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A Note on Trigonometric Moments of Stereographic Circular/Semicircular Generalized Gamma Model

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Abstract— ***Trigonometric moments play a vital role in evaluating population characteristics. This note is aimed at obtaining trigonometric moments of Stereographic Semicircular Generalized Gamma Distribution (SSCGG).***

Keywords— ***Trigonometric moments, Stereographic projection, GGD, Gradshteyn & Ryzhik (2007)***

I. INTRODUCTION

A random variable X is said to follow Generalized Gamma distribution with shape parameter $c > 0$, index parameter $\alpha > 0$ and scale parameter $\lambda > 0$, if its pdf is given by

$$f(x;c,\alpha,\lambda) = \frac{c}{\lambda\,\Gamma(\alpha)}\left(\frac{x}{\lambda}\right)^{\alpha c-1}\exp\left(-\left(\frac{x}{\lambda}\right)^{c}\right),\quad x \ge 0,\ c,\alpha,\lambda > 0 \tag{1.1}$$

By applying modified inverse stereographic projection on Generalized Gamma Distribution we get Stereographic Semicircular Generalized Gamma model.

A random variable Φ_{sc} on unit semicircle is said to have Stereographic Semicircular Generalized Gamma distribution with shape parameter $c > 0$, index parameter $\alpha > 0$ and scale parameter $\sigma > 0$ denoted by $SSCGG(c,\alpha,\sigma)$, if the probability density is given by

$$g(\theta;c,\alpha,\sigma) = \frac{c}{2\sigma^{\alpha c}\Gamma(\alpha)}\sec^{2}\left(\frac{\theta}{2}\right)\left(\tan\left(\frac{\theta}{2}\right)\right)^{\alpha c-1}\exp\left(-\left(\frac{\tan\left(\frac{\theta}{2}\right)}{\sigma}\right)^{c}\right) \tag{1.2}$$

$$\text{where } 0 \le \theta < \pi,\ \sigma = \frac{\lambda}{v},\ c,\alpha > 0$$

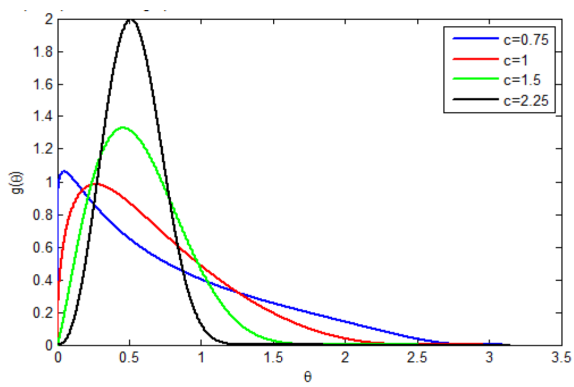

Fig 1.1 The graph of the pdf of the Stereographic Semicircular Generalized Gamma model is plotted for σ =0.25 and α =1.5

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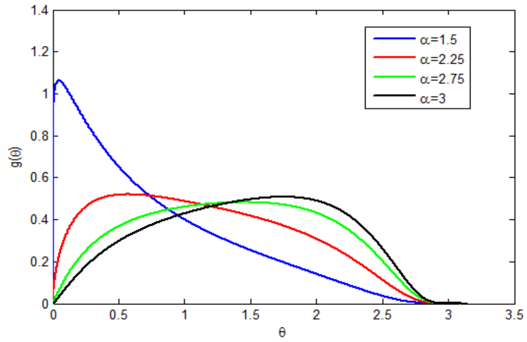

Fig 1.2 The graph of the pdf of the Stereographic Semicircular Generalized Gamma model is plotted for σ =0.25 and c=0.75

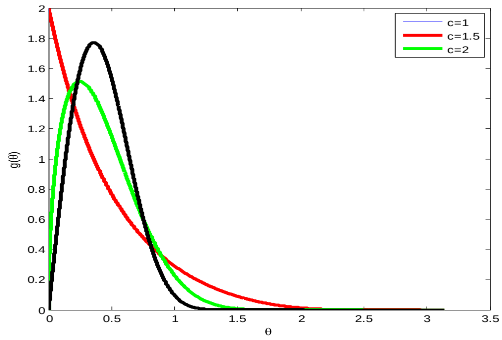

Fig 1.3 The graph of the pdf of the Stereographic Semicircular Generalized Gamma model is plotted for σ = 0.25 and α = 1.00

The Characteristic function of Stereographic Semicircular Generalized Gamma model is

$$\phi_p(\theta) = \int_0^{\pi} e^{ip\theta} g(\theta)\, d\theta$$

$$= \int_0^{\pi} e^{ip\theta} \frac{c}{2\sigma^{\alpha c}\Gamma(\alpha)} \sec^2\left(\frac{\theta}{2}\right) \left(\tan\left(\frac{\theta}{2}\right)\right)^{\alpha c - 1} \exp\left(-\left(\frac{\tan\left(\frac{\theta}{2}\right)}{\sigma}\right)^{c}\right) d\theta \qquad (1.3)$$

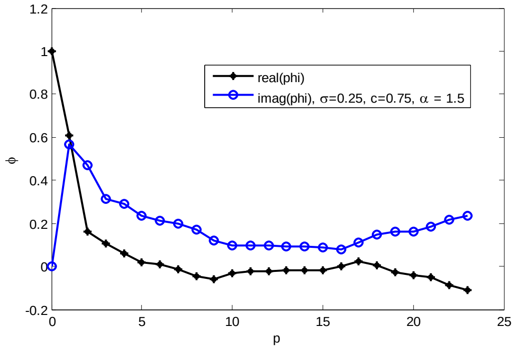

Fig 1.4 The Characteristic Function of the Stereographic Semicircular Generalized Gamma Distribution

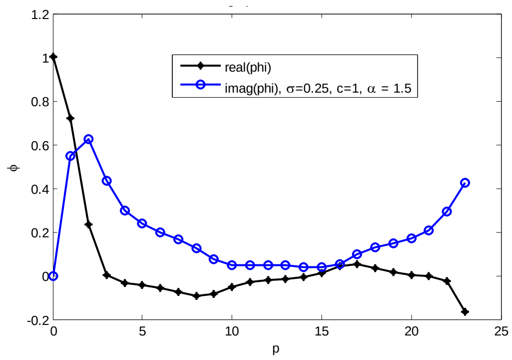

Fig 1.5 The Characteristic Function of the Stereographic Semicircular Generalized Gamma Distribution

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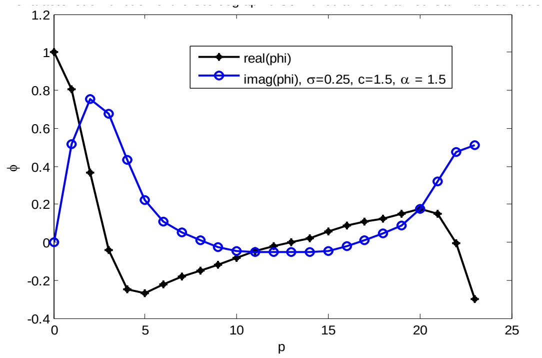

Fig 1.6 The Characteristic Function of the Stereographic Semicircular Generalized Gamma Distribution

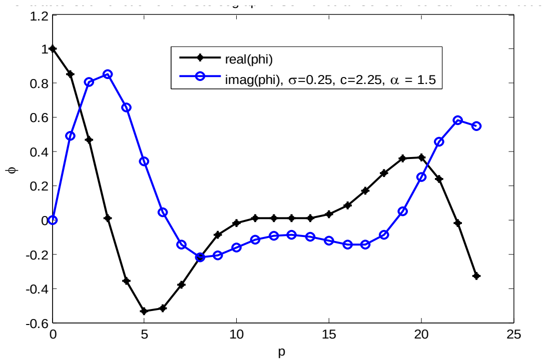

Fig 1.7 The Characteristic Function of the Stereographic Semicircular Generalized Gamma Distribution

II. TRIGONOMETRIC MOMENTS OF STEREOGRAPHIC SEMICIRCULAR GENERALIZED GAMMA DISTRIBUTION

The first two trigonometric moments are derived by us using formula

$$\int_0^\infty x^{\upsilon-1}e^{(-\mu x^p)}dx = \frac{1}{p}\mu^{-\frac{\upsilon}{p}}\Gamma\left(\frac{\upsilon}{p}\right) \quad \text{where} \quad [\operatorname{Re}\mu>0,\ \operatorname{Re}\upsilon>0,\ p>0]$$

[**3.478.1** (Gradshteyn and Ryzhik, 2007)] [2]and are presented in the form of the following result.

Result: Under the pdf of Stereographic Semicircular Generalized Gamma Distribution with $\mu=0$, the first two $\alpha_p = E(\cos p\theta)$ and $\beta_p = E(\sin p\theta)$, $p=1,2.$ are given as follows:

$$\alpha_1 = 1 - \frac{2}{\Gamma(\alpha)}\sum_{n=0}^{\infty}(-1)^n\sigma^{2n+2}\Gamma\left(\frac{2n+\alpha c+2}{c}\right),$$

$$\beta_1 = \frac{2}{\Gamma(\alpha)}\sum_{n=0}^{\infty}(-1)^n\sigma^{2n+1}\Gamma\left(\frac{2n+\alpha c+1}{c}\right),$$

$$\alpha_2 = 1 - \frac{8}{\Gamma(\alpha)}\sum_{n=0}^{\infty}(-1)^n(n+1)\sigma^{2n+2}\Gamma\left(\frac{2n+\alpha c+2}{c}\right),$$

$$\beta_2 = \frac{4}{\Gamma(\alpha)}\sum_{n=0}^{\infty}(-1)^n\sigma^{2n+1}\Gamma\left(\frac{2n+\alpha c+1}{c}\right) - \frac{8}{\Gamma(\alpha)}\sum_{n=0}^{\infty}(-1)^n(n+1)\sigma^{2n+3}\Gamma\left(\frac{2n+\alpha c+3}{c}\right),$$

III. CONCLUSION

Having defined new Semicircular model called Stereographic Semicircular Generalized Gamma Distribution (SSCGG), first two trigonometric moments are derived using these trigonometric moments one can easily compute population characteristics.

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