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Compressive Sensing– A Survey

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Abstract – Compressive sensing (CS) is a novel method of acquiring signals or images with minimum number of measurements compared to traditional method of acquiring the same which follows Nyquist/Shannon's sampling theorem. CS relies mainly on two fundamental premises which are sparsity and incoherence. Procedure of compressive sensing contains three main steps which are sparse representation, sensing using measurement matrix and reconstruction procedure. This article gives a brief idea about the various mathematical methods of compressive sensing images and reconstruction, details of various algorithms used in the field of compressive sensing.

Keywords- Compressive sensing, sparsity, sensing matrix, wavelet packets, compression.

I. INTRODUCTION

According to the traditional Nyquist/Shannon's sampling theorem to perfectly reconstruct a signal from its sample the signal has to be sampled at a rate twice the bandwidth. But using CS the same can be done using fewer numbers of samples.

In conventional image compression the image is transformed from spatial domain to transform domain. A sparse representation of the image is obtained. Coefficients with close to zero entries are discarded retaining the remaining which results in compression. Compression techniques such as JPEG, MPEG, or MP3 work well in practice based on this idea. For instance, JPEG relies on the sparsity of images in the discrete cosine basis or wavelet basis and achieves compression by only storing the largest discrete cosine or wavelet coefficients. The other coefficients are simply set to zero.

In compressive sensing, instead of obtaining all coefficients which requires huge memory and discarding the major coefficients near to zero which results in wastage of computational time, we directly sample very less number of coefficients from which efficient reconstruction can be done.

II. COMPRESSIVE SENSING THEORY

Let **x** Є R^N is a one-dimensional signal. In ψ domain let s be the representation of **x**. i.e.,

$$x = \psi s = \sum_{i=1}^{N} s_i \psi_i$$

Where s = $\{s_1,s_2,\dots,s_N\}$ is an N-vector of weighted coefficients. $s_i = \langle \mathbf{x}, \psi_i \rangle = \psi_i^T \mathbf{x}$ and where $\cdot^T$ is the transpose or hermitian operation. x is the representation of the given signal in time domain and s is the representation of the same signal in ψ domain. $\boldsymbol{\psi} = [\psi_1|\psi_2| \dots|\psi_N]$ is an NxN basic matrix with ψ_i being the i^{th} basic column vector.

A vector **x** is said to K-sparse in ψ-Domain where K<<N and K out of N coefficients of s are nonzero. In traditional compression the complete signal is first acquired, transformed to ψ-Domain with N coefficients, N minus K (N-K) coefficients are discarded and the remaining are encoded.

In compressive sensing the above indicated huge redundancy is removed by acquiring only M samples of the signal instead of acquiring N samples which consumes lot of memory, then discarding (N-K) samples which is waste of computational time, where K<M<<N. let y = Φx a M-length vector, where Φ is a MxN measurement matrix. Then y = Φψs = Θs where Θ = Φψ is an MxN matrix. High incoherence between sensing matrix and signals guarantees the minimum number of measurements required.

III. SENSING

Here some stable measurement matrices Φ are used to perform dimensionality reduction of the signal from **x** Є R^N down to y Є R^M. The maximum correlation between any two elements of two different matrices is called Coherence. The matrices might represent two different basis/representation domains. In compressive sensing the Incoherence of the measurement matrix **Φ** and the matrix representing the basis i.e. the representation matrix **ψ** is used. The low coherence between **Φ and ψ** translates the signal to fewer samples required for reconstruction of the signal. In [1,2] it has been shown that x can be recovered losslessly from M measurements, if Φψ satisfies the RIP i.e Restricted Isometric Property which is possible when Φ and ψ are incoherent.

IV. RECONSTRUCTION

The algorithms are broadly classified into six types.

1) *Convex relaxation*: Some of the algorithms under this category are Basis pursuit (BP), Basis Pursuit De-Noising (BPDN), modified BPDN, Least Absolute Shrinkage and Selection Operator (LASSO) and Least Angle Regression (LARS).
2) *Greedy Iterative Algorithm*: Most used greedy algorithms are Matching Pursuit and its derivative Orthogonal Matching Pursuits (OMP) because of their low implementation cost and high speed of recovery. For such situations, improved versions of OMP have been devise like Regularized OMP Stage wise OMP. Compressive Sampling Matching Pursuits (CoSaMP) Subspace Pursuits, Gradient Pursuits and Orthogonal Multiple Matching Pursuit.
3) *Iterative Thresholding Algorithms*: Iterative approaches to CS recovery problem are faster than the convex optimization problems. For this class of algorithms, correct measurements are recovered by soft or hard thresholding starting from noisy measurements given the signal is sparse. The thresholding function depends upon number of iterations and problem setup at hand. Message Passing (MP) algorithm, Expander Matching Pursuits, Sparse Matching Pursuit, Sequential Sparse Matching Pursuits, Belief Propagation are some algorithms coming under this category.
4) *Combinatorial / Sublinear Algorithms*: This class of algorithms recovers sparse signal through group testing. They are extremely fast and efficient, as compared to convex relaxation or greedy algorithms but require specific pattern in the measurements; Φ needs to be sparse. Representative algorithms are Fourier Sampling Algorithm, Chaining Pursuits, Heavy Hitters on Steroids (HHS).
5) *Non Convex Minimization Algorithms*: Non-convex optimization is mostly utilized in medical imaging tomography, network state inference, streaming data reduction. There are many algorithms proposed in literature that use this technique like Focal Underdetermined System Solution (FOCUSS), Iterative Re-weighted Least Squares, Sparse Bayesian Learning algorithms, Monte-Carlo based algorithms.
6) *Bregman Iterative Algorithms*: These algorithms provide a simple and efficient way of solving l_1 minimization problem. The computational speed of these algorithms is particularly appealing compared to that available with other existing algorithms.

V. RECENT ADVANCES

In [3] a method in which the classical DCT technique is combined with global noiselet measure, which is then solved using SOCP(second order cone programming). Here DCT is performed blockwise and the DCT information is transmitted to the decoder. Then Noiselet coefficients of the same image are calculated in the encoder which is then transmitted to the decoder. Using the DCT coefficients both at encoder and decoder the locations of the noiselet coefficients are found which eliminates the need of them to be transmitted to the decoder thereby improving the efficiency. At the decoder both DCT coefficients and Noiselet coefficients are combined to recover the original image using SOCP. When compared with the normal JPEG for obtaining the same PSNR of 33.85 dB with a file size of 9,741 bytes in JPEG this algorithm requires 9,598 bytes, i.e. slightly more efficient than JPEG. To compress an example image to a file size of 11,754 bytes, a PSNR of 43.01dB results. But by using JPEG 39.77dB is required.

In [4] to compress images and videos compressive sensing using pseudorandom sequence, Vector quantization and Arithmatic coding methods are combined. Better quality images are obtained at the same compression rate and as compared to JPEG/MPEG techniques on the transmit side, the compression of the images is eleven times faster. A set of basis functions that corresponds to the randomly selected radial frequencies are used to sense the images using linear projection technique. The 2D FFT is then performed, the measurements of which are compressed using Vector Quantization techniques. Arithmetic coding is used next for further compression and then transmitted. Reverse procedure is followed with TV minimization with equal constraint technique along with convex optimization problem to obtain inverse CS. Here main image is segmented and processing is done. The idea behind this is to use parallel processing to reduce computational time of 0.2 seconds at 0.47 bpp v/s 0.5 bpp requiring 22 seconds.

In [5] technique based on CS-VQ is used where weighted sequence is used instead of pseudorandom sequence. CS with weighted sequence is performed, then VQ encoding followed by huffman coding is performed and then transmitted. In the receiving side reverse operation is performed along with CS based signal recovery l_1 minimization Norm as this is better than some optimization reconstruction algorithms which include OMP and iterative threshold. As a comparison for a fixed bpp the proposed CS-VQ method shows better PSNR than the CS-VQ and CS-VQ with DCT, but is less than that of JPEQ but the decrease in Queue makes the proposed CS-VQ method better than it. It is also seen that when image block size is increased, PSNR is higher and as the image block size is decreased PSNR goes on decreasing.

In [6] wavelet packet is used to decompose the given image to make it sparse. The Low frequency coefficients are retained in line with the optimal basis of the wavelet packet. Random measurement of all High frequency coefficients is made according to the CS theory using a suitable random measurement matrix and the recovery is made with the OMP and inverse transform of the wavelet packet is implemented to all restored LF and HF coefficients and the original image is reconstructed. The result shows that this algorithm is simple and highly efficient with better reconstructed image quality.

In [7] Discrete wavelet transform is used for obtaining sparse representation and CS measurement is done using complex Hadamard Matrix. At the decoder CoSamp algorithm is used for performing sparse reconstruction. The results show that for a particular measurement PSNR for the present technique is more than the technique where Gaussian matrix is used. The results are as shown in Table I below

Table I PSNR in dB for an example image

M	PFFT	Gaussian	Proposed CHT

1000	9.24	19.36	21.22
1500	11.07	22.36	26.08

In [8] multiwavelet domain is used for obtaining sparse representation while OMP iterative is used as reconstruction algorithm. Gaussian matrix and bernoulli matrices are used as measurement matrices. For both matrices and for a particular value of M, PSNR is better for this technique when compared to the technique using DCT transform and wavelet transform as shown in the Table II below.

Table II PSNR for M = 150

	DCT	Wavelet Transform	Multi-wavelet Transform
Gaussian Matrix	25.3225	28.2376	28.9140
Bernoulli Matrix	24.6866	28.3624	29.0561

In [9] sparsity of image in DCT and contourlet domain with TV minimization in the reconstruction is combined. Result is superior when compared to the conventional TV minimization algorithm and as contourlet transform is used more improvement is observed because of the ability of the algorithm to detect contours and special geometrical structures in the image. The measurement matrix is MxN binary matrix of elements with the same probability. Table III given below shows the PSNR for a particular value of M for all the three methods.

Table III PSNR for M = 20

	8x8 blocks	16x16 blocks
TV	39.81dB	31.08dB
TV + DCT	41.29dB	31.92dB
TV + contourlet	42.83dB	32.63dB

In [10] compressive imaging of natural images is done using Bandlet basis and l_1 minimization algorithm is used for reconstruction. Instead of using conventional sampling matrices, regular point sampling matrix is used for finding out the geometric regularities of the images. This algorithm not only provides higher PSNR value but also produces better visual quality with sharp edges. Table IV presents the comparison of various other techniques with the proposed technique by selecting PSNR as a parameter.

Table IV PSNR for an example image (lena)

BWHT	FFT + wavelet	P.sen*	Proposed
29.6dB	29.8dB	31.2dB	33.7dB

- Refers to Random point sampling matrix, GPRS Algorithm and dB8 wavelets.

In [11] compressive video sampling method is investigated. Each video frame is split into non-overlapping blocks of equal size. Reference frames are sampled fully. Each block of the reference frame is tested for sparsity using compressive sampling test using DCT and i.i.d Gaussian matrix. All blocks of successive non-reference frames that spatially correspond to sparse blocks in the previous reference frame will be compressively sampled. Other blocks are sampled conventionally. The compressively sampled blocks are reconstructed using OMP algorithm.

In [12] compressive sampling of video is investigated. Generally in video sampling methods processing is conducted on a single frame basis. But here the multiple frames are used. This method produces successful results quickly where quality of video stream is also maintained. Here extension of wavelet to 3D along with Noiselet 3D is used and for reconstruction fast reconstruction TV algorithm is used.

In [13] the given image is decomposed with the wavelet packet to make it sparse. The LF coefficients are retained in line with the optimal basis of the wavelet packet, meanwhile measurement encoding is performed on all the HF coefficients in accordance with the CS theory and restoration is done using OMP. Then wavelet packet inverse transform is performed to reconstruct the original image. In subband coding, wavelet transform method is prone to loss of image detailed information with a high compression ratio, deteriorating the reconstructed image quality. Wavelet packet analysis provides a more sophisticated analysis method for the signal frequency band which is partitioned with multilevel divisions. HF subbands are further decomposed to Increase the frequency resolution.

In [14] fast algorithms are introduced to reconstruct the signals from Random Toeplitz and circulant matrices. The result shows that these are as effective as Random matrices and also faster decoding is resulted.

VI. CONCLUSION

A review of the compressed sensing is discussed which indicates the proper selection of sparse representation of the signals, selection of measurement matrices for acquisition and algorithms for reconstruction. First few sparse domains like multiwavelet, contourlets, curvelets, ridgelets, bandlets are used in place of conventional DFT, DCT and Wavelets. First few sensing matrices like Gaussian and Bernoulli matrices are found to be inefficient for hardware implementation.

These can be replaced by Toeplitz, circulant matrics. Basis Pursuit, OMP, Tree based OMP, stage wise OMP and CoAMP are some of the reconstruction algorithms. The aim of this paper is to develop a novel algorithm which includes better selection of measurement matrix and reconstruction algorithm.

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