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On the Selection of a Suitable Fuzzy Operator for Information Combination

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Abstract*— *The purpose of aggregation of information is to meaningfully summarize and simplify voluminous data from multiple sources. Some of the familiar examples of data aggregation techniques are arithmetic averages, geometric averages, majority voting. For aggregation of data from multiple sources, special techniques such as combination rules are used. In fuzzy set theory, the operators used are Context Independent Constant Behavior (CICB) and are grouped into three families: Triangular norms (T-norms), Triangular conorms (T-conorms) and mean operators. The aim of this paper is to discuss on the selection of a suitable aggregation operator to combine data from different sources in the interval [0, 1].

Keywords*— *Data Fusion, Aggregation, Fuzzy Operators, Fuzzy Algebraic Sum, Fuzzy Gamma

I. INTRODUCTION

Data fusion is an important technique used to integrate data from different sources and to make inferences that may be optimal for a given application. Data from multiple sources, when aggregated using an appropriate operator, the result may provide additional information with improved accuracy. Application of data fusion techniques range from remote sensing multi sensor data fusion, multi image classification fusion, medical imaging, robotics, airborne surveillance etc. Generally, data from one source may be of limited accuracy with limited information. The major problem here lies in the approach to combine correctly the information from different sources, which may be imprecise, incomplete and uncertain. A huge number of combination rules and operators developed under different theoretical frame works are in use. These include probabilistic and Bayesian fusion, fuzzy sets and possibility theory, Dempster - Shafer evidence theory [3], ordered weighted average operators [12], endorsement theory [7] and neural networks. A detailed review of combination operators can be found in [2]. The combination rules under different theoretical frame work are appropriate only for certain specific conditions or applications and there is no single, universal and unique rule for data fusion.

For fuzzy sets and possibility theory, three families of operators viz. Triangular norms (T-norms), Triangular conorms (T-conorms) and mean operators are generally used [2]. These operators are called Context Independent Constant Behavior (CICB) operators. CICB operators are those which have the same behavior irrespective of the values of the information to combine and does not use any external or contextual information. The utility of fuzzy operators in integrating different information which are fuzzy in nature has been studied in detail by [11],[1],[10]. Reference[5] used fuzzy operators for linear feature detection in multi temporal SAR images. Reference [6] effectively applied fuzzy operators to integrate various spatial data for landslide hazard mapping. Reference [9] evaluated the performance of fuzzy operators for integrating fuzzy membership values associated with multiple spectral bands for mapping complex urban land cover.

In this paper, five fuzzy operators for information combination have been discussed with the help of hypothetical examples. The performance of these operators have been also been discussed based on the results from hypothetical cases.

II. FUZZY OPERATORS FOR INTEGRATION OF FUZZY MEMBERSHIP VALUES

Fuzzy set theory provides a set of operators to combine fuzzy membership values in the range [0, 1]. Amongst various fuzzy operators, the fuzzy AND, fuzzy OR, fuzzy algebraic product, fuzzy algebraic sum and the fuzzy gamma operators [1], [4] are frequently used to integrate fuzzy membership values.

A. Fuzzy AND Operator.

Fuzzy AND operator combines fuzzy membership values (denoted by μ_i for $i = 1, 2, 3,\ldots .m$) using a fuzzy minimum operator to yield an integrated value denoted by $\mu_{combination}$. It may be defined as,

$$\mu_{combination} = MIN\ (\mu_1, \mu_2, \mu_3, \ldots\ldots\mu_m) \qquad (1)$$

If two or more fuzzy membership values are combined using this operator, the resultant value will be the minimum of the input values, which is a conservative estimate and thus produces small values. Thus, fuzzy AND operator utilises only one of the input values i.e., the minimum value and hence may be biased towards the lowest value of input components.

B. Fuzzy OR Operator.

Fuzzy OR operator combines fuzzy membership values using a fuzzy maximum operator and the resulting value is controlled by the maximum value in the input. It may be defined as,

$$\mu_{combination} = MAX\ (\ \mu_1,\ \mu_2,\ \mu_3,\ \ldots\ldots\mu_m\ ) \quad (2)$$

The fuzzy OR operator also utilises only one of the input values i.e., the maximum value and hence may be biased towards the maximum input value.

C. Fuzzy Algebraic Product Operator.

The fuzzy algebraic product combines fuzzy memberships as,

$$\mu_{combination} = \prod_{i=1}^{m} \mu_i \quad (3)$$

The fuzzy algebraic product is obtained by multiplying the fuzzy membership values. If there are two or more values to be combined, the result of this operator is the product of the input values. The use of this operator yields very small values, due to the effect of multiplying fuzzy membership values in the range [0, 1]. The result will always be lesser than or equal to the smallest value in the input and is therefore, decreasive in nature. Unlike fuzzy AND and fuzzy OR operators, fuzzy algebraic product utilises every input membership value to produce the result.

D. Fuzzy Algebraic Sum Operator.

The fuzzy algebraic sum operator is a modified fuzzy union operator expressed as a probabilistic sum [13] and may be defined as,

$$\mu_{combination} = 1 - \prod_{i=1}^{m} (1 - \mu_i) \quad (4)$$

This operator yields a value which is always greater than or equal to the highest value in the input and, therefore, the result is increasive in nature.

E. Fuzzy GAMMA Operator.

The fuzzy gamma operator is defined in terms of fuzzy algebraic product and the fuzzy algebraic sum as,

$$\mu_{combination} = (\ fuzzy\ algebraic\ sum\ )^{\gamma} \times (\ fuzzy\ algebraic\ product\ )^{(1-\gamma)} \quad (5)$$

The value of γ in Eq. (5) can be in the range [0, 1]. For γ =0, the combination yields fuzzy algebraic product operator and for γ =1, it leads to fuzzy algebraic sum operator. Thus, the outputs from fuzzy gamma operator depend on the value of γ chosen. Therefore, a judicious choice of γ produces values that ensure a flexible compromise between increasive tendency of fuzzy algebraic sum operator and the decreasive nature of fuzzy algebraic product operator.

III. SELECTION OF A SUITABLE FUZZY OPERATOR

Several criteria have been defined to select appropriate fuzzy operator [13] for a specific model or situation.

i. *Axiomatic Strength*: An operator must satisfy certain axioms related to properties of associativity, commutativity, symmetry, non-decreasiveness and continuousness.
ii. *Empirical Fit*: An operator should be representative model of real system behaviour and this can be proven only by empirical testing.
iii. *Numerical Efficiency*: When large problems have to be solved, the computational effort of an operator becomes important. For example, when compared to fuzzy AND operator, fuzzy gamma operator is computationally intensive.
iv. *Compensation*: In the context of aggregation operators for two fuzzy sets A and B, an operator is compensatory if the aggregated value is obtainable for a different value of membership grade in set A by a change in corresponding element in set B. Thus fuzzy AND operator is not compensatory while the product and gamma operators are compensatory.
v. *Range of Compensation*: In general, the larger the range of compensation, the better is the compensatory operator. For example, the product operator allows the compensation in the open interval [0, 1].
vi. *Aggregating Behaviour*: The aggregated value depends on the number of individual values combined. For example, while using fuzzy algebraic product operator, each additional input value decreases the aggregated value, which may not be desirable in certain situations.
vii. *Required Scale Level of Membership Functions*: Different operators may require information at different data types (or scale levels) such as nominal, interval, ratio or absolute. In general, the operator that requires the lowest scale level is the most preferable.

Reference [8] provided guidelines in selecting an operator. Accordingly, a combined operator *C* is a bag mapping (i.e., a mapping symmetric with respect to its arguments) operating on two variables A and B (with A ≥ B), which assigns a real number on [0, 1] under the following four conditions (Eq. 6),

$$(a)\quad A \geq B \Rightarrow C(A) \geq C(B)$$

$$(b)\quad C(A \oplus B) \geq C(A) \qquad (6)$$

$$(c)\quad C(A \oplus <0> = C(A)$$

$$(d)\quad C(\Phi) = 0$$

Although, these guidelines help in the selection of an appropriate fuzzy operator, an operator may not adhere to all the above guidelines in most situations. For example, fuzzy AND operator does not satisfy the condition stated in Eq. 6(b), since it is biased towards the minimum of the input values. Fuzzy OR operator partially satisfies the condition stated in Eq. 6(b), i.e., the resulting value can not be greater than the highest input value. However, fuzzy algebraic product does not satisfy the condition stated in Eq. 6(b) as the resulting value will always be lesser than or equal to the smallest value in the input. Fuzzy algebraic sum and fuzzy GAMMA operators appears to satisfy the conditions given by Eq. (6).

IV. HYPOTHETICAL EXAMPLES AND DISCUSSION

In this section, hypothetical examples are used to aggregate fuzzy membership values in the range [0, 1]. The combined values for each example case have been computed using various fuzzy operators (Equations 1 through 5). To implement fuzzy gamma operator, four values of γ have been selected arbitrarily. The combined values after implementation of various fuzzy operators are given in Tables I and II.

Table I Hypothetical cases to combine data from different sources and orresponding combined values from different fuzzy operators

Cases	**Data values from**			**Combined Values from**			
	Source 1	**Source 2**	**Source 3**	**Fuzzy AND**	**Fuzzy OR**	**Fuzzy Algebraic Product**	**Fuzzy Algebraic Sum**
1	0.00	0.00	0.00	0.00	0.0	0.00	0.00
2	0.21	0.43	0.30	0.21	0.43	0.03	0.68
3	0.37	0.60	0.50	0.37	0.60	0.11	0.87
4	0.50	1.00	0.50	0.50	1.0	0.25	1.00
5	0.66	1.00	0.66	0.66	1.0	0.43	1.00
6	0.69	1.00	0.70	0.69	1.0	0.48	1.00
7	0.75	1.00	0.75	0.75	1.0	0.56	1.00
8	1.00	0.00	1.00	0.00	1.0	0.00	1.00

Table II Hypothetical Cases To Combine Data From Different Sources And Orresponding Values From Fuzzy Gamma Operator

Cases	**Data values from**			**Combined Values from fuzzy GAMMA operator**			
	Source 1	**Source 2**	**Source 3**	**γ=0.25**	**γ=0.75**	**γ=0.9**	**γ=0.95**
1	0.00	0.00	0.00	0.00	0.0	0.00	0.00
2	0.21	0.43	0.30	0.06	0.31	0.49	0.58
3	0.37	0.60	0.50	0.18	0.52	0.71	0.79
4	0.50	1.00	0.50	0.35	0.71	0.87	0.93
5	0.66	1.00	0.66	0.53	0.81	0.92	0.96
6	0.69	1.00	0.70	0.57	0.83	0.93	0.97
7	0.75	1.00	0.75	0.64	0.86	0.94	0.98
8	1.00	0.00	1.00	0.00	0.00	0.00	0.00

It can be seen from Table I that fuzzy AND operator yields a value of zero for case 1, when all the individual components have a zero value. However, in hypothetical case 8, fuzzy AND operator yields a value of zero, even though the individual components from sources 1 and 3 have a value equal to one. This outcome for case 8 is may not be suitable in some applications. In hypothetical case 2, the highest input value is 0.43 and the combined value derived from fuzzy AND operator is equal to the minimum of input values, i.e., 0.21. Thus, the result from using fuzzy AND operator for case 2 does not satisfy Eq.6 (b). Similarly, for cases 3 through 7, fuzzy AND operator does not satisfy Eq. 6 (b).

The implementation of fuzzy OR operator for the hypothetical example cases 1 and 8 given in Table I indicates that the operator does not completely satisfy the condition given in Eq. 6(b), i.e., the combined value is greater than or equal to the highest input value. This can be observed in cases 2 and 3, where the input values are (0.21, 0.43, 0.30) and (0.37, 0.60, 0.50) respectively. The combined values from fuzzy OR operator for cases 2 and 3 are equal to 0.43 and 0.6 respectively and can never exceed the corresponding highest input values i.e., 0.43 or 0.6. Thus, fuzzy OR operator partially satisfies Eq. 6 (b).

The results from using fuzzy algebraic product operator for the cases 2 through 7, reflect the decreasive nature of this operator, where the combined values are less than the minimum input value. In hypothetical case 8, two of the inputs have the value equal to 1, while the other has a value equal to zero. The combined value derived from fuzzy algebraic product operator is equal to zero. This outcome from fuzzy algebraic product for case 8 is contradictory and may not be suitable for some applications.

In Table II, the results after using fuzzy GAMMA operator for the example cases 1 through 8 are given. Since, this operator is compensatory i.e., it provides compensation between intersection and union as expressed by the parameter γ, different values of γ have been selected. The output from fuzzy GAMMA operator is a compromise between the values obtained from fuzzy algebraic product and fuzzy algebraic sum operators. It is observed that, for lower values of γ such as 0.25, the results are close to those obtained from fuzzy algebraic product and for higher values of γ such as 0.9 and 0.95, the results are close to those obtained from fuzzy algebraic sum operator. However, for case 8, the fuzzy GAMMA operator returns zero value, which is again become a result that can be argued in some applications. For cases 4, 5, 6 and 7, fuzzy GAMMA operator has yielded values less than the highest input value, and hence does not fulfil the condition given in Eq.6 (b).

The fuzzy algebraic sum operator is the one which satisfies the required conditions given in Eq. (6). This can be observed from Table I, that for hypothetical cases 1 and 8, fuzzy algebraic sum operator has yielded values of zero and 1, For cases 2 and 3, this operator has yielded values of 0.68 and 0.87, which are higher than the highest input values, thus satisfying the condition given in Eq. 6 (b). For cases 3 through 7, the fuzzy algebraic sum returns satisfactory results.

V. CONCLUSION

The purpose of aggregation of information is to meaningfully summarize and simplify voluminous data from multiple sources. For aggregation of data from multiple sources, special techniques such as combination rules are used. In fuzzy set theory, there are several operators used for the purpose. In this paper, fuzzy AND, fuzzy OR, fuzzy algebraic product, fuzzy algebraic sum and fuzzy GAMMA operators have been discussed with the help of hypothetical examples.

The performance of these operators have been discussed based on the results from hypothetical cases. Results from examples indicate that the selection of a suitable fuzzy operator depends on requirement of the output from data fusion in a specific application. Nevertheless, the basic properties such as commutativity and associativity along with the criteria stated by Zimmermann(1991) and the conditions defined by Herencia and Lamata (1997) may be considered while selecting a fuzzy operator for data fusion.

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