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Random Search Optimization Method for the Design of Low-Pass Digital FIR Filter

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Abstract- This paper establishes methodology for the stable and robust design of finite impulse response FIR Digital Filter by using random search optimization technique. Differential Evolution algorithm has been implemented and five Mutation Strategies have been explored for the design of Low-Pass Digital FIR Filter. DE has been applied to design the stable Digital FIR filter in order to minimize the magnitude errors and minimize the ripple errors of both pass-band and stop-band. Parameters of DE have been varied. This technique can also be implemented for the design of High-pass, Band-Pass, and Band-Stop Digital Filter.

Keywords - Differential evolution, Digital Signal Processing, Finite Impulse Response Filter, Magnitude response Optimization, etc.

I. INTRODUCTION

Signal is the function of any physical quantity which varies with time, space or any other independent variable. Mathematically manipulation of an information signal in to discrete time is known as Digital Signal Processing. Digital signal processing is an area of science and engineering which is more important key feature for role of technology. Today DSP has been used several applications in the field of audio and video communication, image processing, biomedicine and data acquisition etc. Due to its advantage they are simple to use and store, more capable, less expensive and applicability to very low frequency signals. Filters are basically part of all Signal Processing and telecommunication circuit system that are widely used to remove unwanted portion of signal. Filters are widely used in applications like noise reduction, radars, channel equalization, speech synthesis, and biomedical signal processing etc. Basically a filter serves two purposes such as Signal Separation and Signal renovation [17].

(a) Signal separation- This is required when the signal is infected with noise.

(b) Signal renovation- Where as signal renovation comes into use when signal has been lost or distorted like obstacles in way of travelling light.

Filters are categories in to two types Analog Filters and Digital Filters. Analog filters are the device that works on continuous time signals. Whereas Digital filters use a digital processor to perform the numerical calculations. Digital filters are used in a large number of signal processing applications like spectrum analysis, pattern recognition, digital image processing etc. Digital Filters are classified in two main categories:-Finite impulse response filters (FIR) and Infinite impulse response filters (IIR).

(a) IIR filters are known as recursive filters; it means the output depends upon current and past input values and past output values. The response of IIR filter never reaches zero and is an infinite response.

(b) FIR means finite impulse response. FIR filters has an impulse response that is zero outside of some finite interval.

Compare to IIR filters the main advantages of FIR filters are as follows: (1) Finite response (2) Linear phase (3) require more memory to achieve (4) easy to optimize (5) FIR filters are always stable [12]. FIR filters are useful for applications where linear phase response is required. FIR filters could be designed using (1) Window method (2) Frequency sampling technique (3) Optimal filter design methods.

Window method is a simple method which contains well defined equations for computing window coefficients. But this method does not provide sufficient flexibility. But this method does not provide sufficient flexibility. Unlike the window method, this method used for any given magnitude response. However they obtained frequency response matches the desire frequency response only at sampled values [7].

Optimization filter design method is defined as a process of finding the conditions that give the maximum or minimum value of the objective function. Optimization techniques are generally three types [3].

1.1 Direct Search Method

1.2 Gradient Based Method

1.3 Nature Inspired Method

Gradient methods are used for finding the quick optimal solution but are not perfect in non-differential problem. To emerge from these limitations focus came on nature inspired methods Nature inspired methods include Genetic Algorithm (GA), evolutionary programming (EP), evolutionary strategies (ES), genetic programming (GP), differential evolution (DE), particle swarm optimization (PSO), predator prey optimization (PPO) etc. PSO is simple, easy to implement and computationally fast. But if initial parameters of PSO are not chosen correctly it results into local minima. PPO has an additional predator to improve the performance. Differential evolution is a simple method for population based techniques [14].

This paper is organised in five sections: - section ii describes The FIR filter design problem formulation. Section iii discuss the Differential Evolution algorithm used for designing the optimal design of Low-Pass Digital FIR filters. Section iv describes the results of DE parameters. Finally conclusion has been drawn in section v.

II. FIR FILTER DESIGN PROBLEM

FIR filters are digital filters with finite impulse response. FIR filters are implemented using a transversal filter. The transversal filter is also known as tapped delay line filter. It consists of three basic elements unit delay element, multiplier and adder. The difference equation of FIR filter is as given below [16].

$$y(n) = \sum_{k=0}^{M-1} b_k\, x(n-k) \qquad (1)$$

Where $y(n)$ is output sequence, $x(n)$ is input sequence, b_k are coefficients, M is the order of filter.
The transfer function of FIR filter is given as:

$$H(z) = \sum_{k=0}^{M-1} b_k\, z^{-k} \qquad (2)$$

The unit sample response of FIR system is identical to the coefficients (b_k) that are described in Eq. 3

$$h(n) = \begin{cases} b_n & 0 \le n \le M-1 \\ 0 & otherwise \end{cases} \qquad (3)$$

Output sequence$y(n)$ can be expressed as the convolution of unit sample response $h(n)$ of the system with its input signal.

$$y(n) = \sum_{k=0}^{M-1} h(k)\, x(n-k) \qquad (4)$$

FIR filters have symmetric and anti- symmetric properties as illustrated in Eq. 5, 6

$$h(n) = h(N-1-n)$$

For even (5)

$$h(n) = -h(N-1-n) \text{ For odd} \qquad (6)$$

For such a system the number of multiplications is reduced from M to $M/2$ for M even and to $(M-1)/2$ for odd.
Magnitude response is specified at K equally spaced discrete frequency points in pass-band and stop-band. Absolute error of $e_1(x)$ and squared error $e_2(x)$ has been given in Eq. 7 and 8

$$e_1(x) = \sum_{i=0}^{K} \left| H_d(\omega i) - |H(w_i, x)| \right| \qquad (7)$$

$$e_2(x) = \sum_{i=0}^{K} \left(\left| H_d(\omega i) - |H(w_i, x)| \right| \right)^2 \qquad (8)$$

FIR filter's desired magnitude response is given by Eq. 9 and 10

$$\delta_1(x) = \max\{|H(\omega_i, x)|\} - \min\{|H(\omega_i, x)|\} \qquad (9)$$

$$\delta_2(x) = \max\{|H(\omega_i, x)|\} \qquad (10)$$

The multivariable constraints optimization problem is stated as follows:

$$\text{Minimize} f_1(x) = e_1(x) \qquad (11.a)$$

$$\text{Minimize } f_2(x) = e_2(x) \qquad (11.b)$$

$$\text{Minimize } f_3(x) = \delta_1(x) \qquad (11.c)$$

$$\text{Minimize } f_4(x) = \delta_2(x) \qquad (11.d)$$

Multi-criterion optimization problem is describes in Eq. 12

$$\text{Minimize} f(x) = \sum_{j=1}^{4} \omega_j\, f_j(x) \qquad (12)$$

Prescribed conditions for Low Pass Filters are depicted in Table I

Table I: Prescribed design conditions for low pass filters

Filter Type	**Pass-Band**	**Stop-Band**	**Maximum value of $\|H(\omega,x)\|$**
Low-Pass	0≤ω≤0.2π	0.3π≤ω≤π	1

III. DIFFERENTIAL EVOLUTION

Differential evolution is a population based stochastic method introduced by Storn and Price. As this algorithm involves a new sort of differential operator hence the method had been named as Differential Evolution (DE) . This population is successfully improved by applying Mutation, Crossover, Selection operators [14].

DE algorithm describes following characteristics of DE algorithm: (a) Fast Convergence speed. (b) It doesn't get easily trapped in the problem of local minima. By using the component of existing population members to build new trail vector. Differential evolution algorithm uses four parameters Size of Population(N), Dimension length(D), Crossover Rate(CR), Mutation Factor(f_m) [8].

For the Population Vector is denoted in Eq 13

$$x_{i,G} = [x_{1,i,G}, x_{2,i,G}, \ldots\ldots\ldots\ldots x_{D,i,G}]\mathrm{I} \qquad (13)$$

Where i=1, 2 ….N, G is generation matrix, N is population size, D is no of parameters.

A. Mutation

In this new parameter vectors are generated by adding the weighted difference between two population vectors to a third vector. This process is known as mutation. For each target a mutant vector is generated according to, for a given parameter vector $x_{i,G}$ randomly select three vectors $x_{r1,G}$,$x_{r2,G}$ and $x_{r3,G}$ such that indices i, r_1,r_2 and $r_3$are different. Then the weighted difference of two vectors is added to the third as follows [5].

$$v_{i,G+1}=x_{r1,G}+f_m.(x_{r2,G}-x_{r3,G}) \quad (14)$$

Where f_m is mutation factor, varied between [0, 2]. $v_{i,G+1}$ is called the donor vector. Five different mutation strategies considered for study are given in Eq. (15.a-15.e)

$$v_{ij}^{t+1} = x_{Bj}^t + F(x_{Bj}^t - x_{ij}^t) \quad (i= 1, 2, \ldots, N;\ j =1, 2, \ldots, D) \quad (15.a)$$

$$v_{ij}^{t+1} = x_{Bj}^t + F(x_{Bj}^t - x_{ij}^t - x_{r_1 j}^t - x_{r_2 j}^t) \quad (i = 1, 2, \ldots, N;\ j= 1, 2, \ldots, D) \quad (15.b)$$

$$v_{ij}^{t+1} = x_{Bj}^t + f_B(x_{Bj}^t - x_{ij}^t) + F\left(x_{r_1 j}^t - x_{r_2 j}^t\right) \quad (i = 1, 2, \ldots, N;\ j= 1, 2, \ldots, D) \quad (15.c)$$

$$v_{ij}^{t+1} = x_{Bj}^t + F(x_{Bj}^t + x_{ij}^t - x_{r_1 j}^t - x_{r_2 j}^t) \quad (i = 1, 2, \ldots, N;\ j = 1, 2, \ldots, D) \quad (15.d)$$

$$v_{ij}^{t+1} = x_{Bj}^t + F(x_{Bj}^t - x_{Bj}^{t-1}) \quad (i = 1, 2, \ldots, N;\ j = 1, 2, \ldots, D) \quad (15.e)$$

B. Crossover

The predetermined vector is mixed with the mutant vector to produce trail vector. The trail vector $u_{i,G+1}$ is developed from the elements of the target vector $x_{i,G}$ and the elements of the donor vector $v_{i,G+1}$. Elements of the donor vector enter the trail vector with probability CR. Crossover can be mathematically expressed as:

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1} & \text{if (randb(j)} \leq \text{CR) or j} = \text{rnbr(i)} \\ x_{ji,G} & \text{if (randb(j)} > \text{CR) and j} \neq \text{rnbr(i)} \end{cases} \quad (16)$$

Where i=1,2,……..,N and j= 1,2,……..,D

randb(j) is the jth evaluation of a uniform random generator with outcome ϵ [0,1]. CR is the crossover constant ϵ [0,1] which is selected by the user, rnbr(i) is a randomly chosen index ϵ 1,2,……D which ensure that $u_{i,G+1}$ gets minimum of one parameter from $v_{i,G+1}$. rnbr(i) ensures that $v_{i,G+1} \neq x_{i,G}$ [5].

C. Selection

In selection process parameters of target vector should be replaced by trial vector or not. This final step of choosing the parameters is done on the basis of minimum value of cost function is known as Selection.

$$x_{i,G+1} = \begin{cases} u_{i,G+1} & if\ f\left(u_{i,G+1}\right) < f(x_i, G) \\ x_{i,G} & otherwise \end{cases} \quad (17)$$

where i= 1,2,……….N

D. Differential evolution algorithm

1. Read data viz. population size, mutation factor, crossover rate and stopping criteria.
2. Specify maximum no. of iterations.
3. Generate an array of (N×D) size of uniform random number.
4. Calculated objective function is arranged in ascending order and select first half of the population members.
5. Set iteration counter i.e. IT=0
6. Increment iteration counter i.e. IT=IT+1
7. Select best member whose objective function is optimum.
8. Donor vector $v_{i,G+1}$ generated from the target vector as using mutation strategies 1-5.
9. Generate arrays, of random number and apply crossover and selection operation.
10. Choice between trial vector and target vector is done as illustrated in Eq. 17
11. If not go back to step 6
12. Note down the GBEST
13. Check the maximum no. of runs if not go back to step 2.
14. Stop

IV. RESULTS

FIR Low pass digital filter design implementation has been done by using optimization technique, "Differential Evolution (DE) five different Mutation Strategy of DE has been explored.

A. Selection of Order

By using the 5 mutation strategies, mutation strategy-1 has been selected randomly. Differential evolution algorithm has been implemented from order 22 to order 40 on mutation strategy-1. Order of selection was done on the basis of value of pass-band ripple and stop-band ripple was optimum. The graph plotted between the trend of pass-band ripple and stop-band ripple.

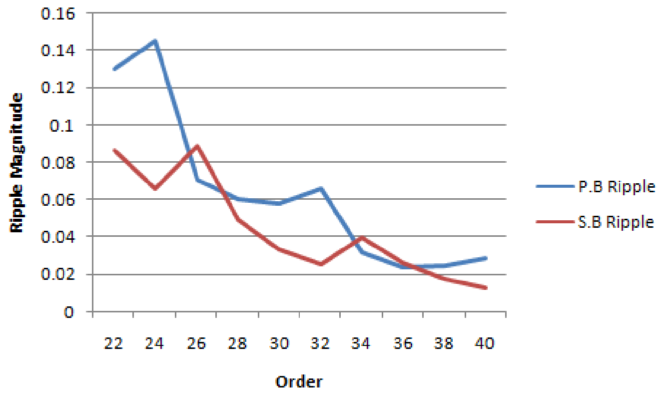

Fig. 1 Pass Band Ripple and Stop Band Ripple Trend

From the Fig.1 it is clear that objective function value is optimum at filter order 36. So order 36 has been selected for design of Low-Pass Digital FIR filter.

B. Differential mutation strategies for filter order 36

The results obtained after exploring all the Five Mutation Strategy for filter order 36have been shown in Table 2

Table II: Objective Function, Magnitude Error 1, Magnitude Error 2 along with Pass-Band Ripple and Stop-Band Ripple achieved for different mutation strategy at filter order 36

S. No	Mutation Strategies	Objective Function	Magnitude Error 1	Magnitude Error 2	Pass-Band Performance	Stop-Band Performance
1	Mutation Strategy 1	1.293560	0.709957	0.085826	0.988019≤|H(ω)|≤1.011088 (.023069)	|H(ω)|≤.0.026709 0.026709
2	Mutation Strategy 2	1.334314	0.715571	0.079095	0.981113≤|H(ω)|≤1.011594 (.030481)	|H(ω)|≤.0.023484 .023484
3	Mutation Strategy 3	1.293828	0.717187	0.086022	0.988299≤|H(ω)|≤1.010650 (.022351)	|H(ω)|≤.0.026711 .026711
4	Mutation Strategy 4	1.293799	0.710008	0.085632	0.988066≤|H(ω)|≤1.011271 (.023204)	|H(ω)|≤.0.026612 .026612
5	Mutation Strategy 5	1.295094	0 .721258	0.085227	0.987942≤|H(ω)|≤1.010443 (.022501)	|H(ω)|≤.0.026360 .026360

As from the results it is clear that Mutation Strategy- 1 gives the optimum objective function. Mutation strategy of Differential Evolution and Order of the FIR filter has been kept fixed, parameters of Differential Evolution algorithm has been varied to observe the performance of algorithm. Population has been varied from 60 to 200. The results are as follows:

Table III: Objective Function Obtained for different values of Population implemented on mutation strategy 1 for filter order 36

S. No.	Population Value	Objective Function
1	60	1.293601
2	80	1.293680
3	100	1.293560
4	120	1.293702
5	140	1.293663
6	160	1.293505
7	180	1.293403
8	200	1.293465

From the Table 3 values graph has been plotted between the Population Size and Objective function shown in Fig 2 below:-

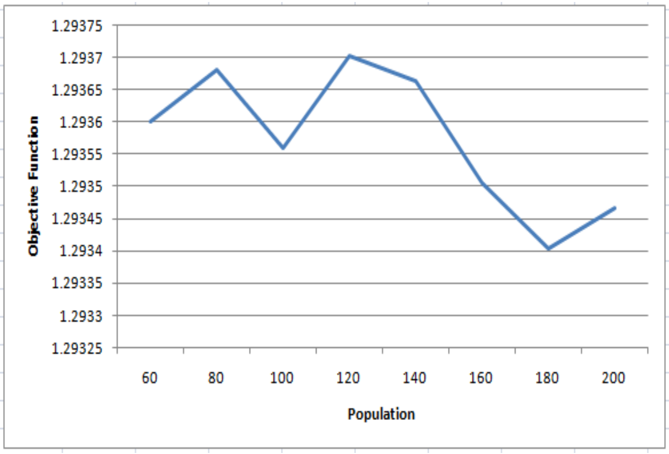

Fig. 2 Objective Function Vs. Population

From Fig. 2, it is clear that objective function value increases from population size 60 to 80. There is a decrease in value of objective function from population size 80 to 100 and from population size 100 to 120, Objective function increases. After the population size 120 objective functions suddenly decrease at the population size 180 and again increase in objective function with population size 180 to 200. It has been concluded that obtained minimum value of objective function 1.293403 at population value 180 for Mutation Strategy-1 at Filter Order 36.

C. Selection of mutation factor (f_m)

From the above Fig 2, it is clear that DE algorithm with Population Size 180 gives the optimum results. Having value of the Mutation factor (f_m) has been varied from 0.5 to 1.0. The values of Objective Function by varying the Mutation Factor (f_m) has been show in Table 4 below.

Table IV: Objective function vs. Mutation Factor values

S. No	Mutation Factor	Objective Function
1	0.5	1.304084
2	0.6	1.294009
3	0.7	1.293704
4	0.8	1.293403
5	0.9	1.293943
6	1.0	1.304084

The graph has been plotted in between the Mutation Factor (f_m) and Objective function has been shown in Fig 3 below:

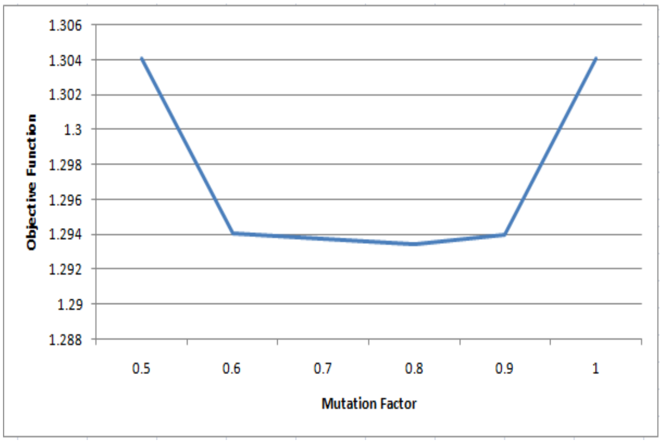

Fig.3 Objective Function vs. Mutation Factor

In Fig. 3 it is observed that value of objective function remains decrease for f_m range 0.4 to 0.6 and it is constant for f_m range 0.6 to 0.8. After the f_m range 0.8 objective functions increase. It has been observed that Differential Evolution algorithm with mutation factor 0.8 gives the minimum value of objective function as compared to other values of mutation factor.

D. Selection of crossover rate (cr)

With the mutation factor 0.8 fixed the value of crossover rate has been changed from 0.1 to 0.5. The value of objective function for this range is as shown below:

Table V Objective Function Vs. Crossover Rate

S. No	Crossover Rate	Objective Function
1	0.1	1.293927
2	0.2	1.293403
3	0.3	1.293371
4	0.4	1.293201
5	0.5	1.293233

The graph has been plotted in between Crossover Rate and Objective function has been shown in Fig 4 below

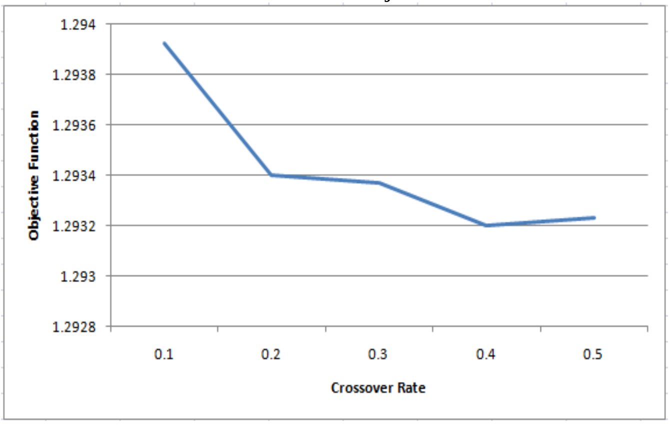

Fig. 4 Objective Functions Vs. Crossover Rate

From the Fig 4 it is clear that objective function decrease with crossover rate variation from 0.1 to 0.2. After that it is continuously decreases from crossover rate variation from 0.2 to 0.4. Once again objective function increase slowly with crossover rate 0.4 to 05. It has been concluded that Differential Evolution with a crossover rate 0.4 gives the best results.

Combining the results it has been concluded that Differential evolution with Population Size 180; Mutation Factor 0.8 and Crossover Rate 0.4 gives that optimum results for designing a Low Pass FIR Digital filter. Program is run 100 times. The optimum result is obtained at run number 60.

Table VI: Design Results for LP Filter

S. No.	Magnitude Error 1	Magnitude Error 2	Pass-Band Performance	Stop-Band Performance
1.	0.716104	0.086687	0.988385≤|H(ω)|≤1.010502 (0.022116)	H(ω)|≤0.026925 (0.026925)

E. Magnitude Response in dB:

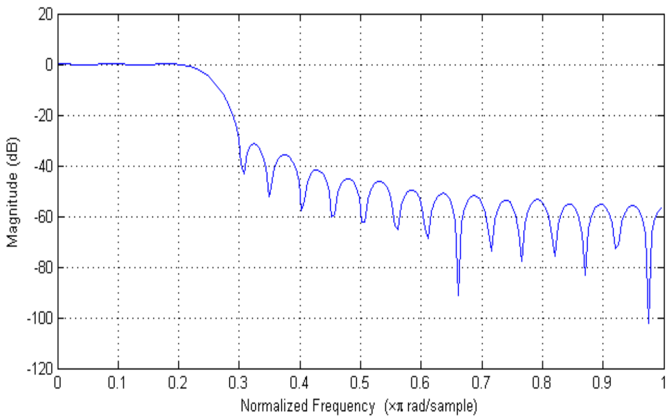

Fig. 5 Magnitude response (in dB) for Low-Pass Digital FIR Filter with filter order 36 and Mutation Strategy-1

The Fig. 5 graph between has been plotted in between the magnitude value in decibels and normalized frequency. In Fig. 5 as frequency increases, magnitude decreases. This is characteristic of low pass filter.

F. Magnitude Response:

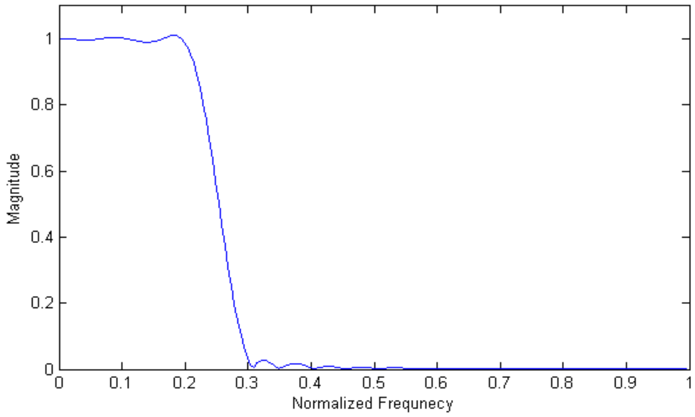

Fig. 6 Magnitude response for Low –pass Digital FIR Filter with filter order 36 and Mutation Strategy-1

The predefined range of pass-band and stop-band has been taken as $0 \leq \omega \leq 0.2\,\pi$ and $0.3 \leq \omega \leq \pi$. The magnitude response of low-pass filter has been show in Fig. 6

G. Phase Response

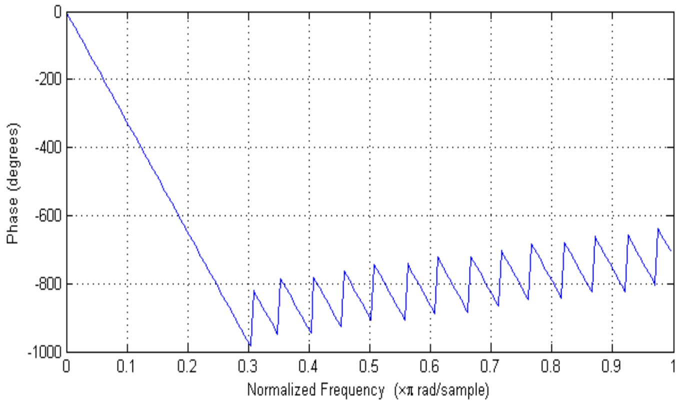

Fig. 7 Phase response for Low –pass Digital FIR Filter with filter order 36 and Mutation Strategy-1

FIR Digital filters have linear response. In Fig. 7 phase of low pass Digital FIR filter is linear from frequency range 0 to 0.3.

Table VII: Maximum, Minimum and Average value of Objective Function along with Standard Deviation for Filter Order 36 with Mutation Strategy-1

S. No.	**Minimum value of Objective Function**	**Maximum value of Objective Function**	**Average value of Objective Function**	**Standard deviation**
1	1.293201	1.301983	1.295183	0.001794

From Table 7 it is clear that achieved Standard deviation 0.001794 which is less than one. So it has been concluded that designed low-pass Digital FIR Filter is robust in design.

V. CONCULSION

The new DE algorithm has been applied to the design of Digital FIR low-pass Filter with different orders. DE Algorithm has been implemented on five Mutation Strategies with filter order 36 and Mutation Strategy-1 gave the best value of Objective Function along with magnitude error and ripple in pass-band and stop-band. For Mutation Strategy with Filter order 36; other DE parameters such as Population size, Mutation factor and Crossover rate have been varied and best results for each have been calculated. It has been concluded that DE with Population Size 180, Mutation Factor 0.8 and crossover Rate 0.4 gives the best results. Total 100 runs were executed and standard deviation in Objective Function is 0.001794 which is less than 1 and this shows the robustness of the designed filter.

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