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# Generalized Wirelength Models for Analytical Placement

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***Absract-In VLSI physical design, placement is a critical stage. At his stage cells are optimally placed within the chip with the condition that blocks do not overlap. Analytical placer minimizes the Half Perimeter Wirelength (HPWL) of the circuit with non-overlapping constraint. Commonly used wirelengths are lp and weighted average (WA) wirelength models. This paper generalizes these two models to their generalized counter parts. Experimental results for global placement on ISPD 2004 benchmarks show that the generalized weighted average model shows reduction in wirelength by 36 % and 52% than WA and lp models respectively.***

***Keywords-wirelength model; analytical placement; VLSI; analytical placer;HPWL***

## I. INTRODUCTION

Placement is considered an important stage in physical design of VLSI chips. At this stage blocks locations in chip are optimally determined such that blocks do not overlap and their total interconnect wirelength is minimum. Analytical placer solves the placement problem as mathematical programming problem, where objective function is HPWL of the circuit and constraints are power, delay, congestion etc.etc. Over past years a bunch of of research papers have been published on analytical placement. Recent widely used analytical placers are Aplace [6], Fast-Place [10], mPL6 [2], FastPlace [10],NTUPlacer [1]. Analyti-cal placers like Aplace [6], mPL6 [2],NTUPlacer [1] use Half Half-perimeter Wirelength (HPWL) as their only objective becase of its simplicity and ease of calculation. -lytical placers, replace max and min functions by their smooth continuous approximations. HPWL uses max and min functions which are non smooth and there are many approximations to max and min functions suggested by various authors in literature [6,8 ]. smoothed HPWL models for analytical placement. The authors in [8] applied the new wirelength model to TSV aware placement. error bounds reported in their work are even less than widely used Log-Sum-Exponent wirelength model.

In this paper we proposed two wirelength models generalizing lp and WA wirelength models. Experimental results show that the generalized WA model outperform than its counter part as well lp model. The rest of the paper is organized as follows: Section 2 discusses the HPWL formulation and existing wirelength models. The proposed wirelength models are presented in Section 3. Section 4 discusses the implementation issues and finally the the paper is concluded in Section 6.

## II. HPWL FORMULATION AND REVIEW OF EXISTING WIRELENGTH MODELS

The circuits in placement are represented as a hypergraph H(V,E), where V is the set of fixed or movable blocks or pads, and E is a set of nets. Each net or hyperedge e ∈ E is a subset of V. The objective of placement problem is to determine the physical locations $(x_i, y_i)$ $(1 \le i \le |V|)$ of blocks in chip so as to minimize the total wirelengths of nets. The constaints imposed in the minimization problem are congestion, routability, amount of overlap, power etc. As finding shortest wirelength of a placement satisfying the aforesaid constaints is an NP-hard problem, HPWL is most commonly used metric for determining the wirelength of a placement.

HPWL of a net e is defined as

$$HPWLe = \max_{i\in e}\{x_i\} - \min_{i\in e}\{x_i\} + \max_{i\in e}\{yx_i\} - \min_{i\in e}\{y_i\} \qquad (1)$$

and HPWL of a placement is given by sum of the HPWL of all nets in the netlist

$$HPWL = \sum_{e\in E} HPWLe \qquad (2)$$

There are various other wirelength models to measure wire length of a net connecting a set of pins. These include the minimum spanning tree cost, half perimeter wirelength, Steiner tree app-roximation, minimum chain, Source sink etc. [9]

### *A. Review of Existing HPWL Wirelength Models*

**p-norm approximation** For p>1, using the p−norm of $l_p$ space another smooth approxi-mation **to** HPWL is given by:

$$PNWLe = \left(\sum_{i\in e}|x_i|^p\right)^{1/p} - \left(\sum_{i\in e}|x_i|^{-p}\right)^{-1/p} + \left(\sum_{i\in e}|y_i|^p\right)^{1/p} - \left(\sum_{i\in e}|y_i|^{-p}\right)^{-1/p}$$

which converges to HPWL as $p \to \infty$. (3)

**Weighted-Average Wirelength Model(WA)** [4] If $x$ coordinates of a net $e$ is denoted by $x_e$, then for calculating the wirelength of a net $e$, the weighted average is defined by

$$X(x_e) = \frac{\sum_{v_i \in e} x_i F(x_i)}{\sum_{v_i \in e} F(x_i)} \quad (4)$$

Choosing $F(x_i) = exp(x_i/\gamma)$ where $\gamma \to 0$the smooth approximation for maximum function is given by

$$X(x_e) = \frac{\sum_{v_i \in e} x_i exp(x_i/\gamma)}{\sum_{v_i \in e} exp(x_i/\gamma)} \quad (5)$$

The minimum estimation function can be defined similarly. Using the above approxi-mations Hsu[8] derived the following WA wirelenghth model for HPWL.

$$X_e^{max} = \frac{\sum_{v_i \in e} x_i exp(x_i/\gamma)}{\sum_{v_i \in e} exp(x_i/\gamma)} \quad X_e^{min} = \frac{\sum_{v_i \in e} x_i exp(-x_i/\gamma)}{\sum_{v_i \in e} exp(-x_i/\gamma)}$$

$$Y_e^{max} = \frac{\sum_{v_i \in e} x_i exp(y_i/\gamma)}{\sum_{v_i \in e} exp(y_i/\gamma)} \quad Y_e^{min} = \frac{\sum_{v_i \in e} x_i exp(-y_i/\gamma)}{\sum_{v_i \in e} exp(-y_i/\gamma)}$$

$$\sum_{e \in E} (X_e^{max} - X_e^{min} + Y_e^{max} - Y_e^{min}) \quad (6)$$

where $\gamma$ tends to 0. If $\epsilon WA(x_i)$ is the estimation error of the WA wirelength model for x co-ordinates of a net, Then the authors in [8] proved the following theorems concerning the bounds of error.

**Theorem 1** $0 \le \epsilon WA(x_i) \le \frac{\gamma \Delta x}{1+\exp(\Delta x)/n}$

$where\ \Delta x = x_{max} - x_{min}$

**Theorem 2** The estmation error upper bound of the WA wirelength model is smaller than that of the LSE wire-length model i.e

$\frac{\gamma \Delta x}{1+\exp(\Delta x)/n} < \gamma \ln\ ,\ \forall n \ge 2$

## III. PROPOSED WIRELENGTH MODELS

This section proposes two wirelength models by generalizing the existing p-norm based approximation, WA approximation discussed in Section 2.

### *A. Proposed Weighted $l_p$-norm Wirelength Model*

Let $p \ge 1\ and\ w = (w_n)$ be a decreasing non-negative sequence of real or complex numbers such that $\lim_{n\to\infty} w_n \to 0$ and $\sum_{n=1}^{\infty} w_n = \infty$Then the weighted sequence space[3] $l_p(w)$ is given by

$$l_p(w) = \left( x = (x_k) : \sum_{k=1}^{\infty} w_k\, |x_k|^p < \infty \right) \quad (7)$$

with a norm

$$\|x\|_{w,p} = \left( \sum_{k=1}^{\infty} w_k\, |x_k|^p \right)^{1/p} \quad (8)$$

It is known that the norm of $l_n^p$ space $(\sum_{i=1}^{n} |x_i|)^{1/p}$ converges to $\|.\|_\infty = \max\{x_1 \dots x_n\}$ as $p \to \infty$. In the following Theorem we claim that the norm of weighted $l_n^p(w)$ space also converges to $\max\{x_1 \dots x_n\}$ as $\to \infty$ .

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**Proposition 1** Weighted $\|.\|_{w,p}$ function is infinitely differentiable function except at x = 0 and strictly convex. For (β > 0), additional β-regularization $(w_1|x_1|^p + \cdots + w_n|x_n|^p + \beta)^{1/p}$ is needed to smooth the function at x = 0.

**Proposition 2** $\|x\|_{w,p} \le \|x\|_p \le n^{1/p}\|.\|_\infty$

**Fact 1**. Gradient computation of smoothed approximation can be derived in $O(n)$ time efficiently.

**Wirelength formulation** Using the wighted lp approximations of max function the HPWL of a net e is given by

$$PNWL_e^w = \left(\sum_{k=1} w_i\, |x_i|^p\right)^{1/p} - \left(\sum_{k=1} w_i\, |x_i|^{-p}\right)^{-1/p} + \left(\sum_{k=1} w_i\, |y_i|^p\right)^{1/p} - \left(\sum_{k=1} w_i\, |y_i|^{-p}\right)^{-1/p} \quad (9)$$

which converges to HPWL as $p \to \infty$

### *B. Proposed Generalized Weighted Average Wirelength Model*

In this section we generalize the wighted average (WA) wirelength model [4] given by equation(8). In Theorem 3(See below) it is shown that the error bounds of the generalized wirelength(GWA) model is less than the error bounds of WA wirelength model.

Let $x_e$ be a vector representing $x$ co-ordinates of a net $e$. For a non-negative non-increasing sequence of real numbers $w = w_n$ with the property that $w_n \to 0$ as n→ ∞ and $w_n = \infty$ . Define weighted average by

$$X^w(x_e) = \frac{\sum_{v_i \in e} w_i x_i F(x_i)}{\sum_{v_i \in e} w_i F(x_i)} \quad (10)$$

For $0 \le \alpha < 1$take$w_i = \frac{1}{i^\alpha}$ and $F(x_i) = exp(x_i/\gamma)$. Clearly $w_i = \frac{1}{i^\alpha} \to 0\ as\ i \to \infty$ and $\sum_{i=1} \frac{1}{i^\alpha} = \infty$where $\gamma \to 0.$ Now the estimation function for the−maximum value is given by

$$X^{w}(x_e) = \frac{\sum_{v_i \in e} x_i \frac{exp\,(x_i/\gamma)}{i^{\alpha}}}{\sum_{v_i \in e} \frac{exp\,(x_i/\gamma)}{i^{\alpha}}} \qquad (11)$$

Similarly estimation function for minimum could be defined. Therefore the GWA wirelength model is given by

$$\sum_{e \in E} (X_e^{max} - X_e^{min} + Y_e^{max} - Y_e^{min}) \qquad (12)$$

Where

$$X_e^{max} = \frac{\sum_{v_i \in e} x_i exp(x_i/\gamma) i^{-\alpha}}{\sum_{v_i \in e} exp(x_i/\gamma) i^{-\alpha}}$$

$$Y_e^{min} = \frac{\sum_{v_i \in e} x_i exp(-y_i/\gamma) i^{-\alpha}}{\sum_{v_i \in e} exp(-y_i/\gamma) i^{-\alpha}}$$

It is clear from Theorem 3 that the GWA wirelength model converges to the HPWL of equation(2), as γ goes to 0. In parallel to the argument in [4], it can be shown that the approximation functions of max and min functions in equation(12) are strictly convex and continuousely differentiable with respect to $x_i$ and $y_i$. Thus we have the following results of GWA wirelength model.

**Lemma 1** The GWA wirelength model is strictly convex and continuously differentiable.

Let the $\epsilon GWA$ be the estimation error of GWA wirelength model with respect to $x$ coordinate. We have the following estimation error bounds of this model.

**Theorem 3** $0 \le \epsilon GWA(x_e) \le \frac{\gamma \Delta x}{1 + \exp\,(\Delta x) / \sum_{i=1}^{n} \frac{1}{i^{\alpha}}}$

Where $\Delta x = x_{max} - x_{min}$ $and$ $0 \le \alpha < 1$

Now for α = 0, Theorem 4 reduces to Theorem 1. Further for 0 < α < 1, the error bounds of Theorem 3 is less than error bounds of Theorem 1.Thus the pro-posed wirelength model is a generalization of weighted average wirelength model [4].

**Proof** : Let $x_{max}$ ($x_{min}$) be the maximum(minimum) value of $x_e$ and $X_{max}^{w}$ $(x_e)\big(X_{min}^{w}$ $(x_e)\big)$ be the estimation function for the maximum(minimum) value of $x_e$. For the maximum function of the GWA wirelength model, the estimation error is given by

$\in_{GWA}^{w*} = x_{max} - X_{max}^{w}\ (x_e) =$

$$xt_{max} - \frac{\sum_{v_i \in e} x_i exp(x_i/\gamma) i^{-\alpha}}{\sum_{v_i \in e} exp(x_i/\gamma) i^{-\alpha}} =$$

$$\frac{(x_{max} - x_1) exp(x_1/\gamma) 1^{-\alpha} + \cdots + (x_{max} - x_n) exp(x_n/\gamma) n^{-\alpha}}{\sum_{v_i \in e} exp(x_i/\gamma) i^{-\alpha}}$$

$$= \gamma \frac{\sum_{v_i \in e} -\Delta^* x_i exp\,(-\Delta^* x_i) i^{-\alpha}}{\sum_{v_i \in e} exp\,(-\Delta^* x_i) i^{-\alpha}} \qquad (13)$$

where $\Delta^* x_i = (x_{max} - x_i)/\gamma$. Since by definition of generalized weighted average, $X_{max}^{w}\ (x_e) \le x_{max}$ and $\in_{GWA}^{*} \ge 0$ and $x_e = \{x_1, x_2, \dots, x_n\}$. Without loss of generality assume $x_1 \ge x_2 \ge x_3 \ge \cdots \ge x_n$ . From eqaution (13) we have,

$\in_{GWA}^{w*}\ (x_e) =$

$$\gamma \frac{\Delta x_{1,2}, \exp(-\Delta x_{1,2})\, 2^{-\alpha} + \cdots + \Delta x_{1,n} \exp(-\Delta x_{1,n})\, n^{-\alpha}}{1 + \exp(-\Delta x_{1,2})\, 2^{-\alpha} + \cdots + \exp(-\Delta x_{1,n})\, n^{-\alpha}} \qquad (14)$$

Inorder to find an upper bound of equation(14), let us differentiate equation(14) with respect to variables $\Delta x_{1,2}$ , $\Delta x_{1,3}$ , … $\Delta x_{1,n}$ and make them equal to 0. For $1 \le i \le n$, we have

$\partial \in_{GWA}^{w*}\ (x_e) / \partial \Delta x_{1,i} = 0$

$\Rightarrow \big(1 - \Delta x_{1,i}\big)(1 + \sum_{k=2}^{n} \exp(-\Delta x_{1,K}) k^{-\alpha}) +$

$\sum_{k=2}^{n} \Delta x_{1,K} \exp(-\Delta x_{1,K}) k^{-\alpha}) = 0$

$\Rightarrow \Delta x_{1,2} = \Delta x_{1,3} = \cdots = \Delta x_{1,n}$ and

$x_{min} = x_1 = x_2 = \cdots = x_n$ Thus

$$0 \le \in_{GWA}^{w*} = \frac{\gamma \Delta x exp(-\Delta x)(\sum_{k=2}^{n} k^{-\alpha})}{1 + exp(-\Delta x)(\sum_{k=1}^{n} k^{-\alpha})} =$$

$$\frac{\gamma \Delta x}{1 + \frac{exp(\Delta x)}{(\sum_{k=1}^{n} k^{-\alpha})}} \qquad (15)$$

where $\Delta x = x_{max} - x_{min}$

In parallel to the above arguements we have the same bound for minimum function as well.

Since $\in_{GWA}^{w}$

$= \Big|(x_{max} - x_{min}) - \Big(X_{max}^{w}\ (x_e) - X_{min}^{w}\ (x_e)\Big)\Big|$

 

$$= \left( \left( x_{max} - X_{max}^{w}\ (x_e) \right) - \left( x_{min} - X_{min}^{w}\ (x_e) \right) \right)$$

$\leq \frac{\gamma \Delta x}{1 + \frac{exp\ (\Delta x)}{\left(\sum_{k=1}^{n} k^{-\alpha}\right)}}$ Hence the Therom.

**Theorem4** The estimation error upper bounds of GWA wirelength model is less than WA wirelength model[4] which in turn less than logarithm-sum-expotial wirelength model's error bounds(Result of Theorem 2). Thus we have the following inequality.

$\in_{GWA}^{w*}\ (x_e)\ \leq \in_{GWA}^{w*}\ (x_e)\ \leq \gamma \ln n\ \forall n \geq 2$

## IV. EXPERIMENTATION AND RESULTS

We compare wirelengths of weighted $lp$ norm, Generalized Weighted Average(GWA) and Weighted Average(WA) wirelength models with NTU-placer[?] wirelength, conducting experi-ments on IBM ISPD 2004 benchmark suite. The num-ber of cells in these benchmarks varies from 12K to 210K. The experiments were performed on a PC with Core 2 Duo processors and 2GB memory. We implemented these wire-length models in C++ and integrated into NTU-placer, which is a leading academic 2D placer and leaves overlap less than 10%. In our experiment $\alpha$ and $\gamma$ were set to values 0.04 and 0.14 respectively. The results of global and detail placement wirelength (of NTU-placer, Weighted $lp$ norm, WA and GWA wirlength 6.based approximations) are presented in table 1. As the result of weighted Log-Sum-Exponent scheme is similar to WA method, we do not present the result from that approximation scheme. It is evident from the table that the proposed weighted $lp$ norm, GWA and existing WA wirelength schemes give better approximation to HPWL compared to the NTU-placer wirelngth. However for a given value of $\alpha$ and $\gamma$ the weighted $lp$ norm scheme gives closest approximation to HPWL compared to all other schemes used in the simulation.

## V. CONCLUSIONS AND FUTURE WORK

We proposed two smoothed wirelength approximations (Weighted $lp$ norm,  GWA, ) to HPWL function which can be used in analytical placers. We studied the convergence propertise of the proposed schemes. The proposed wirelength models when deployed in analytical placer NTUPlace give better wirelength than its counter parts.

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Table 1: HPWL Comparison on Global and Detail Placements Using Different Approximation Schemes

| | Global WL ($\times 10^7$) | | | | Detail WL ($\times 10^7$) | | | |
|---|---|---|---|---|---|---|---|---|
| Circuit | NTU | Weighted lp | WA | GWA | NTU | Weighted lp | WA | GWA |
| ibm01 | 0.17 | 0.176 | 0.28 | 0.18 | 3.69 | 66.84 | 2.73 | 12 |
| ibm02 | 0.37 | 0.381 | 0.55 | 0.38 | 2.66 | 48.31 | 1.97 | 1 |
| ibm03 | 0.5 | 0.511 | 0.76 | 0.51 | 3 | 52.45 | 2.23 | 1 |
| ibm04 | 0.59 | 0.612 | 0.94 | 0.61 | 3.25 | 57.97 | 2.41 | 1 |
| ibm05 | 1.03 | 1.05 | 1.37 | 1.05 | 1.9 | 32.22 | 1.42 | 1 |
| ibm06 | 0.53 | 0.547 | 0.91 | 0.54 | 4.28 | 73.07 | 3.18 | 1 |
| ibm07 | 0.87 | 0.918 | 1.6 | 0.91 | 5.19 | 82.81 | 3.89 | 1 |
| ibm08 | 0.96 | 1.02 | 1.76 | 1 | 5.44 | 82.8 | 4.1 | 1 |
| ibm09 | 0.98 | 1.05 | 2.02 | 1.03 | 6.67 | 105.7 | 4.99 | 1 |
| ibm10 | 1.84 | 1.94 | 3.51 | 1.91 | 5.3 | 90.69 | 3.94 | 1 |
| ibm11 | 1.42 | 1.53 | 3 | 1.5 | 7.43 | 111.05 | 5.61 | 1 |
| ibm12 | 2.4 | 2.5 | 4.13 | 2.47 | 3.93 | 72.09 | 2.9 | 1 |
| ibm13 | 1.77 | 1.91 | 3.87 | 1.87 | 7.47 | 117.97 | 5.59 | 1 |
| ibm14 | 3.36 | 3.68 | 7.91 | 3.6 | 9.34 | 135.32 | 7.05 | 1 |
| ibm15 | 4.08 | 4.47 | 9.77 | 4.38 | 9.54 | 139.36 | 7.19 | 1 |
| ibm16 | 4.35 | 4.87 | 11.4 | 4.75 | 12 | 163.11 | 9.14 | 1 |
| ibm17 | 6.65 | 7.16 | 14.4 | 7.04 | 7.79 | 116.86 | 5.86 | 1 |
| ibm18 | 4.53 | 5.13 | 12.4 | 4.98 | 13.2 | 174.68 | 10 | 1 |
| Avg Error (in%) | 6.22 | 95.74 | 4.68 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |

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