



## Design of Digital Low Pass FIR Filter Using Craziness Based Particle Swarm Optimization

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**Abstract:-** Digital Filters have acquired wide range of applications not only in the field of communications but also in other signal processing areas such as image processing, medical field, pattern recognition, etc. In this paper an optimum design of linear phase digital low pass FIR filter using craziness based particle swarm optimization (CRPSO) algorithm has been proposed. The traditional and conventional gradient based optimization techniques are not efficient for such optimization problems as they get trapped on local optimum solutions. CRPSO algorithm giveses a set of optimum filter coefficients and tries to meet the required specifications. In problems of bird flying and fish schooling, a bird and a fish suddenly changes its direction due to factor known as "craziness". This has been studied by making the use of CRPSO algorithm. In this paper CRPSO technique has been applied on FIR low pass filters of different orders to obtain minimum optimal best order and further changes in different parameters are applied on the best order filter. Design of low pass FIR filter using CRPSO algorithm to minimize the magnitude error of the digital FIR filter and to minimize ripple magnitude in the pass band has been proposed.

**Keywords:** Finite Impulse Response (FIR) filter, Particle Swarm Optimization (PSO), Craziness based Particle Swarm Optimization (CRPSO).

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### I. INTRODUCTION

Processing in electronic terms means operating on a signal in some method to extract some useful information. For example to hear something ears and auditory path ways in the brain extract the information. In this example the system is biological in nature. The signal processor may be electronic system, mechanical system or it might even be a computer program. Signal processing is of two types: Analog signal processing and digital signal processing. Digital signal processing has advantages of flexibility and repeatability over analog signal processing. So it has become dominant now days. Digital signal processing (DSP) is the manipulating mathematically the information signals to modify or improve it in some way. It is characterized by the representation of discrete time, discrete frequency, or other discrete domain signals by a sequence of numbers or symbols and the processing of these signals or symbols.

In signal processing, a filter is a device or process that removes unwanted component or feature from a signal. Filters may be either analog or digital. Analog filters are the device that operates on continuous time signals. Digital filters consist of DSP processors and controller which plays an important role in DSP applications such as signal analysis and estimation. It performs mathematical operations on samples of discrete time signals to diminish or enhance certain aspects of the signal. There are two types of digital filters i.e. Finite Impulse Response (FIR) filters and Infinite Impulse Response (IIR) filters depending on the length of the impulse response [1]. FIR filter has the advantage that it is stable and easy to design. FIR filters are also used when time domain features are specified [2]. By designing the filter taps to be symmetrical about the centre tap position we get an FIR filter with linear phase.

Traditionally there are many other methods for FIR filter design, such as window method, frequency sampling method, etc. in windowing method a theoretically infinite impulse response is truncated by a window function. This method is fast, convenient, robust but generally not optimal. There are various types of window functions namely- Butterworth, Chebyshev, Kaiser, Hamming, Hanning, etc available depending on filter specifications to be met like pass band ripples, stop band ripples, stop band attenuation and transition width. The major disadvantage of this method is lack of accurate control of the critical frequencies. Other drawbacks of such methods are increased computational cost and non existence of theoretical proof of convergence to global optimum in sufficient general conditions. Different heuristic optimization methods such as simulated annealing algorithm[3], artificial bee colony algorithm[4], genetic algorithm(GA)[5], etc. have been widely used for the synthesis of design methods capable of satisfying certain constraints which were otherwise unattainable. GA has been quite successful in global optimization methods. Filters designed by GA have the ability of obtaining near global optimum solution [5]. However it is incapable of determining the global optimum in terms of convergence speed and solution quality. The approach designed in this paper takes the advantage of power of the stochastic global optimization technique called particle swarm optimization (PSO). PSO is an evolutionary algorithm [6-7]. Several attempts have been made toward optimization of the FIR filter using the PSO algorithm. The details of PSO algorithms have been discussed in [8]. PSO is simpler to implement and its convergence can be controlled via few parameters. The limitation of the conventional PSO is that it may be influenced by early

convergence and stagnation problem [9-10]. In order to overcome these problems, the PSO algorithm has been modified in this paper and is used for FIR filter design.

In this paper an alternative technique for the FIR low pass digital filter design using craziness based particle swarm optimization (CRPSO) technique has been described. The CRPSO algorithm aims at finding the best coefficients that match the ideal frequency response closely. Based upon the improved PSO approach this paper presents a set of good and comprehensive set of results. Simulation results demonstrate the effectiveness, robustness and better performance of the proposed design method.

This paper is arranged as follows. In the section 2, the FIR low pass(LP) filter design is formulated. Section 3 discusses the algorithm of CRPSO. Section 4 describes the simulation results obtained for FIR LP filter using CRPSO algorithm. Finally, Section 5 concludes the paper.

## II. PROBLEM FORMULATION

FIR Filter is a non-recursive type of digital filter which means that its output does not depends upon one or more of its previous outputs. Due to this FIR filters are more stable than IIR filters. The realization of FIR filter can be stated by the following equation:

$$y(n) = \sum_{k=0}^{M-1} b_k x(n - k) \tag{1}$$

Where  $b_k$  is the coefficient of the filter.  $x(n)$  and  $y(n)$  are the discrete input and output of the filter respectively.  $M$  gives the number of filter coefficients. FIR filter specifications include the maximum tolerable pass band ripple, maximum tolerable stop band ripple, pass band edge frequency and stop band edge frequency. The difference equation can be expanded as:

$$y(n) = b_0x(n) + b_1x(n - 1) + b_2x(n - 2) + \dots + b_{M-1}x(n - M + 1) \tag{2}$$

Transfer function of FIR filter is described as below:

$$H(z) = \sum_{k=0}^{M-1} b_k z^{-k} \tag{3}$$

The unit sample response of the FIR system is identical to the coefficients  $\{b_k\}$ , that is,

$$h(n) = \begin{cases} b_n, & 0 \leq n \leq M - 1 \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

FIR filters have symmetric and anti-symmetric properties, which are related to their  $h(n)$  under symmetric and asymmetric conditions as described below by equations:

$$h(n) = h(N - 1 - n) \text{ for even (symmetric)} \tag{5}$$

$$h(n) = - h(N - 1 - n) \text{ for odd (anti-symmetric)} \tag{6}$$

for such a system the number of multiplications is reduced from  $M$  to  $M/2$  for  $M$  even and to  $(M-1)/2$  for odd. Then for linear phase FIR filter, the following condition should be satisfied:

$$h(n) = \pm h(M - 1 - n) ; n = 0, 1, \dots, M - 1 \tag{7}$$

### A. Errors

The FIR filter is designed by optimizing the coefficients in such a way that the approximation error function in  $L_p$ -norm for magnitude is to be kept minimal. The magnitude response is specified at  $K$  equally spaced discrete frequency points in pass-band and stop- band.

$e_1(x)$  – absolute error  $L_1$  -norm of magnitude response

$e_2(x)$  – squared error  $L_2$ -norm of magnitude response

$$e_1(x) = \sum_{i=0}^k |H_d(\omega_i) - |H(\omega_i, x)|| \tag{8}$$

$$e_2(x) = \sum_{i=0}^k (|H_d(\omega_i) - |H(\omega_i, x)||)^2 \tag{9}$$

The desired magnitude response  $H_d(\omega_i)$  is given as:

$$H_d(\omega_i) = \begin{cases} 1 & ; \text{for } \omega_i \in \text{passband} \\ 0 & ; \text{for } \omega_i \in \text{stopband} \end{cases} \tag{10}$$

The ripple magnitudes of pass band and stop band are given by  $\delta_1(x)$  and  $\delta_2(x)$  respectively as under:

$$\delta_1(x) = \max_{\omega_i} \{|H(\omega_i, x)|\} - \min_{\omega_i} \{|H(\omega_i, x)|\} ; \omega_i \in \text{passband} \tag{11}$$

And

$$\delta_2(x) = \max_{\omega_i} \{|H(\omega_i, x)|\} ; \omega_i \in \text{stopband} \tag{12}$$

All objectives are included and the multi criterion optimization problem is stated as:

$$\text{Minimize } O_1(x) = e_1(x) \tag{13a}$$

$$\text{Minimize } O_2(x) = e_2(x) \tag{13b}$$

$$\text{Minimize } O_3(x) = \delta_p(x) \tag{13c}$$

$$\text{Minimize } O_4(x) = \delta_s(x) \tag{13d}$$

In multiple-criterion constrained optimization problem for the design of digital IIR filter a single optimal tradeoff point can be found by solving following:

$$\text{Minimize } f(x) = \sum_{j=1}^4 w_j f_j(x) \tag{14}$$

## III. CRPSO ALGORITHM

CRPSO is a global search method originated from PSO. This algorithm mimics the particle behavior of swarm in a very closely manner. CRPSO has some special features like sudden change of velocity, a craziness factor and change of

direction of flying towards an apparently non promising area of food depending on the particles mood. In the craziness based particle swarm optimization technique velocity can be expressed as follows:

$$V_i^{(k+1)} = r_2 * \text{sign}(r_3) * V_i^{(k)} + (1 - r_2) * C_1 * r_1 * \{pbest_i^{(k)} - S_i^{(k)}\} + (1 - r_2) * C_2 * (1 - r_1) * \{gbest^{(k)} - S_i^{(k)}\} \quad (15)$$

Where  $r_1, r_2$  and  $r_3$  are the random parameters uniformly taken from the interval  $[0,1]$  and  $\text{sign}(r_3)$  is a function defined as follows:

$$\begin{aligned} \text{sign}(r_3) &= -1 \text{ where } r_3 \leq 0.05 \\ &= 1 \text{ where } r_3 < 0.05 \end{aligned} \quad (16)$$

$\text{rand}_1$  and  $\text{rand}_2$  are two parameters independent parameters that are used in PSO. If both are small then both the social and personal experiences are not used full and convergence speed is reduced. So instead of using independent parameters single parameter is used so  $r_1$  is large and  $1-r_1$  is small and vice-versa. To control the balance between local and global searches  $r_2$  random parameter is induced. In the bird's flocking, a bird changes its direction suddenly. This is defined by a craziness factor and modeled in the technique by using a craziness variable. A craziness operator is induced. Before updating the position of particles the velocity of particle is crazed by

$$V_i^{(k+1)} = V_i^{(k+1)} + P(r_4) * \text{sign}(r_4) * v^{\text{craziness}} \quad (17)$$

$$S_i^{k+1} = S_i^k + v_i^{k+1} \quad (18)$$

Where  $r_4$  is a random parameter that is chosen between the interval  $[0,1]$

The following steps are involved in the CRPSO algorithm:

1. Initialize the population for a swarm of  $n_p$  vectors, in which each vector represents a solution of filter coefficients values.
2. Compute the initial cost value of the total population,  $n_p$ .
3. Compute the population based minimum cost value that is  $gbest$  and  $pbest$ .
4. Update the velocities as per Eq. 17 and checking against the limits of the filter coefficients position updated as per Eq. 18 and compute the updated cost values of particle vectors and population based minimum cost value.
5. Now updating the  $pbest$  vectors and  $gbest$  vectors, replace the updated particle vectors as initial particle vectors for step 4.
6. Continue the iterations from step 4 till the stopping criterion reduced.

#### IV. SIMULATION RESULTS

A design of cascaded digital FIR high pass filter has been shown by evaluating filter coefficients using Craziness based Particle Swarm Optimization (CRPSO) algorithm. In linear phase FIR filter coefficients are symmetrical so only half of the coefficients have been calculated. The order of filter is taken as 20 which results in 21 coefficients. To design digital FIR filter, 200 equally spaced samples are set within the frequency range  $[0, \pi]$ . The range of pass-band and stop-band has been taken as  $0 \leq \omega \leq 0.2\pi$  and  $0.3\pi \leq \omega \leq \pi$  The CRPSO algorithm is run for 100 times and 200 iterations have been taken to obtain best results at different orders. Initially, the population (IPOP) is taken 100,  $C_1$  &  $C_2$  as 1.5, VCRZ as 0.00001 and PCR as 0.3.

##### A. Selection of Order

Order of filter has been varied from 20 to 30 for the CRPSO algorithm and objective function is observed. The Fig. 1 shows objective function variations with respect to filter order.

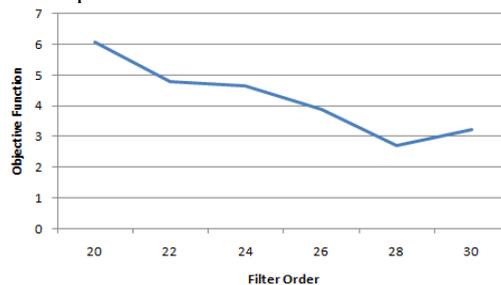


Figure 1: Objective Function v/s Filter Order

Fig. 1 shows that with the increase of filter order objective function decreases continuously and we get the minimum value of objective function at order 28 and then after this objective function starts to increase with increase in filter order. So order 28 has been selected for the design of digital low pass FIR filter. Table 1 shows various parameters corresponding to order 28 FIR Filter.

Table 1 objective function v/s filter order

Sr. No.	Filter Order	Objective Function
1.	20	6.0797
2.	22	4.7894

3.	24	4.6481
4.	26	3.8753
5.	28	2.6915
6.	30	3.2153

**B. Selection of population size**

Population has been varied from 50 to 140 for FIR Filter order 28 using the CRPSO algorithm and objective function is observed. The Fig. 2 shows objective function variations with respect to population.

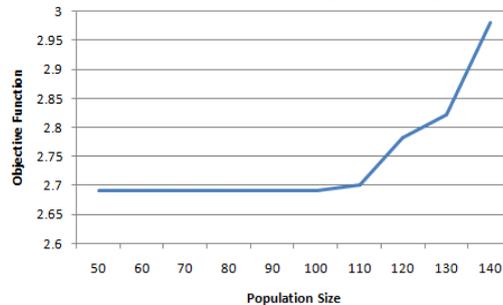


Figure 2: Graph of Objective Function v/s Population Size

Fig. 2 shows that with the increase of population objective function first remains constant and then starts increasing after population is equal to 100. We get the minimum value of objective function at population 100 (considering other parameters). So order 28 with population 100 has been selected for the further design of digital low pass FIR filter. Table 2 shows various parameters corresponding to order 28 FIR Filter with population equal to 100.

Table 2 objective function v/s population size

Sr. No.	Population Size	Objective Function
1.	50	2.6913
2.	60	2.6912
3.	70	2.6913
4.	80	2.6912
5.	90	2.6912
6.	100	2.6912
7.	110	2.7009
8.	120	2.7831
9.	130	2.8224
10.	140	2.9804

**C. Selection of acceleration constants(C<sub>1</sub>&C<sub>2</sub>)**

Acceleration Constants C<sub>1</sub> & C<sub>2</sub> have been taken equal and have been varied from 1.50 to 3.00 for FIR Filter order 28 using the CRPSO algorithm and objective function is observed. The Fig. 3 shows objective function variations with respect to constants.

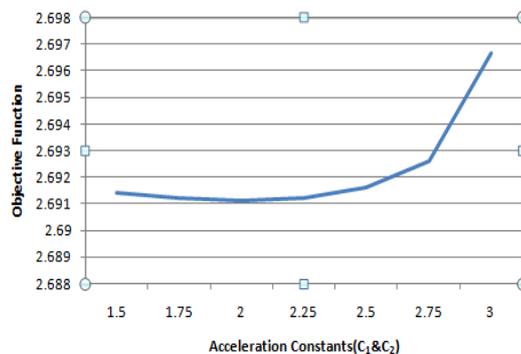


Figure 3: Graph of Objective Function v/s Acceleration Constants (C<sub>1</sub>&C<sub>2</sub>)

Fig. 3 shows that with the increase of the acceleration constants C<sub>1</sub>&C<sub>2</sub> objective function first decreases gradually and then starts increasing after constants are equal to 2. We get the minimum value of objective function at acceleration constants equal to 2. So filter order 28 with C<sub>1</sub>&C<sub>2</sub> equal to 2 has been selected for the further design of digital low pass FIR filter. Table 3 shows various parameters corresponding to order 28 FIR Filter with constants equal to 2.

Table 3 objective function v/s acceleration constants

Sr. No.	Acceleration Constants	Objective Function
1.	1.50	2.6914
2.	1.75	2.6912
3.	2.00	2.6911
4.	2.25	2.6912
5.	2.50	2.6916
6.	2.75	2.6926
7.	3.00	2.6967

**D. Selection of Craziness Factor (VCRZ)**

Craziness Factor (VCRZ) has been varied from 1.50 to 3.00 for FIR Filter order 28 using the CRPSO algorithm and objective function is observed. The Fig. 4 shows objective function variations with respect to VCRZ.

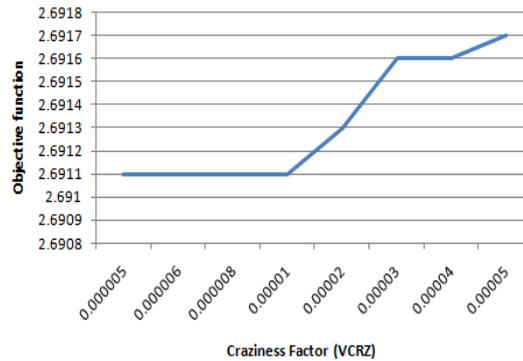


Figure 4: Objective Function v/s Craziness Factor (VCRZ)

Fig. 4 shows that with the increase of the VCRZ objective function first remains constant and then starts increasing after VCRZ is equal to 0.00001. We get the minimum value of objective function at VCRZ equal to 0.000006. So order 28 with VCRZ equal to 0.000006 has been selected for the further design of digital low pass FIR filter. Table 4 shows various parameters corresponding to order 28 FIR Filter with VCRZ equal to 0.000006.

Table 4 objective function v/s craziness factor (VCRZ)

Sr. No.	Craziness Factor (VCRZ)	Objective Function
1.	0.000005	2.6911
2.	0.000006	2.6911
3.	0.000008	2.6911
4.	0.00001	2.6911
5.	0.00002	2.6913
6.	0.00003	2.6916
7.	0.00004	2.6916
8.	0.00005	2.6917

**E. Selection of Predefined Probability of Craziness(PCR)**

PCR has been varied from 0.05 to 0.5 for FIR Filter order 28 using the CRPSO algorithm and objective function is observed. The Fig. 5 shows objective function variations with respect to PCR.

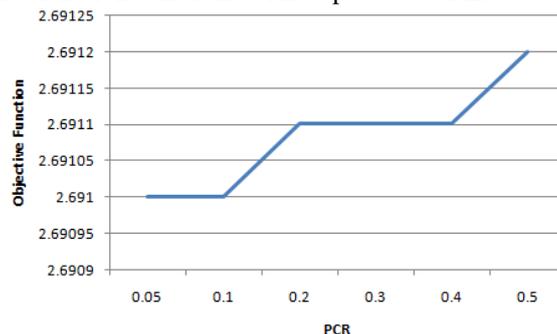


Figure 5: Objective Function v/s PCR

Fig. 5 shows that with the increase of the PCR objective function first remains constant and then starts increasing and again constant and then starts increasing. We get the minimum value of objective function at PCR equal to 0.05. So order 28 with PCR equal to 0.05 has been selected for the further design of digital low pass FIR filter. Table 5 shows various parameters corresponding to order 28 FIR Filter with PCR equal to 0.05.

Table 5 objective function V/S PCR

Sr. No.	PCR	Objective Function
1.	0.05	2.6910
2.	0.1	2.6910
3.	0.2	2.6911
4.	0.3	2.6911
5.	0.4	2.6911
6.	0.5	2.6912

**F. Analysis of Magnitude and Phase Response of Low Pass Digital FIR Filter**

This section shows simulation results performed in MATLAB for design of digital FIR low pass filter. Order of filter is taken as 28 which results in number of coefficients as 29. The calculated coefficients are shown as in Table 6

Table 6 optimized filter coefficients

Sr. No.	h(z)	Coefficients Value
1.	h(0)=h(28)	.006954
2.	h(1)=h(27)	.009994
3.	h(2)=h(26)	.003536
4.	h(3)=h(25)	-.010051
5.	h(4)=h(24)	-.021336
6.	h(5)=h(23)	-.020229
7.	h(6)=h(22)	-.002627
8.	h(7)=h(21)	.025115
9.	h(8)=h(20)	.046273
10.	h(9)=h(19)	.041935
11.	h(10)=h(18)	.000897
12.	h(11)=h(17)	-.071997
13.	h(12)=h(16)	-.156677
14.	h(13)=h(15)	-.224249
15.	h(14)	-.250141

Frequency response of designed filter have been obtained from coefficients and magnitude is noticed across the normalized frequency to analyze the amplification and attenuation values for the different frequency range that is to find pass-band and stop-band range and behavior of filter in these bands. Magnitude response of low pass filter having coefficients as shown in Fig. 6.

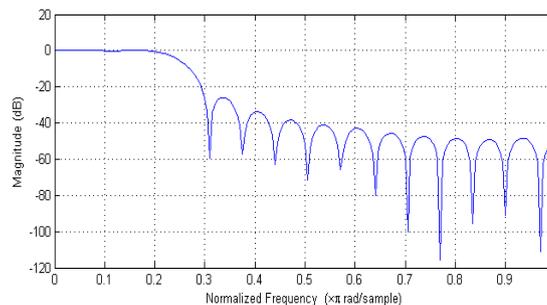


Figure 6: Magnitude vs Normalized Frequency

The Fig.7 shows that magnitude decreases as frequency increases in the digital low pass FIR filter.

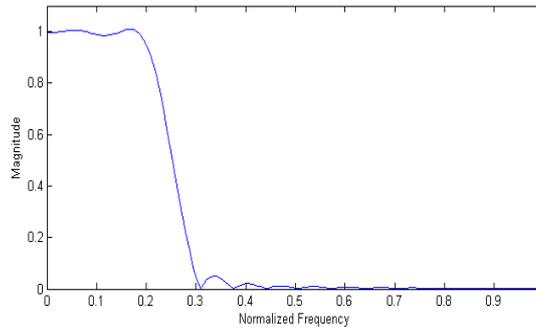


Figure 7: Magnitude response of Low Pass Digital FIR Filter

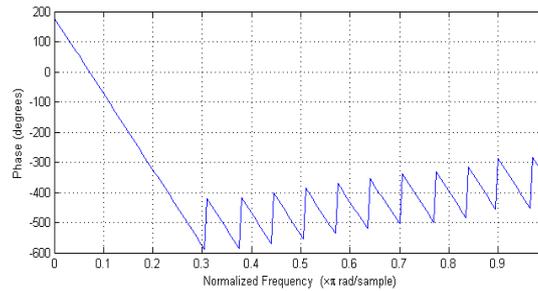


Figure 8: Phase v/s Normalized Frequency

The Fig.8 shows that filter have linear phase response from frequency range 0 to  $0.3\pi$

Table 7 maximum, minimum, average objective function & standard deviation

Sr. No.	Max. Objective Function	Min. Objective Function	Avg. Objective Function	Standard Deviation
1.	2.693146	2.691149	2.691746	0.00039

Table 7 shows that the value of standard deviation is very less than 1 which shows the robustness of the designed filter.

## V. CONCLUSION

This paper presents a novel method for designing a linear phase digital low pass FIR filter using the non linear stochastic global optimization technique based on the CRPSO approach. CRPSO algorithm is very powerful optimization algorithm that exhibits simplicity, robustness using four control parameters. The right choice of parameters is very important in any application. These parameters have great impact on the objective function. Order of filter has been varied from 20 to 30 and filter order 28 gives the best value of objective function. The values of control parameters have been varied to obtain the optimum results for the design of digital low pass FIR filter. The simulation results indicate that the designed filter gives the optimum value of objective function at filter order 28 with population size 100, C1 & C2 value 2.00, VCRZ value 0.000006 and PCR value 0.05. The very small value of standard deviation (0.00039) shows the robustness of the designed filter. Then the magnitude and phase plot have been analyzed. The same algorithm can be applied to design High Pass, Band Pass and Band Stop digital FIR filters.

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