Volume 5, Issue 4, 2015 ISSN: 2277 128X
International Journal of Advanced Research in Computer Science and Software Engineering
Research Paper
Available online at: www.ijarcsse.com

Some Implication Results Related To Intuitionistic Fuzzy Multisets

C. Deepa
Department of Mathematics, PSNA College of Engineering and Technology
Kothandaraman Nagar, Dindigul, Tamilnadu, India

Abstract-Intuitionistic fuzzy mutlisets are generalization of both IFS and FMS . In this paper, we have proved some new implication results related to intuitionistic fuzzy multisets are proved.

Key words-Fuzzy sets, Intuitionistic fuzzy sets, Multisets, Intuitionistic fuzzy multisets, Operations on intuitionistic fuzzy multisets AMS Classification: 03E72,

I. INTRODUCTION

Fuzzy Set (FS) was introduced by Zadeh [6] in 1965, as a generalization of crisp sets. A generalization of fuzzy sets, Yager [5] introduced the concept of Fuzzy Multiset(FMS). An element of a fuzzy multiset can occur more than once with possibly the same or different membership values.The concept of Intuitionistic Fuzzy Sets (IFSs), as a generalization of fuzzy set was introduced by K.Atanassov [1]. Intuitionistic fuzzy sets are characterized by two functions expressing the degree of membership and the degree of non-membership. In 1994 K. Atanassov [2] proposed new operations defined over the intuitionistic fuzzy sets and in 2000 De. S. K., Biswas. R and Roy A. R [3] presented some operations on intuitionistic fuzzy sets. Then years after, Shinoj and Sunil [4]made an attempt to combine the two concepts: IFS and FMS together by introducing a new concept called intuitionistic fuzzy multiset. The concept of intuitionistic fuzzy sets and intuitionistic fuzzy multiset has been successfully applied in numerous fields, such as pattern recognition, machine learning, image processing and multiple criteria decision making, and etc. In the present communication, I have proved some implication results related to intuitionistic fuzzy multisets based on operations (denoted by$\cup$,$\cap$, $+$, $\cdot$, @ , \$, # , $*$, $\rightarrow$).This paper proceeds as follows: In section 2 some basic definitions related to fuzzy set, fuzzy multiset, intuitionistic fuzzy set, intuitionistic fuzzy multiset and basic relations and operations on intuitionistic fuzzy multisets are presented. In section 3 some implication results related to intuitionistic fuzzy multisets are proved. In section 4 conclusion are given.

II. SOME NEW IMPLICATION RESULTS RELATED TO INTUITIONISTIC FUZZY MULTISETS

Definition: 2.1FUZZY SET

Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A = \{[x, \mu_A(x)] \mid x \epsilon X\}$, where the functions $\mu_A : X \rightarrow [0\,,1]$ define the degree of membership of the element $x \in X$ to the set A which is a subset of X, respectively, and for every $x \in X$: $0 \leq \mu_A(x) \leq 1$.

Definition: 2.2FUZZY MULTISET

Let X be a nonempty set. A fuzzy multiset A drawn from X is characterized by a function, 'count membership' of A denoted by CM_A such that CM_A: X→ Q where Q is the set all crisp multisets drawn from the unit interval [0,1]. Then for any $x \epsilon X$, the value $CM_A(x)$ is a crisp multisets drawn from [0,1]. For each $x \epsilon X$, the membership degree is defined as the decreasingly ordered sequence of elements in $CM_A(x)$. It is denoted by $\left(\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^n(x)\right)$, where $\mu_A^1(x) \geq \mu_A^2(x) \geq \cdots \geq \mu_A^n(x)$.

Definition: 2.3INTUITIONISTIC FUZZY SET

Let X be a nonempty set.. An intuitionistic fuzzy set Ain $X$is an object having the form $A = \{[x, \mu_A(x), \gamma_A(x)] \mid x \epsilon X\}$, where the functions $\mu_A : X \rightarrow [0\,,1]$ and $\gamma_A : X \rightarrow [0\,,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ to the set A which is a subset of X, respectively, and for every $x \in X$: $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.

Definition:2.4 INTUITONISTIC FUZZY MULTISET

Let X be a nonempty set. Anintuitonistic fuzzy multiset A drawn from X is characterized by a functions: 'count membership' of A denoted by CM_A and 'count non-membership' of A denoted by CN_Asuch that CM_A: X→ Q and CN_A: X→ Q where Q is the set all crisp multisets drawn from the unit interval [0,1]. For each $x \epsilon X$, the membership degree is defined as the decreasingly ordered sequence of elements in $CM_A(x)$. It is denoted by $\left(\mu_A^1(x), \mu_A^2(x), \ldots, \mu_A^n(x)\right)$, where $\mu_A^1(x) \geq \mu_A^2(x) \geq \cdots \geq \mu_A^n(x)$. Whereas the corresponding non-membership degrees of element in $CN_A$is denoted by

$\left(\gamma_A^1(x), \gamma_A^2(x), \dots, \gamma_A^n(x)\right)$ such that $0 \le \mu_A^i(x) + \gamma_A^i(x) \le 1$ for every $x \epsilon X$ and i = 1,….,n. This means an IFMS A is defined as: $A = \{[x, CM_A(x), CN_A(x)] \mid x \epsilon X\}$ or $A = \{[x, \mu_A^i(x), \gamma_A^i(x)] \mid x \epsilon X\}$ for i = 1,….,n.

Definition: 2.5 BASIC RELATIONS AND OPERATIONS ON INTUITIONISTIC FUZZY MULTISETS

Let A and B be two intuitionistic fuzzy multisets on the universe X,
where

$A = \{[x, \mu_A^i(x), \gamma_A^i(x)] \mid x \epsilon X\} \& B = \{[x, \mu_B^i(x), \gamma_B^i(x)] \mid x \epsilon X\}$

$A \cup B = \left\{\left[x, max\left(\mu_A^i(x), \mu_B^i(x)\right), min\left(\gamma_A^i(x), \gamma_B^i(x)\right)\right] \mid x \epsilon X\right\}$

$A \cap B = \left\{\left[x, min\left(\mu_A^i(x), \mu_B^i(x)\right), max\left(\gamma_A^i(x), \gamma_B^i(x)\right)\right] \mid x \epsilon X\right\}$

$A^c \ or \ \neg A = \{[x, \gamma_A^i(x), \mu_A^i(x)] \mid x \epsilon X\}$

$A \oplus B \ or \ A + B = \{[x, \mu_A^i(x) + \mu_B^i(x) - \mu_A^i(x)\mu_B^i(x), \gamma_A^i(x)\gamma_B^i(x)] \mid x \epsilon X\}$

$A \otimes B = \{[x, \mu_A^i(x)\mu_B^i(x), \gamma_A^i(x) + \gamma_B^i(x) - \gamma_A^i(x)\gamma_B^i(x)] \mid x \epsilon X\}$

$$A \ @ \ B = \left\{x, \left[\left(\frac{\mu_A^i(x) + \mu_B^i(x)}{2}, \frac{\gamma_A^i(x) + \gamma_B^i(x)}{2}\right)\right] \mid x \in X\right\}$$

$$A \ \$ \ B = \left\{x, \left[\left(\sqrt{\mu_A^i(x)\mu_B^i(x)}, \sqrt{\gamma_A^i(x)\gamma_B^i(x)}\right)\right] \mid x \in X\right\}$$

$$A \ \# \ B = \left\{x, \left[\left(\frac{2\mu_A^i(x)\mu_B^i(x)}{\mu_A^i(x) + \mu_B^i(x)}, \frac{2\gamma_A^i(x)\gamma_B^i(x)}{\gamma_A^i(x) + \gamma_B^i(x)}\right)\right] \mid x \in X\right\}$$

For which we shall accept that if $\mu_A^i(x) = \mu_B^i(x) = 0$ then $\frac{\mu_A^i(x)\mu_B^i(x)}{\mu_A^i(x)+\mu_B^i(x)} = 0$,
if $\gamma_A^i(x) = \gamma_B^i(x) = 0$ then $\frac{\gamma_A^i(x)\gamma_B^i(x)}{\gamma_A^i(x)+\gamma_B^i(x)} = 0$

$$A * B = \left\{x, \left[\left(\frac{\mu_A^i(x) + \mu_B^i(x)}{2\left(\mu_A^i(x)\mu_B^i(x) + 1\right)}, \frac{\gamma_A^i(x) + \gamma_B^i(x)}{2\left(\gamma_A^i(x)\gamma_B^i(x) + 1\right)}\right), \right] \mid x \in X\right\}$$

$$A \to B = \left\{x, \left[Max\left(\gamma_A^i(x), \mu_B^i(x)\right), Min\left(\mu_A^i(x), \gamma_B^i(x)\right)\right] \mid x \in X\right\}$$

III. SOMEIMPLICATION RESULTS RELATED TO INTUITIONISTIC FUZZY MULTISETS ARE PROVED

Theorem 3.1: Let X be nonempty. For every two IFMSs A and B in X;

(a) $(A^c \to B)@(A \to B^c)^c = (A@B)$
(b) $(A^c \to B)\oplus(A \to B^c)^c = (A\oplus B)$
(c) $(A^c \to B)\otimes(A \to B^c)^c = (A\otimes B)$
(d) $(A^c \to B)\$(A \to B^c)^c = (A\$B)$
(e) $(A^c \to B)\#(A \to B^c)^c = (A\#B)$
(f) $(A^c \to B) * (A \to B^c)^c = (A * B)$.

Proof: (a)

$$(A^c \to B) = \begin{pmatrix} \left\{x, \left[\left(\gamma_A^i(x), \mu_A^i(x)\right)\right] \mid x \in X\right\} \\ \to \\ \left\{x, \left[\left(\mu_B^i(x), \gamma_B^i(x)\right)\right] \mid x \in X\right\} \end{pmatrix}$$

$$= \left\{x, \left[\left(Max\left(\mu_A^i(x), \mu_B^i(x)\right), Min\left(\gamma_A^i(x), \gamma_B^i(x)\right)\right), \right] \mid x \in X\right\} (1)$$

$$(A \to B^c)^c = \begin{pmatrix} \left\{x, \left(\mu_A^i(x), \gamma_A^i(x)\right) \mid x \in X\right\} \\ \to \\ \left\{x, \left[\left(\gamma_B^i(x), \mu_B^i(x)\right)\right] \mid x \in X\right\} \end{pmatrix}^C$$

$$= \left(\left\{x, \left[\left(Max\left(\gamma_A^i(x), \gamma_B^i(x)\right), Min\left(\mu_A^i(x), \mu_B^i(x)\right)\right), \right] \mid x \in X\right\}\right)^C$$

$$= \left(\left\{x, \left[\left(Min\left(\mu_A^i(x), \mu_B^i(x)\right), Max\left(\gamma_A^i(x), \gamma_B^i(x)\right)\right)\right] \mid x \in X\right\}\right) (2)$$

From (1) and (2), we get
$(A^c \to B)@(A \to B^c)^c$

$$= \begin{pmatrix} \left\{x, \left[\left(Max\left(\mu_A^i(x), \mu_B^i(x)\right), Min\left(\gamma_A^i(x), \gamma_B^i(x)\right)\right), \right] \mid x \in X\right\} \\ @ \\ \left(\left(\left\{x, \left[\left(Min\left(\mu_A^i(x), \mu_B^i(x)\right), Max\left(\gamma_A^i(x), \gamma_B^i(x)\right)\right)\right] \mid x \in X\right\}\right)\right) \end{pmatrix}$$

$$= \left\{ x, \begin{bmatrix} \dfrac{Max\left(\mu_A^i(x), \mu_B^i(x)\right) + Min\left(\mu_A^i(x), \mu_B^i(x)\right)}{2}, \\ \dfrac{Min\left(\gamma_A^i(x), \gamma_B^i(x)\right) + Max\left(\gamma_A^i(x), \gamma_B^i(x)\right)}{2} \end{bmatrix}, \mid x \in X \right\}$$

$$= \left\{ x, \left[\left(\frac{\mu_A^i(x) + \mu_B^i(x)}{2}, \frac{\gamma_A^i(x) + \gamma_B^i(x)}{2}\right)\right] \mid x \in X \right\}$$

$= (A@B)$. Hence theorem (a) is proved.

The remaining theorem (b), (c), (d), (e) and (f) can be proved by analogously.

Theorem 3.2: Let X be nonempty. For every two IFMSs A and B in X;

(a) $(A \to B)^c @ (B \to A) = (A@B^c)$
(b) $(A \to B)^c \oplus (B \to A) = (A \oplus B^c)$
(c) $(A \to B)^c \otimes (B \to A) = (A \otimes B^c)$
(d) $(A \to B)^c \$ (B \to A) = (A \$ B^c)$
(e) $(A \to B)^c \# (B \to A) = (A \# B^c)$
(f) $(A \to B)^c * (B \to A) = (A * B^c)$.

Proof :(a)

$(A \to B)^c @ (B \to A) = (A@B^c)$

$$(A \to B)^c = \begin{pmatrix} \left\{x, \left[\left(\mu_A^i(x), \gamma_A^i(x)\right)\right] \mid x \in X\right\} \\ \to \\ \left\{x, \left[\left(\mu_B^i(x), \gamma_B^i(x)\right)\right] \mid x \in X\right\} \end{pmatrix}^c$$

$$= \left(\left\{x, \left[\left(Max\left(\gamma_A^i(x), \mu_B^i(x)\right)\right), \left(Min\left(\mu_A^i(x), \gamma_B^i(x)\right)\right)\right] \mid x \in X\right\}\right)^C$$

$$= \left(\left\{x, \left[\left(Min\left(\mu_A^i(x), \gamma_B^i(x)\right)\right), \left(Max\left(\gamma_A^i(x), \mu_B^i(x)\right)\right)\right] \mid x \in X\right\}\right) \quad (3)$$

$$(B \to A) = \begin{pmatrix} \left\{x, \left[\left(\mu_B^i(x), \gamma_B^i(x)\right)\right] \mid x \in X\right\} \\ \to \\ \left\{x, \left[\left(\mu_A^i(x), \gamma_A^i(x)\right)\right] \mid x \in X\right\} \end{pmatrix}$$

$$= \left(\left\{x, \left[\left(Max\left(\gamma_B^i(x), \mu_A^i(x)\right)\right), \left(Min\left(\mu_B^i(x), \gamma_A^i(x)\right)\right)\right] \mid x \in X\right\}\right) \quad (4)$$

From (3) and (4), we get

$(A \to B)^c @ (B \to A)$

$$= \begin{pmatrix} \left(\left\{x, \left[\left(Min\left(\mu_A^i(x), \gamma_B^i(x)\right)\right), \left(Max\left(\gamma_A^i(x), \mu_B^i(x)\right)\right)\right] \mid x \in X\right\}\right) \\ @ \\ \left(\left\{x, \left[\left(Max\left(\gamma_B^i(x), \mu_A^i(x)\right)\right), \left(Min\left(\mu_B^i(x), \gamma_A^i(x)\right)\right)\right] \mid x \in X\right\}\right) \end{pmatrix}$$

$$= \left\{ x, \left[\frac{Min\left(\mu_A^i(x), \gamma_B^i(x)\right) + Max\left(\gamma_B^i(x), \mu_A^i(x)\right)}{2}, \frac{Max\left(\gamma_A^i(x), \mu_B^i(x)\right) + Min\left(\mu_B^i(x), \gamma_A^i(x)\right)}{2}\right] \mid x \in X \right\}$$

$$= \left\{x, \left[\left(\frac{\mu_A^i(x)+\gamma_B^i(x)}{2}, \frac{\gamma_A^i(x)+\mu_B^i(x)}{2}\right)\right] \mid x \in X\right\} \quad (5)$$

$$(A@B^c) = \begin{pmatrix} \left\{x, \left[\left(\mu_A^i(x), \gamma_A^i(x)\right)\right] \mid x \in X\right\} \\ @ \\ \left\{x, \left[\left(\gamma_B^i(x), \mu_B^i(x)\right)\right] \mid x \in X\right\} \end{pmatrix}$$

$$= \left\{x, \left[\left(\left(\frac{\mu_A^i(x)+\gamma_B^i(x)}{2}, \frac{\gamma_A^i(x)+\mu_B^i(x)}{2}\right)\right)\right] \mid x \in X\right\} \quad (6)$$

From (5) and (6), we get

$(A \to B)^c @ (B \to A) = (A@B^c)$. Hence (a) is proved.

The remaining theorem (b), (c), (d), (e) and (f) can be proved by analogously.

Theorem 3.3: Let X be nonempty. For every two IFMSs A and B in X;

$(a). \left((A \oplus B) \to (A@B)^c\right)^c = \left((A@B) \to (A \oplus B)^c\right)^c = (A@B)$

$(b). \left((A \otimes B) \to (A@B)^c\right)^c = \left((A@B) \to (A \otimes B)^c\right)^c = (A \otimes B)$

$(c). \left((A \oplus B) \to (A\#B)^c\right)^c = \left((A\#B) \to (A \oplus B)^c\right)^c = (A\#B)$

(d).$\left((A \otimes B) \to (A\#B)^c\right)^c = \left((A\#B) \to (A \otimes B)^c\right)^c = (A \otimes B)$

$(e). \left((A \oplus B) \to (A\$B)^c\right)^c = \left((A\$B) \to (A \oplus B)^c\right)^c = (A\$B)$

(f).$\left((A \otimes B) \to (A\$B)^c\right)^c = \left((A\$B) \to (A \otimes B)^c\right)^c = (A \otimes B)$.

Proof:(a)

$\left((A \oplus B) \to (A@B)^c\right)^c$

$$= \begin{pmatrix} \left\{x, \left[\left(\mu_A^i(x)+\mu_B^i(x)-\mu_A^i(x)\mu_B^i(x), \gamma_A^i(x)\gamma_B^i(x)\right)\right] \mid x \in X\right\} \\ \to \\ \left\{x, \left[\left(\frac{\gamma_A^i(x)+\gamma_B^i(x)}{2}, \frac{\mu_A^i(x)+\mu_B^i(x)}{2}\right)\right] \mid x \in X\right\} \end{pmatrix}^c$$

$$= \left(\left\{x, \left[\begin{pmatrix} Max\left(\gamma_A^i(x)\gamma_B^i(x), \frac{\gamma_A^i(x)+\gamma_B^i(x)}{2}\right), \\ Min\left(\mu_A^i(x)+\mu_B^i(x)-\mu_A^i(x)\mu_B^i(x), \frac{\mu_A^i(x)+\mu_B^i(x)}{2}\right) \end{pmatrix}, \right] \mid x \in X \right\}\right)^c$$

$$= \left(\left\{x, \left[\left(\frac{\gamma_A^i(x)+\gamma_B^i(x)}{2}, \frac{\mu_A^i(x)+\mu_B^i(x)}{2}\right)\right] \mid x \in X\right\}\right)^c$$

$$= \left\{x, \left[\left(\frac{\mu_A^i(x)+\mu_B^i(x)}{2}, \frac{\gamma_A^i(x)+\gamma_B^i(x)}{2}\right)\right] \mid x \in X\right\}$$

$$= (A@B). \quad (7)$$

$\left((A@B) \to (A \oplus B)^c\right)^c$

$$= \begin{pmatrix} \left\{x, \left[\left(\frac{\mu_A^i(x)+\mu_B^i(x)}{2}, \frac{\gamma_A^i(x)+\gamma_B^i(x)}{2}\right)\right] \mid x \in X\right\} \\ \to \\ \left\{x, \left[\left(\gamma_A^i(x)\gamma_B^i(x), \mu_A^i(x)+\mu_B^i(x)-\mu_A^i(x)\mu_B^i(x)\right), \right] \mid x \in X\right\} \end{pmatrix}^c$$

$$= \left(\left\{x, \left[\begin{pmatrix} Max\left(\frac{\gamma_A^i(x)+\gamma_B^i(x)}{2}, \gamma_A^i(x)\gamma_B^i(x)\right), \\ Min\left(\frac{\mu_A^i(x)+\mu_B^i(x)}{2}, \mu_A^i(x)+\mu_B^i(x)-\mu_A^i(x)\mu_B^i(x)\right) \end{pmatrix}, \right] \mid x \in X \right\}\right)^c$$

$$= \left(\left\{ x, \left[\left(\frac{\gamma_A^i(x) + \gamma_B^i(x)}{2}, \frac{\mu_A^i(x) + \mu_B^i(x)}{2} \right) \right] \mid x \in X \right\} \right)^c$$

$$= \left\{ x, \left[\left(\frac{\mu_A^i(x) + \mu_B^i(x)}{2}, \frac{\gamma_A^i(x) + \gamma_B^i(x)}{2} \right) \right] \mid x \in X \right\}$$

$= (A@B).$ (8)

From (7) and (8), we get $\left((A \oplus B) \rightarrow (A@B)^c\right)^c = \left((A@B) \rightarrow (A \oplus B)^c\right)^c = (A@B)$

Hence (a) is proved.

The remaining theorem (b), (c), (d), (e) and (f) can be proved by analogously.

Theorem3.4: Let X be nonempty. For every two IFMSs A and B in X;

$[(A^c \rightarrow B) \oplus (A \rightarrow B^c)^c] @ [(A^c \rightarrow B) \otimes (A \rightarrow B^c)^c] = (A@B)$

Proof: It follows from the definition.

IV. CONCLUSION

In this paper, some implication results related to intuitionistic fuzzy multisetswere introduced and proved. In future, the application of these identities will be proposed.

REFERENCES

[1] K. Atanassov. Intuitionistic fuzzy sets. *Fuzzy Sets & Systems*, 20 (1986), 87-96.

[2] K. Atanassov. New operations defined over the intuitionistic fuzzy sets. *Fuzzy Sets & Systems*. 61 (1994), 137-42.

[3] De. S. K., Biswas. R and Roy A. R. Some Operations on intuitionistic fuzzy sets. *Fuzzy Sets & Systems*, 114 (2000), 477-484.

[4] T.K. Shinoj, J. J. Sunil, Intuitionistic fuzzy multisets and its application in medical diagnosis, *Int. J. of Mathematical and Computational Sciences* 6 (2012).

[5] R. R. Yager, On the theory of bags, *Int. J. of General Systems* 13 (1986) 23-37.

[6] L. A. Zadeh. Fuzzy Sets. *Information & Control*. 8(1965), 338-353.

