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High Pass Digital FIR Filter Design Using Differential Evolution

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Abstract-- Digital Filters are generally used in the present era of communication and computation. Good performance of Digital filters is demanding one & hence it's a challenge to design a digital finite impulse response (FIR) filter satisfying all the necessary and required conditions. Various artificially intelligent optimization techniques evolve for optimized design of digital filter. This paper demonstrates the optimal design of a linear phase digital high pass FIR filter using most suitable mutation strategy of differential evolution (DE) method. DE is a stochastic, population based evolutionary search algorithm used to determine the frequency response of digital FIR filters. Simplicity, fast convergence speed and robustness of algorithm strengthen it and optimal filter coefficients are obtained by its capability of exploring and exploiting the search space locally as well as globally. Multiparameter optimization is employed as the design criterion to obtain the optimal digital FIR filter that minimize the $L_p$norm-approximation error and ripple magnitudes of both pass band and stop band. Simulation results for the employed DE method for digital high pass FIR filter authenticates that results are comparable to other evolutionary algorithms and can be applied for higher order filter design.

Keywords-- Digital FIR filter, Differential Evolution algorithm, Magnitude response, Pass band ripples and Stop band ripples.

I. INTRODUCTION

A filter extracts useful information by suppressing the noise and interfered signals from the desired information bearing signal. There are two categories of filters as: analog filter and digital filter. Analog filters works on analog electronic circuits having components such as resistor, capacitor and op-amp etc. to perform the desired filtering operation. Digital filter uses the digital processor to perform the numerous mathematical calculations on the sampled values of signal to perform the filtering operation. Digital processor may be the generalized computer or a programmed processor chip. They have number of advantages over analog filters like more flexibility, better performance and time response, environment stability and less production cost of equipment's etc. So they are the essential constructional lumps of digital signal processing schemes having applications in communication and control systems, aerospace, medical and defense equipment's and image dispensation etc. Digital filters can be classified as infinite impulse response (IIR) and finite impulse response (FIR) filters based on the duration of impulse response. FIR or non-recursive filters are those whose impulse response is of finite duration because it settles down to zero in finite duration of time. IIR or recursive filters are those whose impulse response continues forever in time. FIR filters are more stable than IIR filters because their present input does not depend upon previous output values and the entire poles lie at the origin so they are within the unit circle whereas IIR filters consists of both poles and zeros. Due to simplicity, guaranteed stability, linear phase response at all frequencies and low coefficient sensitivity FIR filters are preferable over IIR filters in most of the cases [1,2].

There are many traditional techniques used for the design of digital FIR filters, like window based methods, frequency sampling method and least mean square error etc. [3]. Of these, window method is most popular. There are variety of windows (Blackman, Hamming, Rectangular, Kaiser etc.), which limits the infinite impulse response of ideal filter into finite window to design actual response. But various window methods do not allow sufficient and precise control of frequency response in different frequency bands and transition width etc.[4, 5]. Parks and McClellan developed chebyshev approximation method, that is better than other traditional techniques, but it too has limitation of computational complexity and high pass band ripples [6]. An iterative computer program was also developed for the design of FIR filter [7]. Due to the non-linear, multimodal nature of error surface conventional gradient based methods get easily stuck in the local minima. These methods cannot optimize and converge the global minimum solution [8]. To overcome these shortcomings, nature based heuristic and stochastic optimization methods have been implemented for the optimal design of digital filters with better parameter control and highest stop band attenuation such as genetic algorithms [9,10,11], simulated annealing [12], particle swarm optimization [15], differential evolution [13], artificial bee colony [14], tabu search [18], ant colony optimization [16], immune algorithm [17] etc.

The idea of Genetic algorithm was pioneered by Holland [19].GA operates with chromosomes (population) which are randomly initialized. To improve quality of solution over successive iterations the realization of biological crossovers and mutations of chromosomes was demonstrated in the algorithm [20]. In crossover, two parent chromosomes are combined to form new offspring chromosome. In mutation, random changes are introduced into characteristics of

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chromosome. GA is capable of searching multidimensional and multimodal spaces and has ability to optimize complex and discontinuous functions that are difficult to analyze mathematically [21]. GA gives better results than window method and Parks and McClellan optimization technique [22]. But when there are number of parameters and numerous local optima of the high dimensional optimization problems then it may get trapped in the local optima of the objective function. Sometimes it poses very slow convergence [23]. So further development was worthy in evolutionary algorithm for designing optimal digital FIR filters.

Eberhart and Kennedy [24] gave an alternative solution to complex non-linear optimization problem inspired by animal's social behavior such as fish schooling, bird's flocking etc. known as particle swarm optimization (PSO). It is simple, easy to implement, better parameter control, fast computational capability and has robust search ability. Instead of many advantages it has some shortcomings also because its convergence behavior depends upon the parameters. Divergent particle trajectories may occur if in any case the parameters of PSO have been chosen incorrectly, which results trapping of local minimum value [27]. The problem of premature convergence may occur when PSO is applied on high-dimensional optimization functions. Due to this low optimization precision or failure takes place. Researchers have made many efforts to improve performance of PSO.

Price and Storn [25] presented meta-heuristic approach named as differential evolution for minimizing non-linear, non-differentiable and non-convex continuous space function. Classical operators of crossover and mutation have replaced by the suitable differential operators to handle the problem. Many engineering application have benefited from powerful nature of DE [26]. DE has number of advantages such as fast convergence, simplicity, parallel processing nature, few control parameters, ability to find true global minima regardless of initial parameter values and provides multiple solutions in a single run. DE gives the optimal solution for non-linear constrained problem with penalty functions [27-33].

The objective of this paper is to purpose DE for the optimal design of linear phase digital FIR filter. It randomly explores and exploits the search space locally as well as globally to determine the frequency response. Optimized Filter coefficients are obtained, that closely match the ideal frequency response, with differential evolution.L_p-norm approximation errors and magnitudes of both pass band and stop band have calculated as objective function for optimization problem. Simulation result illustrates the effectiveness and better performance of the proposed algorithm.

The rest Paper is organized in four sections. In section II the design problem of digital FIR is formulated. Section III briefly discusses the details regarding DE algorithm for optimal design of digital FIR high pass filter. The results and performance of DE method has been discussed in section IV. Finally section V concludes the paper.

II. DESIGN FORMULATION OF DIGITAL FIR FILTER

Digital FIR filter is a non-recursive type of filter having exactly linear phase frequency response. The response of filter does not depend upon the previous output values, so they are very stable. The difference equation for digital FIR filter is stated as in following equation:

$$y(n) = a_0 x(n) + a_1 x(n-1) + \ldots + a_n x(n-M) \quad (1)$$

$$y(n) = \sum_{n=0}^{M} a_n x(n-M) \quad (2)$$

Where $y(n)$ and $x(n)$ gives the digital output and input of FIR filter respectively. a_n represents the coefficients of the filter. The digital output sequence can also be expressed as convolution between the unit impulse response $h(n)$ and input sequence as shown in the following equation:

$$y(n) = \sum_{n=0}^{M} h(n) x(n-M) \quad (3)$$

Transfer function of the digital FIR filter is described as:

$$H(z) = \sum_{n=0}^{M} h(n) z^{-n} \quad (4)$$

Where M is the order of filter which gives number of impulse response coefficients as $(M+1)$. $h(n)$ represents the impulse response of digital filter which is to be determined in the design process. $h(n)$ determines the filter type (i.e. high pass).

FIR filters exhibits the symmetry and anti-symmetry properties corresponding to their $h(n)$ as described in the equations given below:

$$h(n) = h(M-n) \text{ for even} \quad (5)$$

$$h(n) = -h(M-n) \text{ for odd} \quad (6)$$

Hence FIR filter has linear phase only if it would satisfy the following equation:

$$h(n) = \pm h(M-n)\,;\; n = 0,1 \ldots\ldots\ldots M \quad (7)$$

This paper shows the design of digital high pass FIR filter with even order. So linear phase FIR filter is symmetrical due to this its coefficients are also symmetrical. Only half of the coefficients $\left(\frac{M}{2}+1\right)$ have been updated by the DE

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algorithm hence dimension of problem is halved. Due to symmetric property of digital FIR filter other half of coefficients are also obtained by concatenation.

The frequency response of the digital FIR filter can be represented as:

$$H(e^{j\omega_k}) = \sum_{n=0}^{M} h(n)e^{-j\omega_k n} \tag{8}$$

Where $H(e^{j\omega_k})$ is the Fourier transform vector and $\omega_k = \frac{2\pi k}{N}$; $k = 0,1,2,\ldots.N$

Where $N = 200$ samples

For high pass filter

$$H_d(\omega_k) = \begin{cases} 0 \; if \;\; 0 \le \omega_k \le 0.7\pi \quad (stopband)(k = 0-140) \\ 1 \; if \;\; 0.8\pi \le \omega_k \le \pi \; (passband)(k = 160-200) \end{cases} \tag{9}$$

The equally spaced samples have been set within the frequency range $[0, \pi]$. Hence the position of each particle vector in D-dimensional search space corresponds to the coefficients same as given by the transfer function in Eq. 4.

The error function is calculated from the difference between the desired frequency response and actual frequency response of designed digital FIR filter. In this paper DE algorithm is applied so as to obtain the actual filter response as close as possible to the ideal filter response. $Er_1(x)$ and $Er_2(x)$ are the absolute error L_1-norm of magnitude and the squared error L_2-norm of magnitude response of digital FIR filter respectively that are defined as:

$$Er_1(x) = \sum_{k=0}^{N} \left\{ H_d(\omega_k) - \left| H_i(\omega_k, x) \right| \right\} \tag{10}$$

$$Er_2(x) = \sum_{k=0}^{N} \left\{ \left| H_d(\omega_k) - \left| H_i(\omega_k, x) \right| \right|^2 \right\}^{1/2} \tag{11}$$

$\delta_p(x)$ and $\delta_s(x)$ are the ripple magnitudes of both pass band and stop band respectively. Ripple magnitudes are calculated as:

$$\delta_p(x) = \sum_{k=0}^{N} \max\left\{ \left| H_i(\omega_k, x) \right| \right\} - \min\left\{ \left| H_i(\omega_k, x) \right| \right\} \; for \;\; \omega_k \in passband \tag{12}$$

$$\delta_s(x) = \sum_{k=0}^{N} \max\left\{ \left| H_i(\omega_k, x) \right| \right\} \; for \;\; \omega_k \in stopband \tag{13}$$

After the aggregation of all objectives for the digital filter design, the multiple-criterion constrained optimization technique is given as:

$$Minimize \; F_1(x) = Er_1(x) \tag{14}$$

$$Minimize \; F_2(x) = Er_2(x) \tag{15}$$

$$Minimize \; F_3(x) = \delta_p(x) \tag{16}$$

$$Minimize \; F_4(x) = \delta_s(x) \tag{17}$$

In the multiple-criterion constrained optimization technique for the design of digital FIR filter, a single optimal non inferior point can be found by solving the following equation:

$$Minimize \; F(x) = \sum_{j=1}^{4} w_j F_j \tag{18}$$

Hence the overall objective function as given in Eq. 18 has been minimized for the optimal design of digital FIR high pass filter using DE algorithm.

III. DIFFERENTIAL EVOLUTION: THEORETICAL BACKGROUND AND FORMULATION IN CONTINUOUS REAL SPACE

To deal with search and optimization problems of non-linear and non-differentiable functions innovation of an optimal design technique was necessary. Optimization means to find the global optimal solution of any problem while incorporating its objectives and constraints. Storn and Price in 1995 introduced the encoded floating point evolutionary search technique i.e. differential evolution (DE), for global optimization. DE is a population based stochastic and powerful optimization algorithm which solve real parameter and real valued problems. In DE algorithm simple arithmetical operators have been combined with the classical operators of mutation, recombination and selection so as to evolve from a randomly generated initial population to optimal final solution. Performance of DE algorithm greatly depends upon its control parameters. It incorporates very few control parameters as population size (NP), mutation scale factor (f_M) and crossover rate (CR), with parameter tuning. DE is a simple real parameter algorithm which has simple cycle of stages as shown in Fig.1

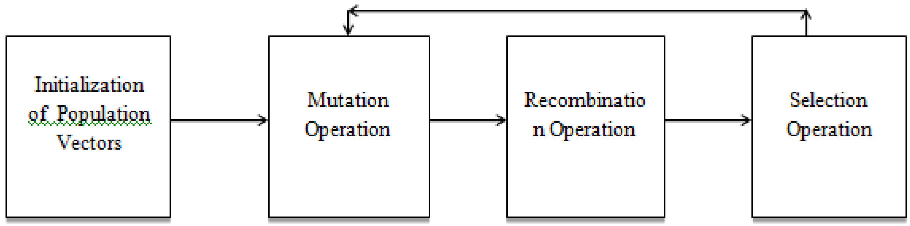

Figure 1: Cycle of Main Stages of DE Algorithm

The DE algorithm is described more briefly in the following sub-sections:

A. Parameter Setup

The key parameters have been selected to control the DE algorithm i.e. population size (N_P), boundary constraints of optimization variables (D), mutation factor (f_M), crossover rate (CR) and stopping criterion of maximum number of iterations (generations) G and maximum number of runs. $\vec{X}_{ij}(G)$ is the j^{th} element of D set of filter coefficients giving i^{th} individual of population. The set of real digital FIR filter coefficient of all generations is represented as the population. For a system with D filter coefficients, the population is represented as a vector of length D. If there are N_P members in the population, the complete population can be represented as a matrix.

B. Initialization

Set iteration (generation) equal to one. The evolutionary algorithm searches for a global optimum solution (point) in a D- dimensional search space. So, population is initiated randomly with suitable range of N_P D-dimensional real valued parameter vectors. Each vector can also be named as genome/chromosome. These vectors form a candidate solution to the multi-dimensional optimization problem. The generations of DE can be denoted as $G = 1,2,\ldots,G_{Max}$. Over the successive generations parameter vectors also gets changed. So, i^{th} vector of population can be represented as: $\vec{X}_i(G) = \left(x_{i,1}(G), x_{i,2}(G),\ldots, x_{i,D}(G)\right)$. There should be a certain range within which the values of parameters lie. Because each parameter represents physical parameters or measures that have certain natural bounds (for example if any parameter represents length or mass then it can never be negative). Thus the initial population (at $G = 1$) should cover the desired range as much as possible with uniform probability distribution within the search space that is constrained by the minimum and maximum limits: $X^{Min} = \left\{x_1^{Min}, x_2^{Min},\ldots, x_D^{Min},\right\}$and $X^{Max} = \left\{x_1^{Max}, x_2^{Max},\ldots x_D^{Max},\right\}$

Hence j^{th} component of i^{th} vector can be initialized as:

$$x_{i,j}(1) = x_j^{Min} + rand[0,1](x_j^{Max} - x_j^{Min}) \;\; (i = 1,2,\ldots,N_P;\; j = 1,2,\ldots,D) \tag{19}$$

Where $rand[0,1]$ is uniformly distributed random number between 0 and 1.

C. Mutation Operation with Difference Vectors

Biological meaning of "mutation" is the sudden change in characteristics of a gene that is related to chromosome. Mutation operation actually explores the search space to find the global optimal solution for any optimization problem. For expansion of search space various parameter vectors of existing population are combined to create a new population vector.

In DE literature, a parent vector that is taken from the current generation is known as target vector. A mutant vector $\vec{V}_i(G+1) = [v_{i,1}(G+1), v_{i,1}(G+1),\ldots v_{i,1}(G+1)]$ that is obtained from the differential mutation operation is called as donor vector. Finally an offspring is produced by recombination of donor vector with target vector. There exist various mutation strategies for DE algorithm that are described as:

$$\vec{V}_i(G+1)_{Best/1} = \vec{X}_{Best}(G) + f_M\left(\vec{X}_{Best}(G) - \vec{X}_i(G)\right) \tag{20}$$

$$\vec{V}_i(G+1)_{Best/2} = \vec{X}_{Best}(G) + f_M\left(\vec{X}_{Best}(G) - \vec{X}_i(G) - \vec{X}_{r1}(G) - \vec{X}_{r2}(G)\right) \tag{21}$$

$$\vec{V}_i(G+1)_{Best/3} = \vec{X}_{Best}(G) + f_B\left(\vec{X}_{Best}(G) - \vec{X}_i(G)\right) + f_M\left(\vec{X}_{r1}(G) - \vec{X}_{r2}(G)\right) \tag{22}$$

$$\vec{V}_i(G+1)_{Best/4} = \vec{X}_{Best}(G) + f_M\left(\vec{X}_{Best}(G) + \vec{X}_i(G) - \vec{X}_{r1}(G) - \vec{X}_{r2}(G)\right) \tag{23}$$

$$\vec{V}_i(G+1)_{Best/5} = \vec{X}_{Best}(G) + f_M\left(\vec{X}_{r1}(G) + \vec{X}_{r2}(G) - \vec{X}_{r3}(G) - \vec{X}_{r4}(G)\right) \tag{24}$$

Where f_M is the mutation factor, generally lies in the range [0.4, 1], used to control amplification of differential evolution. f_B is the another control parameter apart from the mutation factor. $r1, r2, r3, r4 \in N_P$ which are randomly

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chosen mutually exclusive integers and also different from base vector index $i = 1,2,\ldots,N_P$ $(i.e. i \neq r1 \neq r2 \neq r3 \neq r4)$. N_P is the set of population vectors of evolutionary algorithm.

D. Recombination/Crossover Operation

Once donor vector is generated in the mutation phase, then crossover (recombination) plays an essential role to enhance the potential diversity of population member. In DE literature there exit two kinds of crossover methods namely exponential (or two point modulo) crossover and binomial (or uniform) crossover, representing scheme of crossover where mutant vector exchanges its components. In exponential crossover a number is chosen randomly from the interval [1,D]. This random number indicates the particular component (gene) of target vector from where the exchange of genes with mutant vector will start in actual. In binomial crossover, the integer is chosen from the interval [1,D], signifies the number of components of mutant vector that will actually contribute to the target vector. This integer is a uniformly generated random number between 0 and 1 and less than or equal to the CR value. So the parameter crossover rate (CR) plays a significant role along with the mutation factor (f_M).

The donor vector and the target vector subjected to binomial recombination operation and trail vector $\vec{U}_i(G+1) = \left(u_{i,1}(G+1), u_{i,2}(G+1), \ldots u_{i,D}(G+1)\right)$ is generated as follows:

$$u_{i,j}(G+1) = \begin{cases} v_{i,j}(G+1) & if \quad rand_{i,j}[0,1] \leq CR \; or \quad j = j_{rand} \\ x_{i,j}(G+1) & otherwise \qquad j \neq j_{rand} \end{cases} \tag{25}$$

Where $rand_{i,j}[0,1]$ is uniformly distributed independent random number. j_{rand} is a randomly chosen index $\in 1,2,\ldots,D$, which ensures that $u_{i,j}(G+1)$ will get at least one component from $v_{i,j}(G+1)$. An input parameter crossover rate $CR \in [0,1]$, influences number of components to be exchanged by crossover.

E. Selection Operation

To keep population size fixed over subsequent generations, selection process determines whether the target vector or trail vector survives to the next generation. For this the trail vector is compared with the target vector of previous generation as described in the following equation:

$$\vec{X}_i(G+1) = \begin{cases} \vec{U}_i(G+1) & iff \quad F\left(\vec{U}_i(G+1) \leq \vec{X}_i(G)\right) \\ \vec{X}_i(G) & iff \quad F\left(\vec{U}_i(G+1) > \vec{X}_i(G)\right) \end{cases} \tag{26}$$

Where $F(\vec{X})$ is the overall objective function that has to be minimized. If the trail vector exhibits equal or smaller value of objective function then it would replace the target vector in the next generation otherwise target vector is retained in the population. Hence population will never be deteriorated, it will either be equal or gets better in the fitness status (minimization of objective function).

F. Stopping Criterion Verification

It depends upon the type of problem. Whole procedure is repeated until the stopping criterion is met i.e. maximum number of runs.

G. Algorithm

The algorithm of the proposed method has been shown as under.

1. Read data viz. population size, mutation factor, crossover rate and maximum number of iterations, seed number, upper and lower limits of population coefficients.
2. Generate an array of uniform random numbers.
3. Generate initial population individual and compute augmented objective function. Again generate initial population individual using opposition and compute augmented objective function.
4. Arrange calculated objective function in ascending order and select first half of the population members.
5. Set iteration counter.
6. Increment iteration counter.
7. Select best member.
8. Generate an array of uniform random numbers and generate five different integer random numbers then apply mutation operation using different mutation strategies. Compute augmented objective function and find mutant vector based on minimum augmented objective function.
9. Generate arrays of random numbers and apply crossover and selection operations.
10. Is stopping criteria met?
11. No- Go to 6
12. Write GBEST
13. Maximum number of runs done or not, if not go back to step 2
14. Stop

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IV. SIMULATION RESULTS AND DISCUSSIONS

The design of cascaded digital FIR high pass filter has been demonstrated by evaluating filter coefficients using differential evolution (DE) algorithm. The order of filter is taken as 20 which results in number of coefficients as 21. In linear phase FIR filter coefficients are symmetrical so only half of the coefficients have been calculated in the design problem. To design digital FIR filter, 200 equally spaced samples are set within the frequency range $[0,\pi]$. The range of pass-band and stop-band are taken as $0.8\pi \le \omega \le \pi$ and $0 \le \omega \le 0.7\pi$ .The Evolution algorithm is run for 100 times and 200 iterations have been taken to obtain best results at different orders. Initially, the mutation factor (f_M) and crossover (CR) rate has been taken as 0.8 and 0.2 respectively.

A. Selection of Order

Order of filter has been varied from 20 to 52 for the DE algorithm and objective function is observed. The Table 1 shows objective function, L_p – norm magnitude errors, ripple magnitudes variations of both pass band and stop band with respect to filter order.

Table 1 Objective function values at different filter orders

Sr. No.	Filter Order	Objective Function	Magnitude Error 1	Magnitude Error 2	Pass Band Performance	Stop Band Performance
1	20	5.520913	2.670811	.329154	$1.016299 \le \lvert H(e^{j\omega k})\rvert \le .892045$ .124254	$0 \le \lvert H(e^{j\omega k})\rvert \le .125993$ .125993
2	22	4.429688	2.010767	.254507	$1.026706 \le \lvert H(e^{j\omega k})\rvert \le .896879$ .129828	$0 \le \lvert H(e^{j\omega k})\rvert \le .086264$ .086264
3	24	4.177427	1.839925	.264177	$1.030025 \le \lvert H(e^{j\omega k})\rvert \le .881960$ .148065	$0 \le \lvert H(e^{j\omega k})\rvert \le .057178$ .057178
4	26	3.770174	1.926275	.248817	$1.009461 \le \lvert H(e^{j\omega k})\rvert \le .938828$ .070632	$0 \le \lvert H(e^{j\omega k})\rvert \le .088876$ .088876
5	28	2.669447	1.395376	.170265	$1.014769 \le \lvert H(e^{j\omega k})\rvert \le .953747$ .061022	$0 \le \lvert H(e^{j\omega k})\rvert \le .049359$ .049359
6	30	2.140997	1.095225	.131491	$1.020813 \le \lvert H(e^{j\omega k})\rvert \le .962336$ .058477	$0 \le \lvert H(e^{j\omega k})\rvert \le .032948$ .032948
7	32	2.003781	.949002	.130300	$1.023777 \le \lvert H(e^{j\omega k})\rvert \le .957699$ .066078	$0 \le \lvert H(e^{j\omega k})\rvert \le .026298$ .026298
8	34	1.82731	.995875	.125735	$1.008605 \le \lvert H(e^{j\omega k})\rvert \le .978269$ .030336	$0 \le \lvert H(e^{j\omega k})\rvert \le .040234$ .040234
9	36	1.293628	.713539	.086395	$1.010679 \le \lvert H(e^{j\omega k})\rvert \le .988120$ .022559	$0 \le \lvert H(e^{j\omega k})\rvert \le .026810$ .026810
10	38	1.027968	.536560	.065539	$1.011545 \le \lvert H(e^{j\omega k})\rvert \le .986335$ .025210	$0 \le \lvert H(e^{j\omega k})\rvert \le .017377$ .017377
11	40	0.958694	.465468	.064451	$1.013129 \le \lvert H(e^{j\omega k})\rvert \le .984578$ .028551	$0 \le \lvert H(e^{j\omega k})\rvert \le .014326$ .014326
12	42	0.891991	.485153	.060220	$1.006743 \le \lvert H(e^{j\omega k})\rvert \le .991096$ .015647	$0 \le \lvert H(e^{j\omega k})\rvert \le .019015$ .019015
13	44	0.63571	.351838	.042889	$1.004248 \le \lvert H(e^{j\omega k})\rvert \le .993907$ .010341	$0 \le \lvert H(e^{j\omega k})\rvert \le .013757$ .013757

Sr. No.	Filter Order	Objective Function	Magnitude Error 1	Magnitude Error 2	Pass Band Performance	Stop Band Performance
14	46	0.51075	.254324	.031409	$1.004872 \le \lvert H(e^{j\omega k}) \rvert \le .991904$ .012968	$0 \le \lvert H(e^{j\omega k}) \rvert \le .00953$ .009533
15	48	0.634462	.344374	.043442	$1.003103 \le \lvert H(e^{j\omega k}) \rvert \le .995497$ .007606	$0 \le \lvert H(e^{j\omega k}) \rvert \le .01705$ .017059
16	50	0.446177	.232537	.031330	$1.002719 \le \lvert H(e^{j\omega k}) \rvert \le .996137$ .006582	$0 \le \lvert H(e^{j\omega k}) \rvert \le .01164$ .011649
17	52	1.014541	.497534	.070180	$1.010014 \le \lvert H(e^{j\omega k}) \rvert \le .991564$ .018451	$0 \le \lvert H(e^{j\omega k}) \rvert \le .02623$ .026232

Hence it is observed that the filter order 50 gives the minimum value of objective function, L_p-norm magnitude errors and ripple magnitudes of pass band. So filter order 50 has been selected for the design of digital high pass FIR filter. Now these variations are observed in the plot as shown in Fig. 2

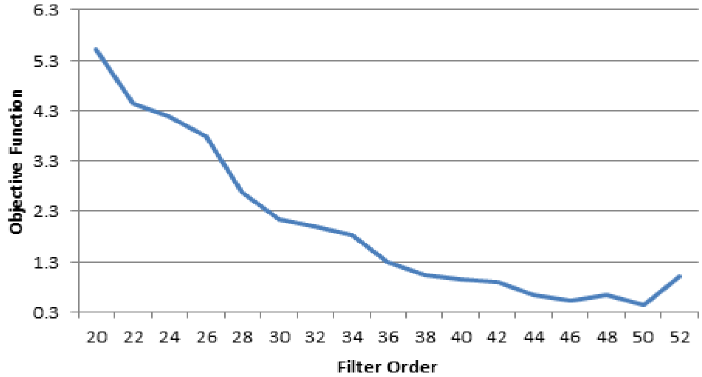

Figure 2: Order vs. Objective Function

Fig. 2 depicts that with the increase of filter order objective function decreases continuously up to filter order 46. From filter order 46 to 48 objective function increases linearly. Then there is linear decrease in value of objective function from filter order 48 to 50. After filter order 50 objective function starts increasing. Filter order 50 gives the best value of objective function. So order 50 has been selected for the design of digital high pass FIR filter.

B. Different Mutation Strategies at Order 50

Different mutation strategies have been applied at filter order 50 for the design of digital high pass FIR filter. These mutation strategies have been described as Eq. 20, Eq. 21, Eq. 22, Eq. 23 and Eq. 24.The results are shown in Table 2.

Table 2 Objective function variations at different mutation strategies for filter order 50

Sr. No.	Mutation Strategy	Objective Function
1	Mutation Strategy 1	0.440201
2	Mutation Strategy 2	0.605793
3	Mutation Strategy 3	0.440244
4	Mutation Strategy 4	0.440997
5	Mutation Strategy 5	0.444375

From the obtained results the mutation strategy-1 gives the minimum value of objective function. Now, Parameters of differential evolution algorithm have been varied, to obtain the best results, for mutation strategy 1 by keeping the order of filter fixed at 50. There are three main control parameters of differential evolution algorithm: population size, mutation factor and crossover rate. Global optimum searching capability and convergence speed are very sensitive to choice of these control parameters. Hence we will focus on the effect of each of these parameters on the performance of DE algorithm.

First of all population has been varied from 20 to 180 and the observed values of objective function at different populations are shown in Table 3

Table 3 Objective function at different population values

Sr. No.	**Population**	**Objective Function**
1	20	0.443612
2	40	0.442108
3	60	0.440638
4	80	0.440201
5	100	0.440201
6	120	0.439943
7	140	0.440131
8	160	0.439877
9	180	0.440300

The Table 3 depicts that the minimum value of objective function has been achieved at population value 160. Fig. 3 shows the graph of objective function variation with increase in population.

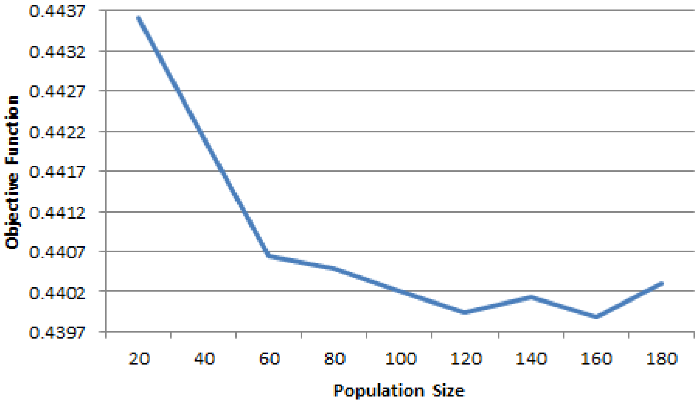

Figure 3: Population vs. Objective Function

The Fig. 3 shows that the objective function value rapidly decreases from population size 20 to 60.Then there is gradual decrease in objective function from population size 60 to 120. After population size 120 objective function increases up to value of population 140 then again there is linear decrease in objective function up to population size 160. Objective function starts increasing after population size 160. It is evident that minimum objective function has been obtained at population 160 for DE algorithm.

Now the impact of mutation factor on the objective function has been studied as shown in Table 4. The effective range of f_m is between 0.4 and 1. So f_M is varied in the steps of 0.1, by keeping population fixed at 160.

Table 4 Objective function variations at different f_M values

Sr. No.	**Mutation factor (f_M)**	**Objective Function**
1	0.4	0.439920
2	0.5	0.468381
3	0.6	0.439920
4	0.7	0.442151
5	0.8	0.439877
6	0.9	0.441059
7	1.0	0.468381

The Table 4 depicts that mutation factor value 0.8 yields the minimum value of objective function. Fig. 4 displays the variations of objective function with respect to the mutation factor.

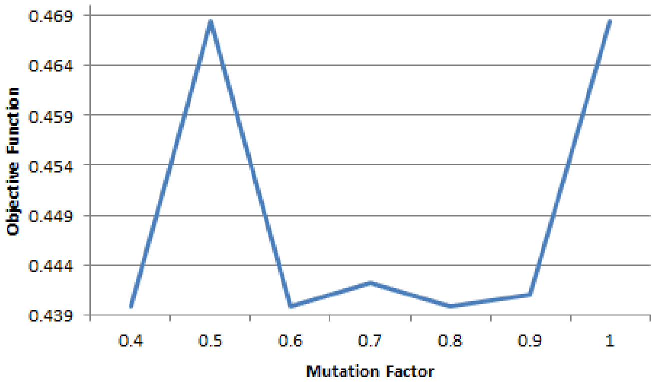

Figure 4: Mutation Factor vs. Objective Function

It is evident from the Fig 4 that there is rapid increase in objective function for f_M value 0.4 to 0.5. For f_m value 0.5 to 0.6 objective function decreases rapidly. Then there are linear variations in objective function with respect to f_m value 0.6 to 0.9. After f_M value 0.9 objective function increases rapidly. Hence DE algorithm at f_M value 0.8 gives the minimum value of objective function.

Now value of crossover rate has been changed from 0.1 to 0.5 with f_M value 0.8 and Population size 160. The observed values of objective function are shown in Table 5.

Table 5 Objective function for different values of crossover rate

Sr. No.	Crossover Rate	Objective Function
1	0.1	0.440325
2	0.2	0.439877
3	0.3	0.439685
4	0.4	0.439319
5	0.5	0.439169
6	0.6	0.439043
7	0.7	0.439188
8	0.8	0.440321

The Table 5 shows that crossover rate 0.6 gives the minimum value of objective function. Now the plot is drawn to show the variations of objective function with respect to the crossover rate

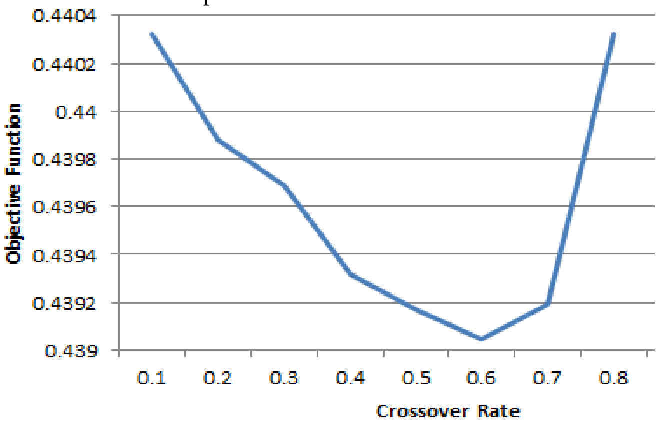

Figure 5: Crossover rate vs. Objective Function

From the graph shown in Fig. 5 it has been observed that objective function have gradually decreasing trend up to crossover rate value 0.6 and after this objective function values start increasing. So, at crossover rate 0.6 differential algorithm gives the best results.

So detailed analysis of above the figures indicates that differential evolution with population size-160, mutation factor-0.8 and crossover rate 0.6exhibits the optimum design of digital FIR high pass filter at filter order 50.

C. Magnitude and Phase Response Analysis of High Pass Digital FIR Filter

This section shows simulation results performed in MATLAB for design of digital FIR high pass filter. Order of filter is taken as 50 which results in number of coefficients as 51. The coefficients obtained from the filter design have been listed as in Table 7

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Table 7 Optimized coefficients of the digital high pass fir filter of order 50

Sr. No.	Coefficient No.	Coefficient Values
1	A(0)=A(50)	-.001086
2	A(1)=A(49)	.001044
3	A(2)=A(48)	.000558
4	A(3)=A(47)	-.002728
5	A(4)=A(46)	.003541
6	A(5)=A(45)	-.001583
7	A(6)=A(44)	-.002702
8	A(7)=A(43)	.006720
9	A(8)=A(42)	-.007109
10	A(9)=A(41)	.002185
11	A(10)=A(40)	.006251
12	A(11)=A(39)	-.013074
13	A(12)=A(38)	.012645
14	A(13)=A(37)	-.002777
15	A(14)=A(36)	-.012584
16	A(15)=A(35)	.024155
17	A(16)=A(34)	-.022287
18	A(17)=A(33)	.003284
19	A(18)=A(32)	.025741
20	A(19)=A(31)	-.048066
21	A(20)=A(30)	.044539
22	A(21)=A(29)	-.003645
23	A(22)=A(28)	-.070548
24	A(23)=A(27)	.157410
25	A(24)=A(26)	-.227141
26	A(25)	.253789

The calculated coefficients have been interpolated in MATLAB to obtain frequency response of designed filter and magnitude is noticed across the normalized frequency to analyze the amplification and attenuation values for the different frequency range that is to find pass-band and stop-band range and behavior of filter in these bands. Magnitude response of digital high pass filter, having coefficients as shown in Table 7, is shown in Fig. 6.

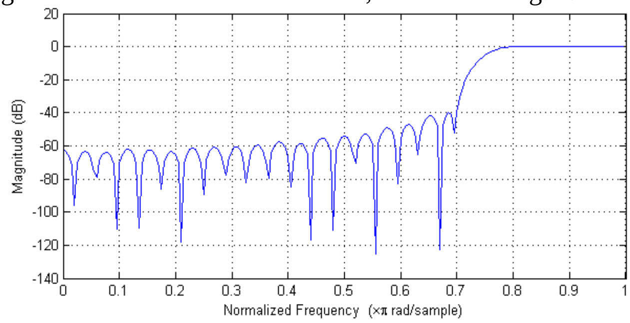

Figure 6: Magnitude vs Normalized Frequency

The Fig. 6 shows that magnitude (in dB) increases as frequency increases in the digital high pass FIR filter. Maximum stop band attenuation achieved for digital high pass FIR filter using DE algorithm is 40 dB.

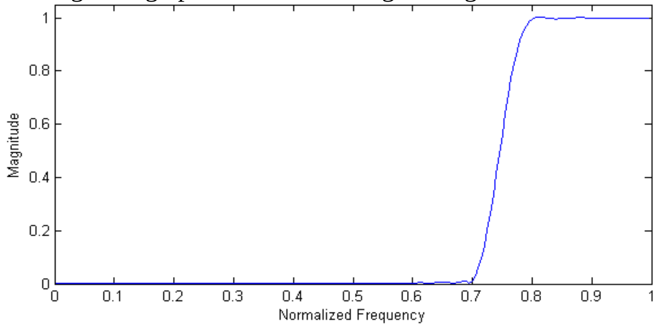

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Figure 7: Magnitude response of High Pass Digital FIR Filter

The Fig. 7 indicates that the signals having frequency range in stop band are attenuated and those having frequency range in pass band are transmitted.

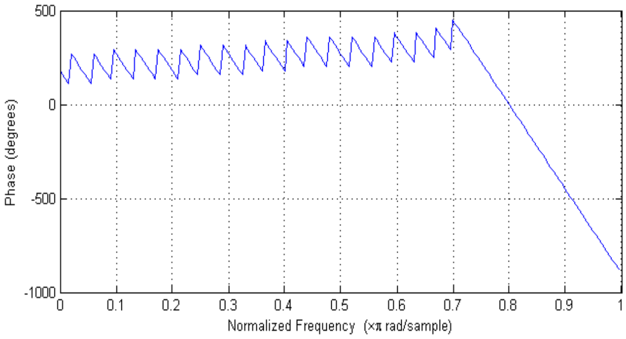

Figure 8: Phase response of High Pass FIR Filter

The Fig. 8 shows that filter have linear phase response from frequency range 0.7 to 1. To check the robustness of filter standard deviation is calculated as shown in Table 8

Table 8 maximum, minimum and average value of objective function along with standard deviation

Sr. No.	Maximum Objective Function	Minimum Objective Function	Average Value	Standard Deviation
1	0.458524	0.439043	0.443319	0.003395

In Table 8 the value of standard deviation is very much less than one, which shows the robust nature of designed filter.

V. CONCLUSION

DE algorithm is very powerful optimization algorithm that exhibits simplicity, robustness and fast convergence using few control parameters. In any application that utilized differential evolution algorithm, the right choice of parameters is very important as convergence speed and objective function have great impact of these control parameters namely population (NP), mutation factor (f_M) and crossover rate (CR). In this high order digital FIR filter is designed using differential evolution algorithm. The designed DE algorithm has been implemented on mutation strategy-4 of DE and best results have been achieved at filter order 50. Then all the mutation strategies have been applied at filter order 50 and it is observed that mutation strategy-1 gives the best result. After analyzing results, it is concluded that for higher order digital FIR filter design problem, population at 160, mutation factor value 0.8 and crossover rate value 0.6 gives the optimum value of objective function. Magnitude and phase response are analyzed for this filter and the same technique can be applied to design the low pass, band pass and band stop filters also.

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