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Peak to Average Power Ratio Reduction Using Zadoff-Chu Matrix Transform Precoding / Post Coding Techniques

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Abstract-- Orthogonal Frequency Division Multiplexing (OFDM) is considered to be a promising technique against the multipath fading channel for wireless communications. However, OFDM faces the Peak-to-Average Power Ratio (PAPR) problem that is a major drawback of multicarrier transmission system which leads to power inefficiency in RF section of the transmitter. In this paper,we present a Zadoff-Chu matrix transform precoding/ post coding techniques for reducing PAPR in OFDM system.We also simulate the Zadoff-Chu matrix techniques for different number of q and r, so that there variation can lead to a significant change in the output.

Keywords-- Orthogonal Frequency Division Multiplexing (OFDM), Peak-to-Average Power Ratio (PAPR), Complementary Cumulative Distribution Function (CCDF).

I. INTRODUCTION

OFDM has various properties that make it desirable over existing single carrier systems; the main advantage is OFDM's immunity to frequency discriminating fading. Single carrier systems can increase their data rate by shortening the symbol time, therefore increasing the occupied bandwidth. Wide-band channels are sensitive to frequency selective fading which require complex equalizers in the receiver to recover the origins signal. Therefore only 1 tap equalizers are required in the receiver, reducing complexity greatly. Despite the many advantages of OFDM it still suffers from some limitations such as sensitivity to carrier frequency offset and a large Peak to Average Power Ratio (PAPR) [1]. The large PAPR is due to the superposition of N independent equally spaced sub-carriers at the output of the Inverse Fast Fourier Transform (IFFT) in the transmitter. A large PAPR is a problem as it requires increased complexity in the word length at the output of the IFFT and the Digital to Analogy Converter (DAC) [2].If the high PAPR is allowed to saturate the HPA out of band radiation is produced affecting adjacent channels and degrading the BER at the receiver. However, one of the main disadvantages of OFDM is its sensitivity against carrier frequency offset which causes attenuation and rotation of sub-carriers, and inter-carrier interference (ICI) [3], [4]. The undesired ICI degrades the performance of the system.

II. IMPLEMENTION OF OFDM

To design the ZCT pre-coding and post-coding system for reducing PAPR, consider the block diagram of an OFDM system shown in Figure 1.1. OFDM signal consists of *N* subcarriers that are modulated by *N* complex symbols selected from a particular QAM constellation.

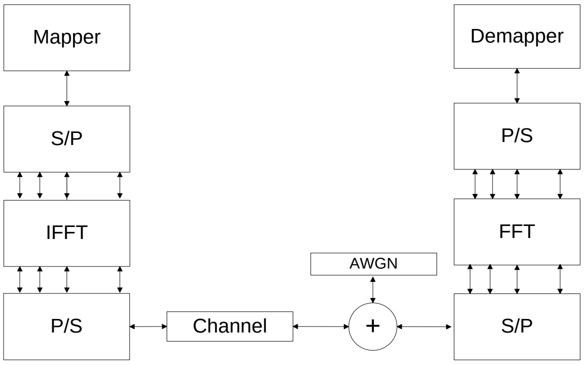

Fig 1 Block diagram of general OFDM system [5].

In Figure1, the baseband modulated symbols are passed through serial to parallel converter which generates complex vector of size N. This complex vector of size N can be written as

$$X = \{X_0, X_1, \dots\dots\dots\dots X_{N-1}\}^T \tag{1}$$

X is then passed through the IFFT block to give

$$x= WX \qquad (2)$$

Where, *W* is the $N \times N$ IFFT matrix. Thus, the complex baseband OFDM signal with subcarriers Can be written as:

$$x_n = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_k \,.\, e^{j2\pi k \frac{n}{N}} \qquad n = 0,1, \ldots..N-1 \qquad (3)$$

After parallel-to-serial conversion, a cyclic prefix (CP) with a length of N_g samples is appended before the IFFT output to form the time-domain OFDM symbol, $s = [s_0, \ldots, s_{N+Ng-1}]$, where, $s_i = x_{\left(i-N_g\right)_N}$ and $\langle i \rangle_N \triangleq i \bmod N$. The useful part of OFDM symbol does not include the N_g prefix samples and has duration of T_u seconds. The samples (*s*) are then amplified, with the amplifier characteristics is given by function *F*. The output of amplifier produces a set of samples given by:

$$Y = \{Y_0, Y_1, \ldots\ldots Y_{N-1}\} \qquad (4)$$

At the receiver front end, the received signal is applied to a matched filter and then sampled at a rate $T_s = T_u/N$. After dropping the CP samples (N_g), the received sequence z, assuming an additive white Gaussian noise (AWGN) channel, can be expressed as

$$Z = F(Wd) + \eta \qquad (5)$$

Where, the noise vector η consists of N independent and normally distributed complex random variables with zero mean and variance $\sigma_n^2 = E\{|\eta_n|^2\}$. Subsequently, the sequence z is fed to the fast Fourier transform (FFT), which produces the frequency-domain sequence *r* as

$$r = W^H z \qquad (6)$$

Where, k_{th} element of r is given by

$$r_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} Z \,.\, e^{-j2\pi k \frac{n}{N}} \qquad k=0, 1, 2,\ldots, N-1 \qquad (7)$$

One of the main disadvantages of OFDM systems is the high PAPR of transmitted signal due to the combination of N modulated subcarriers. The PAPR for a continuous time signal x (t) is defined as:

$$PAPR = \frac{\max|x(t)|^2}{E(|x(t)|^2)} \qquad 0 \le t \le T_u \qquad (8)$$

The PAPR for discrete time signals can be estimated by oversampling the vector *d* by a factor *L* and computing *NL*-point IFFT. The PAPR in this case is defined as:

$$PAPR = \frac{\max|x(n)|^2}{E(|x(n)|^2)} \qquad n = 0,1, \ldots \ldots NL-1 \qquad (9)$$

The denominator in above equation represents the average power per OFDM symbol at the amplifier input, which is denoted as P_{in}. PAPR, in quantitative terms, is usually expressed in terms of Complementary Cumulative Distribution function (CCDF) for an OFDM signal, and is mathematically given by

$$P(PAPR > PAPR_0) = 1 - \left(1 - e^{-PAPR_0}\right)^N \qquad (10)$$

Where, $PAPR_0$ is the clipping level. This equation can also be read as the probability that the PAPR of a symbol block exceeds some clip level $PAPR_0$.

III. RESEARCH DESIGN

In the proposed research, the key part is the Zadoff-Chu sequences, which can be mathematically defined for a sequence of length L as:

$$a_k = \begin{cases} e^{\frac{j2\pi r}{L}\left(\frac{k^2}{L}+qk\right)} & L \text{ even} \\ e^{\frac{j2\pi r}{L}\left(\frac{k(k+1)}{L}+qk\right)} & L \text{ odd} \end{cases} \qquad (11)$$

Where, $k = 0, 1, 2 \ldots L-1$, *q* is any integer and *r* is any integer relatively prime to *L*.

A. ZCT Pre-coding Based OFDM System

In figure 2. In the ZCT pre-coding based OFDM system, the baseband modulated data is passed through S/P convertor which generates a complex vector of size N that can be written as $X = [X_0, X_1, X_2 \ldots X_{N-1}]^T$. ZCT pre-coding is then applied to this complex vector which transforms this complex vector into new vector of length N that can be written as $Y=RX= [Y_0, Y_1, Y_2 \ldots Y_{N-1}]^T$; where, *R* is a ZCT based row-wise pre-coding matrix of size L=N*N. By reordering [6],

$$k = nN + l \qquad (12)$$

And the matrix R with row wise reshaping can be written as

$$R = \begin{bmatrix} r_{00} & \cdots & r_{0(N-1)} \\ \vdots & \ddots & \vdots \\ r_{(N-1)0} & \cdots & r_{(N^2-1)} \end{bmatrix} \qquad (13)$$

Where, R is a N*N, ZCT complex orthogonal matrix with length L2 =N*N. By letting, q=1 and r=1, the ZCT for even L can be written as $r_k = \exp[(j*pi*k2) / L2]$.

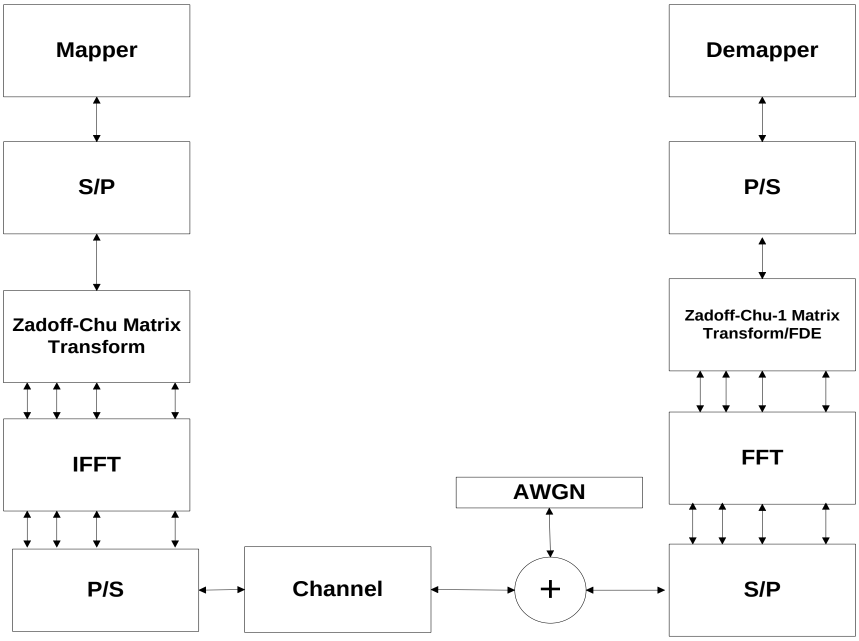

Fig 2 Block Diagram of an OFDM System with ZCT Pre-coding Approach Accordingly, pre-coding *X* gives rise to *Y* as follows:

$$Y = RX \tag{14}$$

$$Y_m = \sum_{l=0}^{N-1} r_{m,l} \cdot X_l \qquad m = 0,1,2 \ldots\ldots\ldots N-1 \tag{15}$$

Where, $r_{m,l}$ means the m^{th} row and l^{th} column of pre-coding matrix

Therefore, the complex baseband OFDM signal with N subcarriers with ZCT pre-coding is given by

$$\widehat{x_n} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \left\{ e^{j2\pi m \frac{n}{N}} \left[e^{j\pi m n^2} \right] \sum_{l=0}^{L-1} \left(Y_l \cdot e^{\frac{j\pi l}{L}} \right) \cdot e^{\frac{j2\pi m l}{L}} \right\} \tag{16}$$

B. ZCT Post-coding Based OFDM System

In figure 3. In the ZCT post-coding based OFDM system, the baseband modulated data is passed through S/P convertor which generates a complex vector of size *N* that can be written as $X = [X_0, X_1, X_2\ldots X_{N-1}]^T$. Then IFFT is performed to this complex vector which transforms this complex vector into new vector of length N that can be written as $y = \text{IFFT}\{X\} = [y_0, y_1, y_2\ldots y_{N-1}]^T$. Similarly, re-ordering as in pre-coding to have:

$$k = m + lN \tag{17}$$

And, ZCT matrix *C* with column wise reshaping is written as

$$C = \begin{bmatrix} c_{00} & \cdots & c_{0(N-1)} \\ \vdots & \ddots & \vdots \\ c_{(N-1)0} & \cdots & c_{(N^2-1)} \end{bmatrix} \tag{18}$$

In other words, the N^2 point long Zadoff-Chu sequence fills the pre-coding matrix column-wise. The matrix *C* is N*N, ZCT complex orthogonal matrix with length $L^2 = N*N$. By letting, *q*=1 and r=1, the ZCT for even *L* can be written as $c_k = \exp[j*pi*k^2/L^2]$. Accordingly, postcoding *y* gives rise to *w* as follows:

$$W = C.\, y \tag{19}$$

Where, *C* is a ZCT column wise post-coding matrix of size $L^2 = N*N$. For $q = 1$ and $r = 1$, the complex baseband ZCT postcoding based OFDM signal with N subcarriers can be written as:

$$x_n = e^{\frac{j\pi m^2}{L^2}} \sum_{n=0}^{N-1} \left[e^{j\pi l^2} \cdot x_l \right] \cdot e^{j2\pi m \frac{n}{L}} \quad m = 0,1,\ldots.N-1 \tag{20}$$

Where, x_n is the IFFT of constellation data X_l premultiplied with quadratic phase and IFFT postcode [7], and then alternated with ±1. The PAPR of this ZCT postcoding based OFDM signal
can be written as:

$$\text{PAPR} = \frac{\max|w(m)|^2}{E(|w(m)|^2)} \qquad n = 0,1,\ldots\ldots NL-1 \tag{21}$$

Where, E [.] denotes expectation and the CCDF for an ZCT postcoding based OFDM signal can be written as:

$$P(\text{PAPR} > \text{PAPR}_0) = 1 - \left(1 - e^{-\text{PAPR}_0}\right)^N \tag{22}$$

Where, PAPR_0 is the clipping level. This equation can be read as the probability that the PAPR of a symbol block exceeds some clip level PAPR_0.

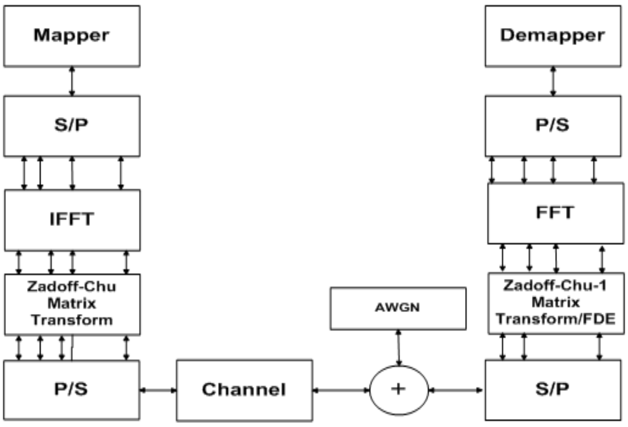

Fig 3. Block Diagram of an OFDM System with ZCT Post-coding Approach.

IV. RESULT

In reported research, the proposed algorithm in Figures 2 and 3 is implemented using MATLAB. It is preferred as it provides a better computational and interactive environment that enables to perform computationally intensive tasks faster and easier than traditional programming languages such as C, C++, and fortan.

To do the PAPR analysis and to evaluate performance of the ZCT pre-coding and post-coding based OFDM systems, the following key steps are undertaken.

1. To do the PAPR analysis of ZCT pre-coding and post-coding based OFDM, firstly binary data is generated randomly.
2. The generated binary data is then converted into symbols and is then modulated by M-QAM (where M=4). The ZCT matrix is calculated for length of the data.
3. The serial data is then converted into parallel data and IFFT is performed. The ZCT matrix (Transform) is applied before and after IFFT, respectively precoding and postcoding. The PAPR is then calculated and compared.
4. The parallel signal is then converted to serial data and is passed through a multipath channel with AWGN noise added.
5. The received signal is again converted from serial to parallel data. ZCT inverse matrix is applied before and after the FFT. Final demodulated signal is parallel data, which converted into serial data. The serial data is demodulated to retrieve back the signal .

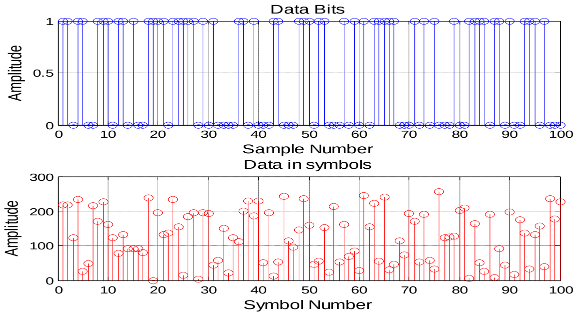

Fig 4 Representation of binary data and its conversion into symbols.

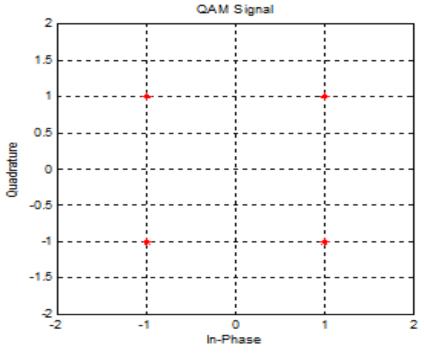

Fig 5 The simulated data modulated Using QAM (M=16)

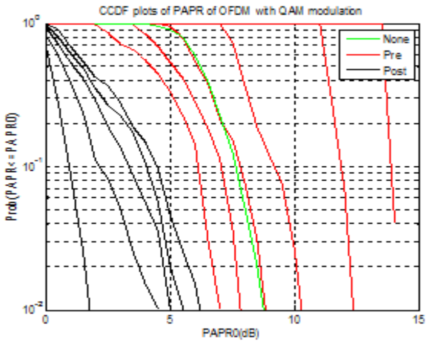

Fig 6. CCDF plots of PAPR of OFDM with QAM modulation for constellation size 4and with ZCT precoding & postcoding.The values are: q=1 and r=2, 13, 31, 53, 73, 101

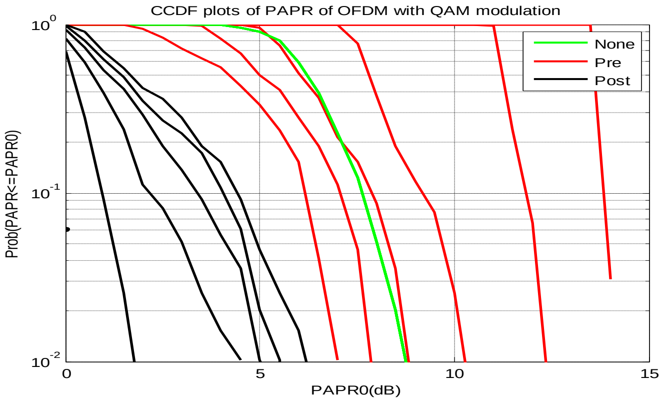

Fig 7. CCDF plots of PAPR of OFDM with QAM modulation for constellation size 16 and with ZCT precoding & postcoding.The values are: *q*=1 and *r*=2, 13, 31, 53, 73, 101

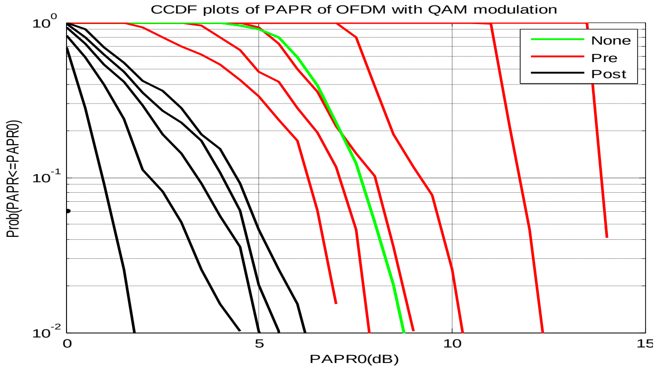

Fig 8 CCDF plots of PAPR of OFDM with QAM modulation for constellation size 64and with ZCT pre-coding & post-coding. The values are: *q*=1 and *r*=2, 13, 31, 53, 73, 101

V. CONCLUSIONS

In this paper we presented the research methodology along with the design to reduce PAPR using the ZCT based pre-coding and post-coding algorithm. From the mathematical expressions, it is found that the complex baseband OFDM signal with N subcarriers with ZCT pre-coding and post-coding is directly proportional to the exponentially raised *q* and *r*. This means the coefficients of ZCT will significantly change with change in values of *q* & *r*; which will further change the values of PAPR. The experimental analysis of this approach is performed it presented the results obtained on applying a ZCT pre-coding and post-coding for the purpose to reduce PAPR in OFDM systems with M-QAM modulation, where as M=4, 16, 64. It is observed from the graphs that the response of both ZCT pre-coding and post-coding is different for *r*>1 and *q*>1. The PAPR significantly varies with the values of *r* and *q*. For any particular value of *r* and *q*, the PAPR reduction is more for ZCT pre-coding than for ZCT pre-coding. From the results, it is preferred to choose a small value of *q* and large value of *r* for ZCT post coding and both small values of *q* and *r* for ZCT pre-coding. It is preferred ZCT post coding because of its sharp roll-off; however, at the expense of less reduction in PAPR or requirement of large values of r if results comparable to ZCT pre-coding needs to be achieved.

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