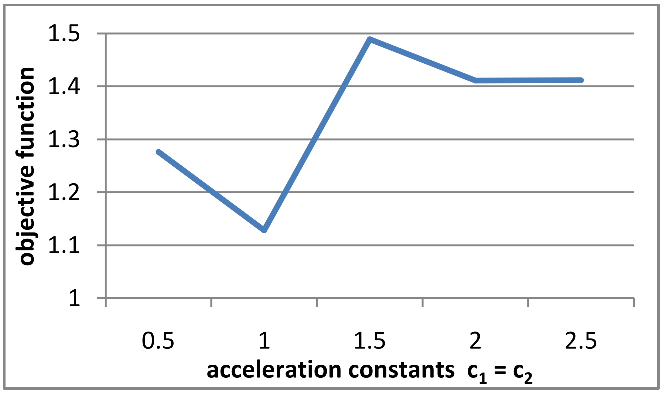

Figure 4:Acceleration Constants v/s Objective Function

Fig. 4 shows that with the increase of the constants c_1 and c_2 objective function first decreases and then starts increasing after constants are equal to 1.5.Then again it increases and decreases when constants are having value of 2.0 and 2.5 respectively. We get the minimum value of objective function at constants equal to 0.5. So order 40 with c_1 and c_2 equal to 0.5 has the minimum value of objective function. Table 2 shows the different values of objective function corresponding to different values of acceleration constants.

Table 2: Acceleration Constants v/s Objective Function

Sr. No.	Acceleration constants ($c_1 = c_2$)	Objective Function
1	0.5	1.276477
2	1.0	1.12826

3	1.5	1.48874
4	2.0	1.41084
5	2.5	1.41181

3) ***Selection of maximum weight:*** Maximum weight w_{max} has been varied from 0.40 to 0.70 in steps of 0.05 for FIR Filter order 40 using the HPSO algorithm, other parameters have initial values and objective function has been observed. The Fig. 5 shows objective function variations with respect to w_{max}.

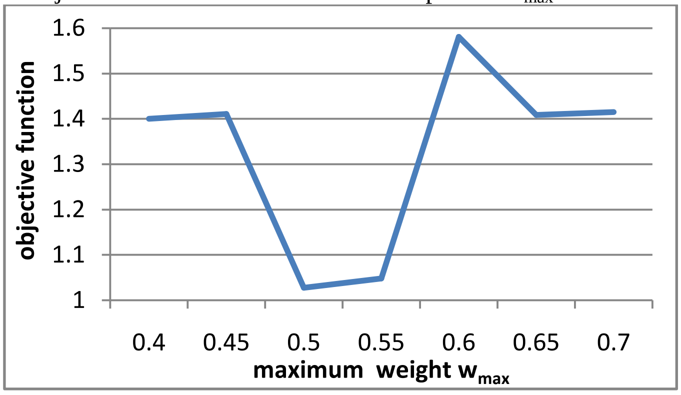

Figure 5: maximum weight v/s Objective Function

Fig. 5 shows that with the increase of the w_{max} objective function varies randomly. Initially it increases and then decreases at the value of maximum weight is 0.50.Now again the value of objective function start increasing with the increase in value of w_{max}. We get the minimum value ofobjective function at equal to 0.5. Table 3 shows various parameters corresponding to order 40 FIR Filter with different values of w_{max} while other parameters are having their initial values.

Table 3: w_{max} v/s Objective Function

Sr. No.	**Maximum Weight(w_{max})**	**Objective Function**
1	0.4	1.39986
2	0.45	1.41084
3	0.50	1.02754
4	0.55	1.048231
5	0.60	1.58082
6	0.65	1.40837
7	0.70	1.41501

D. Analysis of magnitude and phase response

This section shows simulation results performed in MATLAB for design of digital FIR low pass filter. Order of filter is taken as 40 which results in number of coefficients as 41. Frequency response of designed filter have been obtained from coefficients and magnitude is noticed across the normalized frequency to analyze the amplification and attenuation values for the different frequency range that is to find pass-band and stop-band range and behavior of filter in these bands. Magnitude response of low pass filter having coefficients as shown in Figure 6.

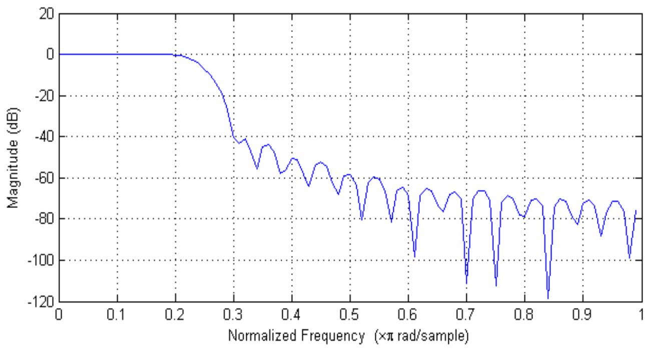

Figure 6: Magnitude vs Normalized Frequency

Fig. 6 shows that magnitude decreases as frequency increases in the digital low pass FIR filter

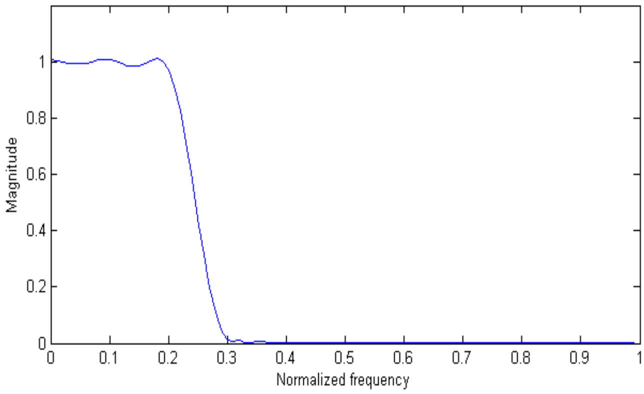

Figure 7: Magnitude response of Low Pass Digital FIR Filter

Fig. 7 shows that signals of frequency range in the stop band have been attenuated by the stop band and the signals of frequency range in the pass band have been passed by the pass band.

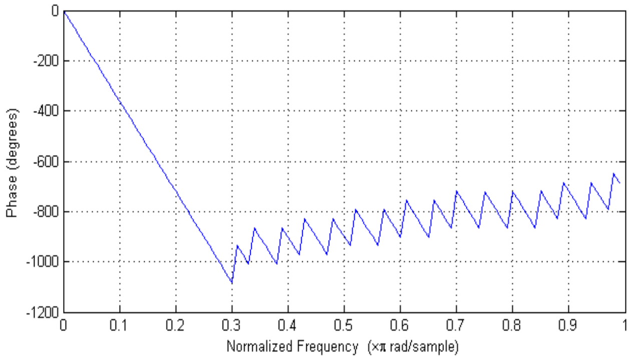

Figure 7: Phase v/s Normalized Frequency

Fig. 8 shows that the filter has a linear phase response from frequency range 0 to 0.3.

V. CONCLUSION

HPSO algorithm shows the fast convergence speed greatly overcome the tendency of trapping in a local minima as compared to PSO. So it is very powerful optimization algorithm that exhibits simplicity, robustness using its control parameters. This algorithm achieves effective trade-off between exploration and exploitation. The right choice of parameters is very important in any application. These parameters have great impact on the objective function. In this thesis digital FIR filter is designed using HPSO algorithm as it has superior performance at higher orders. Order of filter has been varied from 20 to 50 and filter order 40 gives the best value of objective function. The values of control parameters have been varied to obtain the optimum results for the design of digital low pass FIR filter. The simulation results indicates that the designed filter gives the optimum value of objective function at filter order 40 with population size 100, c_1 & c_2 value 2.00, w_{max} value 0.45. Then the magnitude and phase plot have been analyzed. The same algorithm can also be applied to design high pass, band pass and band stop filters.

REFERENCES

[1] Nanda Dulal Jana, and Jaya Sil," Particle Swarm Optimization with Exploratory Move", P.Maji et al. (Eds.): PReMI 2013,LNCS 8251, pp 614-621,2013 @ Springer –verlag Berlin Heidelberg 2013.

[2] Balraj Singh, J.S. Dhillon and Y.S. Brar,"A Hybrid Differential Evolution Method for the Desin of IIR Digital Filter",ACEEE Int. J. on Signal & Image Processing Vol. 4,No. 1, Jan 2013.

[3] Suman Kumar Saha, Rajib Kar,Durbadal Mandal and S.P. Ghosal,"IIR Filter design with Craziness based Particle Swarm Optimization Tehnique",World Academy of Science,Engneering and Technology ,vol:5 2011-12-23.

[4] Sangeea Mondal, Vasundhara, Rajib Kar, Durbadal Mandal, and S.P. Ghosal,"Linear Phase High Pass FIR Filter Design using Improved Particle Swarm Optimization Tehnique", World Academy of Science, Engneering and Technology ,vol:5 2011-12-26.

[5] Vasundhara, Durbadal Mandal, Rajib Kar and Sakti Prasad Ghosal," Digital FIR filter design using fitness based hybrid differential evolution with particle swarm optimization",Nat Comput (2014) 13:55-64.

[6] Elnur Aliyev,"Use of Hybrid Approaches and Metaoptimization For Well Placement Problms" , M. Sci. thesis, Department of Energy Resources Engineering of Stanford University, May 2011.

[7] Ranjit Kaur, Manjit Singh Patterh and J.S.Dhillon, "Digital IIR Filter Design using Real Coded Genetic Algorithm", I.J. Information Technology and Computer Sciences, 2013,07,27-35.

[8] A. Anand Kumar,"Digital Signal Processing", second edition,PHI Learning Pvt. Limitd,2014.
[9] John G. Proakis and Dimitris G. Manolakis, "Digital Signal Processing", Pearson, fourth edition,2013.
[10] J. Kennedy and R.Eberhart,"Particle Swarm optimization" in Proc. IEEE Int. Conf. on Neutral Network, vol. 4, pp. 1942-1948, Australia 1995.
[11] Amandeep Kaur Mann, Balraj Singh Sidhu, Darshan Singh Sidhu," Design of High Order Digital IIR Filter using Heuristic Optimization Technique" International Journal of Advanced Research in Computer Science and Software Engineering, Volume 4, Issue 10, October 2014.
[12] Amrik Singh, Narwant Singh Grewal," Review on FIR Filter Designing by Implementations of Different Optimization Algorithms", International Journal of Advanced Information Science and Technology (IJAIST), Vol.31, No.31, November 2014.

