



An Approach for Image Restoration using Group-based Sparse Representation

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Abstract— This paper presents an approach for image restoration which uses group as the fundamental unit of sparse representation instead of using a single patch as the fundamental unit of sparse representation. Hence this approach for image restoration is termed as group-based sparse representation (GSR). In this approach the natural images is represented within the field of group that successively force the intrinsic local sparsity and nonlocal self-similarity of images in a combined framework at the same time. This approach has low complexity since for every group there is a self-adaptive dictionary learning technique is used. An alternative to dictionary learning from the natural images we use self-adaptive dictionary learning. For solving the GSR-driven ℓ_0 -minimization problem which makes GSR tractable and sturdy, a split Bregman-based technique is developed. Using GSR we will deal with three image restoration problems those are image inpainting, deblurring, compressive sensing (CS) recovery.

Keywords— Image restoration, sparse-representation, non-local self similarity, inpainting, deblurring, compressive sensing.

I. INTRODUCTION

In the past several years image restoration has been widely studied. Image restoration can be stated as restoring the high quality image from the degraded low quality image. The generalized image degradation model and image restoration model can simply be represented diagrammatically as follows.

Mathematically, the image degradation model can be formulated as follows

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad (1)$$

If original image is represented by \mathbf{x} , degraded image is represented by \mathbf{y} , \mathbf{n} represents noise and \mathbf{H} represents the degradation operator. The image restoration problem is changes according to the \mathbf{H} . If \mathbf{H} is mask then the problem becomes image inpainting [2], [3], if \mathbf{H} is a blur operator then the problem becomes image deblurring [4], if \mathbf{H} is a random projection then the problem becomes image compressive sensing (CS) recovery [5],[6],[14].

To deal with the poorly presented nature of image restoration, image prior knowledge [1] is typically used for standardizing the solution to the following minimization problem [1]:

$$\arg \min_{\mathbf{x}} \frac{1}{2} \|\mathbf{H}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda\Psi(\mathbf{x}) \quad (2)$$

where $\frac{1}{2} \|\mathbf{H}\mathbf{x} - \mathbf{y}\|_2^2$ is l_2 data fidelity term, $\Psi(\mathbf{x})$ is called regularization term which denotes image prior and λ represents regularization parameter. To optimize the above regularization- based image inverse problems many approaches have been developed previously [1], [7].

Thus, image prior knowledge is an important character in the performance of image restoration algorithms, designing effective regularization terms to reproduce the image [1] prior is at the center of image restoration [1].

If we know that how the original image is converted into the degraded image then only we can able to apply the better algorithms and techniques on best of our knowledge about various techniques and the algorithms available for the image restoration for restoring the image. Also we can overcome some of the flaws present in the previous image restoration techniques and makes it work better for the image restoration task.

II. RELATED WORK

The conventional method to deal with image deblurring suffers from the problem of recovering discontinuities, choosing the model parameters and in addition problem of model validation. To overcome these problems constrained restoration is used which not only capable of recovering the discontinuities but also contribute in determining the parameters and validating the model [8]. For image denoising a constrained optimization based numerical type algorithm is presented in which constraints are set using Lagrange multipliers and the solution is obtained using the gradient-projection methodology [2], [30]. For the minimization of the overall variation in an image an algorithm is developed [9]. Image denoising, zooming, and the computation of the mean curvature motion of interfaces are the applications of this algorithm [9]. These methods use native structural patterns and are based on the supposition that images are locally

smooth except at edges [1], [15]. These regularization terms [1] shows high effectiveness in protecting edges and recovering smooth regions however they typically smear out image details and cannot deal well with fine structures [1].

Although sparse representation could be a poorly outlined problem and a normally impractical goal, nonetheless some mathematical results shows that if the conditions are certain one will get results of a positive nature, guaranteeing uniqueness, stability, or computational practicality [1], [29]. Problem of gathering and using over-complete dictionary is self-addressed by using K-SVD algorithm and it used to solve similar but constrained problem [10].

Thus, sparsity which is most vital property of natural images is extensively studied in past many years and also the regularization based on the sparsity has achieved great success in numerous image processing applications. A dictionary is a few elements from a basic set, which is used to represent each patch of an image and it is learned from natural images. The advantage of using learned dictionary is that it is better adapted to the images and thus it improves the sparsity which results in better performance [1], [15].

Nonlocal self-similarity is another vital property [1], [28] shows by the natural images, which shows the repetitiveness of higher level patterns (e.g., textures and structures) [1] entirely positioned in images [15]. Based on a non local averaging of all pixels in the image, a new non-local means algorithm is [17] presented whose performance is better than the previous image denoising algorithm. Taking motivation from NLM chain of nonlocal regularization terms for inverse problems makes use of nonlocal self-similarity property of natural images [1], [15]. Nonlocal regularization terms produce better results over the local ones, with sharper image edges and more image [1], [11] details because of the utilization of self-similarity prior by [1] adaptive nonlocal graph [1], [11]. Since, the weighted graph adopted by the nonlocal regularization terms unavoidably leads to disturbance and inaccuracy, due to the inaccurate weights [1], plenty of image details and structures that cannot be recovered accurately [1].

To reach higher performance, the sparsity and the self- similarity of natural images are [1] typically combined [15]. Two regularization terms are used to characterize the sparsity and the self similarity [12], [15], which is jointly included into the final cost of image restoration solution to enhance the [1] image quality. To ensure that similar patches should share the identical dictionary elements in their sparse disintegration [18], simultaneous sparse coding [1] is utilized which results in impressive denoising and demosaicking [1], [13]. A nonlocally centralized sparse representation (NCSR) model is proposed [1], [19], which obtain fine estimates of the sparse coding coefficients of the original image by the principle of NLM [20] in the domain of sparse coding coefficients. After this, it centralizes the sparse coding coefficients of the observed image to [1] the estimates [14] of original image.

Low-rank modeling based approaches has additionally attained grand success in image or video restoration [21]. To get rid of the faults in a video, defective pixels in the video are first detected and [1] tagged as missing [1], [21]. Similar patches are grouped such that the patches in each group share analogous fundamental structure and form a low-rank matrix approximately [21]. At last, the matrix completion is carried out on all patches to restore the image [1], [21]. A low-rank approach toward modeling nonlocal similarity [22] denoted by SAIST is proposed in [1], [2], which present a conceptually easy interpretation for concurrent sparse coding from a bilateral variance assessment viewpoint. It achieves very good performance as compared to some state-of-the-art methods [23].

In [1], the concept of group as the fundamental unit of sparse representation instead of using patch as the fundamental unit of sparse representation [24], [27], which results in a new sparse representation modeling of natural images, termed as group-based sparse representation (GSR) [24], [27] is presented.

III. METHODOLOGY

The main objective of our approach is to restore an image which is approximately similar to original image from a degraded image provide as the input. Here by using our approach we can deals with three image restoration problems that are, image inpainting, image deblurring, and image compressive sensing [38] (CS) recovery.

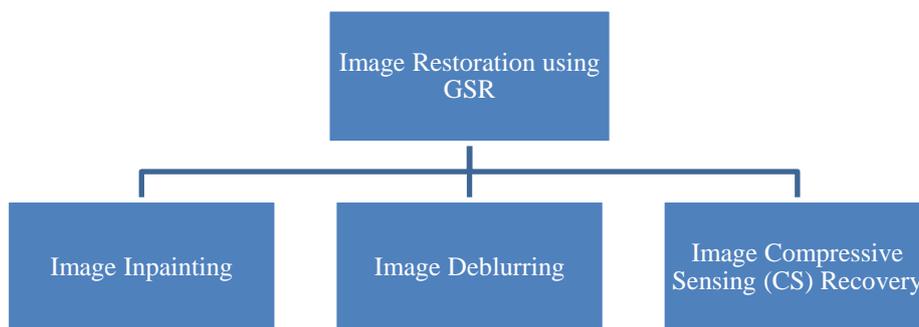


Fig. 1 Main modules of the work

To restore the images from various types of degraded images such as from inpainted, blurred, compressive sensed image we need to perform some operations on original image and degraded image as follows.

A. Operations on original images

On original image we have to perform three main operations namely group construction, GSR modelling and self-adaptive dictionary learning which are explained in detail in following subsections.

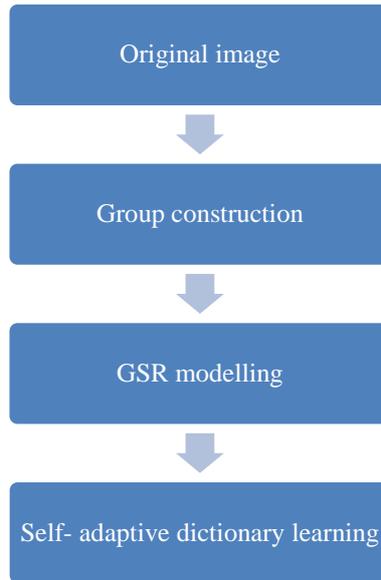


Fig. 2 Operations on original image

1. Group Construction

This subsection describes how to construct the group. The group construction is carried out by following the four steps described below:

Step 1: Divide the image x with size N into n overlapped patches of size $[1] \sqrt{B_s} \times \sqrt{B_s}$ and the vector $x_k \in \mathbb{R}^{B_s}$ is used to represent each patch where $k = 1, 2, \dots, n$.

Step 2: For each patch x_k , within the training window of size $L \times L$, search its c best matched patches which contain the set S_{x_k} . Euclidian distance is used to find out the similarity between the patches.

Step 3: All the patches in the set S_{x_k} into a matrix of size $B_s \times c$, denoted by $x_{G_k} \in \mathbb{R}^{B_s \times c}$, which includes every patch in S_{x_k} as its columns, $x_{G_k} = \{x_{G_k \otimes 1}, x_{G_k \otimes 2}, \dots, x_{G_k \otimes c}\}$. The matrix x_{G_k} is called as group. This can be represented by equation as

$$x_{G_k} = R_{G_k}(x) \quad (3)$$

where $R_{G_k}(\cdot)$ is the operator which is used to extract the group x_{G_k} from x . To put the group back into its k -th position in the reconstructed image padded with zeros elsewhere, the $[1]$ transpose of the operator $R_{G_k}^T(\cdot)$ is used. Thus one can recover whole image x from $\{x_{G_k}\}$.

$$x = \sum_{k=1}^n R_{G_k}^T(x_{G_k}) ./ \sum_{k=1}^n R_{G_k}^T(I_{B_s \times c}) \quad (4)$$

where $./$ represents element-wise division of the two vectors and $I_{B_s \times c}$ represents the matrix of size $B_s \times c$ with all element being 1.

2. Group-Based Sparse Representation Modeling

The GSR model assumes that by using few atoms of the self-adaptive learning dictionary D_{G_k} each group x_{G_k} can be represented accurately [1].

Thus, $D_{G_k} = \{D_{G_k \otimes 1}, D_{G_k \otimes 2}, \dots, D_{G_k \otimes m}\}$ is supposed to be known. D_{G_k} is of size $(B_s \times c) \times m$, that is, $x_{G_k} \in \mathbb{R}^{(B_s \times c) \times m}$.

The rationale behind the sparse coding process of every group is to seek sparse vector $\alpha_{G_k} = \{\alpha_{G_k \otimes 1}, \alpha_{G_k \otimes 2}, \dots, \alpha_{G_k \otimes m}\}$ such that $\alpha_{G_k} \approx \sum_{i=1}^m \alpha_{G_k \otimes i} D_{G_k \otimes i}$. For simplicity here $D_{G_k} \alpha_{G_k}$ is used to represents $\sum_{i=1}^m \alpha_{G_k \otimes i} D_{G_k \otimes i}$. After this, the whole image can be sparsely represented by the $\{\alpha_{G_k}\}$ that is the set of the sparse codes. Thus, x can be constructed from $\{\alpha_{G_k}\}$ which can be represented by

$$x = D_{G \circ} \alpha_G \stackrel{\text{def}}{=} \sum_{k=1}^n R_{G_k}^T(\alpha_{G_k}) ./ \sum_{k=1}^n R_{G_k}^T(I_{B_s \times c}) \quad (5)$$

where D_G denotes the concatenation of all α_{G_k} .

Thus, the GSR can be formulated as:

$$\hat{\alpha}_G = \underset{\alpha_G}{\text{argmin}} \frac{1}{2} \|HD_G \circ \alpha_G - y\|_2^2 + \lambda \|\alpha_G\|_0 \quad (6)$$

3. Self-Adaptive Group Dictionary Learning

While learning the dictionary for each group following points must be considered.

- 1) Computational cost must be minimized
- 2) The learnt dictionary must be adaptive for a group x_{G_k} , that is all the groups $\{x_{G_k}\}$ are represented by the same dictionary D_x .
- 3) It must consider the characteristics of each group x_{G_k} , containing the patches with similar patterns.

The adaptive dictionary D_{G_k} for each group is directly learnt from its estimate r_{G_k} which is naturally selected in the process of optimization. After obtaining r_{G_k} , apply K-SVD. This can be formulated as follows:

$$\mathbf{r}_{G_k} = \mathbf{U}_{G_k} \Sigma_{G_k} \mathbf{V}_{G_k}^T = \sum_{i=1}^m \gamma_{r_{G_k \otimes i}} (\mathbf{u}_{G_k \otimes i} \mathbf{v}_{G_k \otimes i}^T) \quad (7)$$

where $\gamma_{r_{G_k}} = [\gamma_{r_{G_k \otimes 1}}; \gamma_{r_{G_k \otimes 2}}; \dots; \gamma_{r_{G_k \otimes m}}]$ $\Sigma_{G_k} = \text{diag}(\gamma_{r_{G_k}})$ is a diagonal matrix with elements on its main diagonal $\mathbf{u}_{G_k \otimes i}$, $\mathbf{v}_{G_k \otimes i}$ are the columns of $\mathbf{U}_{G_k \otimes i}$ and $\mathbf{V}_{G_k \otimes i}$.

For the group \mathbf{x}_{G_k} , each atom in dictionary D_{G_k} is defined as,

$$\mathbf{d}_{G_k \otimes i} = \mathbf{u}_{G_k \otimes i} \mathbf{v}_{G_k \otimes i}^T, \quad i = 1, 2, \dots, m, \quad (8)$$

where $\mathbf{d}_{G_k} \in \mathbb{R}^{B_s \times c}$. Thus, the learned dictionary for the group \mathbf{x}_{G_k} is given by

$$\mathbf{D}_{G_k} = [\mathbf{D}_{G_k \otimes 1}, \mathbf{D}_{G_k \otimes 2}, \dots, \mathbf{D}_{G_k \otimes m}] \quad (9)$$

B. Operations on degraded image

Now for restoring the image which is approximately similar to original image from the degraded image we have to perform following operations on degraded image.

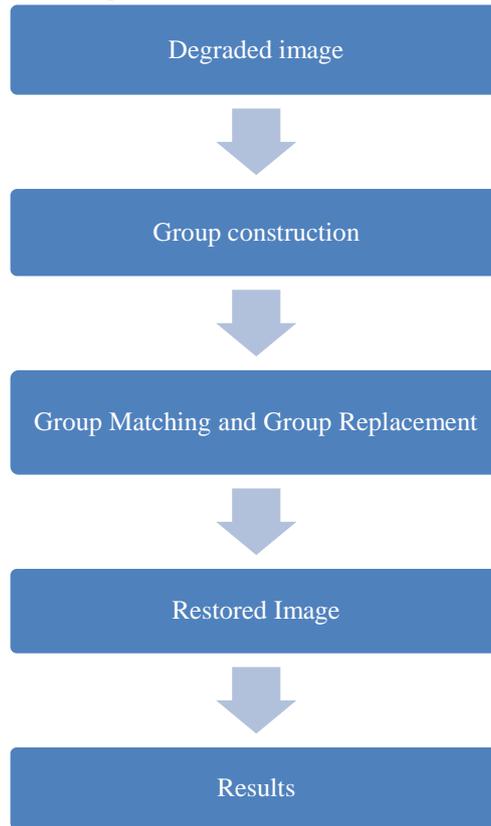


Fig. 3 Operations on degraded image

1. Group Construction

Similar procedure which we applied on original image for group construction is carried out on degraded images too and the groups are constructed.

2. Group Matching and Group Replacement

In group matching the constructed group of degraded image is compared and matched with the constructed group of original image. After the group is matched the group of degraded image is replaced with the group of original image.

3. Restored Image

After replacing all the groups of the degraded image we get the restored image which is approximately similar to the original image.

IV. SUMMARY OF GSR

The GSR can be summarised as follows. All the equations used in GSR are derived after applying the Split-Bregman Iteration (SBI) algorithm which is used for optimization of GSR.

Input: Degraded image \mathbf{y} and degradation operator \mathbf{H} .

Initialise $t = 0$, $\mathbf{b}^{(0)} = 0$, $\alpha_G^{(0)} = 0$, $\mathbf{u}^{(0)}$, B_s , c , λ , μ

Repeat

if H is a mask operator

Update $\mathbf{u}^{(t+1)}$ by $\hat{\mathbf{u}} = (\mathbf{H}^T \mathbf{H} + \mu \mathbf{I})^{-1} \mathbf{q}$

else if H is a blur operator

Update $\mathbf{u}^{(t+1)}$ by $\hat{\mathbf{u}} = (\mathbf{H}^T \mathbf{H} + \mu \mathbf{I})^{-1} \mathbf{q}$

else if H is random projection operator

Update $\mathbf{u}^{(t+1)}$ by $\hat{\mathbf{u}} = \mathbf{u} - \eta(\mathbf{H}^T \mathbf{H} \mathbf{u} - \mathbf{H}^T \mathbf{y} + \mu(\mathbf{u} - \mathbf{D}_{G_o} \alpha_G - \mathbf{b}))$

end if

$\mathbf{r}^{(t+1)} = \mathbf{u}^{(t+1)} - \mathbf{b}^{(t)}$; $\tau = (\lambda K) / (\mu N)$;

for Each group \mathbf{x}_{G_k}

Construct dictionary \mathbf{D}_{G_k} by computing $\mathbf{D}_{G_k} = [\mathbf{D}_{G_k \otimes 1}, \mathbf{D}_{G_k \otimes 2}, \dots, \mathbf{D}_{G_k \otimes m}]$

Reconstruct $\hat{\alpha}_{G_k}$ by computing $\hat{\alpha}_{G_k} = \mathbf{hard}(\gamma_{r_{G_k}}, \sqrt{2\tau}) = \gamma_{r_{G_k}} \odot \mathbf{1}(\mathbf{abs}(\gamma_{r_{G_k}}) - \sqrt{2\tau})$

end for

Update $\mathbf{D}_{G_k}^{(t+1)}$ by concatenating all \mathbf{D}_{G_k} ;

Update $\hat{\alpha}_{G_k}^{(t+1)}$ by concatenating all $\hat{\alpha}_{G_k}$;

Update $\mathbf{b}^{(t+1)}$ by computing $\mathbf{b}^{(t+1)} = \mathbf{b}^{(t)} - (\mathbf{u}^{(t+1)} - \mathbf{D}_{G_o} \alpha_G^{(t+1)})$;

$\mathbf{t} \leftarrow \mathbf{t} + \mathbf{1}$;

Until maximum iteration number is reached

Output: Final restored image $\hat{\mathbf{x}} = \mathbf{D}_G \circ \hat{\alpha}_G$

V. RESULTS AND DISCUSSIONS

To verify the performance of the GSR we have performed experiments on three types of image restoration problem which are image inpainting, image deblurring and image CS recovery. All the experiments are performed in Visual Studio 2010 on Dell Inspiron 1545 PC with Intel(R) Core(TM) 2 Duo CPU, 3GB RAM, and Windows 7 operating system.

We have used PSNR to evaluate the objective image quality in addition to this a powerful perceptual quality metric FSIM to evaluate the visual quality. Higher the PSNR means better the image quality and higher the FSIM means better visual quality. Since we have to compare the performance of GSR with previous algorithms we have worked on standard images only.

We have provided the results for image Barbara for all the three image restoration problems.

A. Image inpainting

There are two cases of inpainting image restoration from random samples and text removal. For image inpainting application, $\mu=0.0025$ and $\lambda=0.082$. The proposed GSR is compared with five recent representative methods for image inpainting: SKR (steering kernel regression) [3], NLTV [31], BPFA [32], HSR [33] and SAIST [22].

Following bar graphs shows the comparison of performance of GSR with five recent representative methods for image inpainting.

a. Random Samples

1) Color image Barbara with only 20% random samples

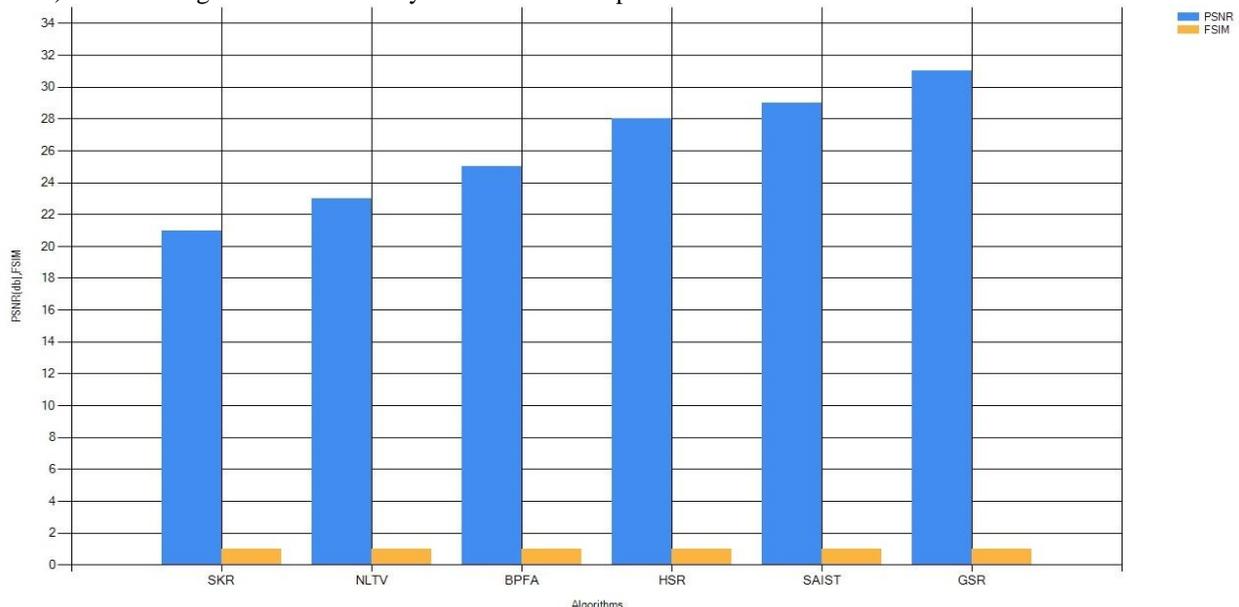


Fig. 4 Comparison of performance of GSR for color image Barbara in case of random samples

b. Text Removal
 1) Color image Barbara

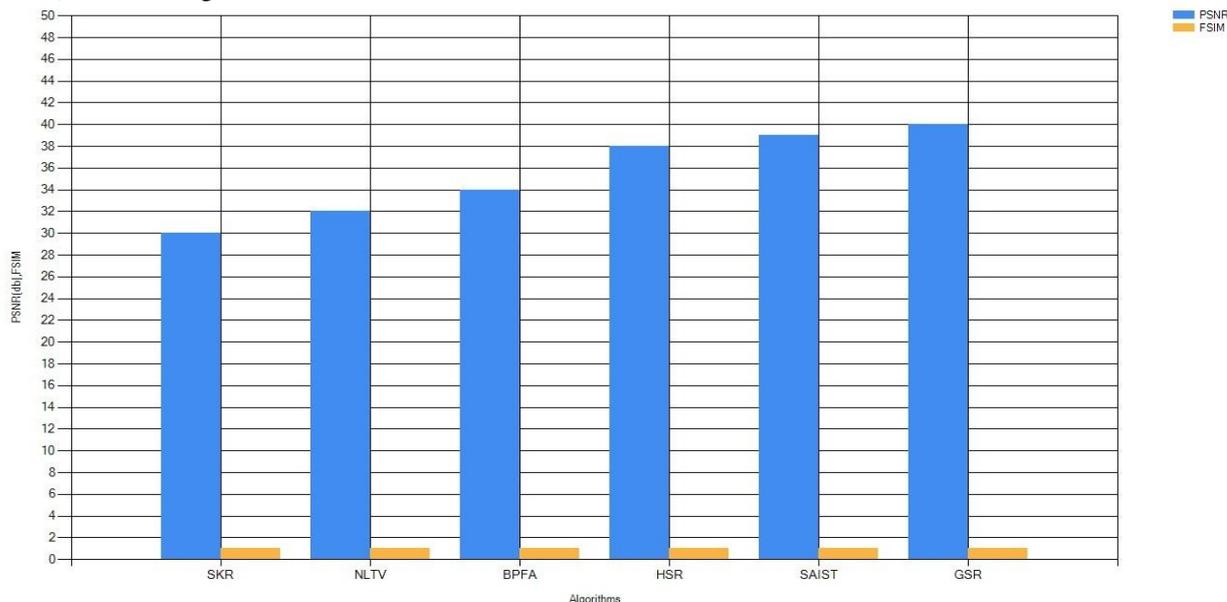


Fig. 5 Comparison of performance of GSR for color image Barbara in case of Text Removal

B. Image Deblurring

Experiments on blurred images is carried out and the results of our GSR is compared with recently developed deblurring approaches such as TVMM [2], L0_ABS [34], NCSR [14], and IDDBM3D [35]. Also the GSR is able to remove mixed and impulse noise which is our one of the contribution. For removing the mixed Gaussian and impulse noise we have applied Wiener filter and median filter respectively.

Following bar graphs shows the comparison of performance of GSR with recently developed deblurring approaches.

1) Gray Image Barbara: noisy and blurred image (uniform kernel: $9 \times 9, \sigma = \sqrt{2}$)

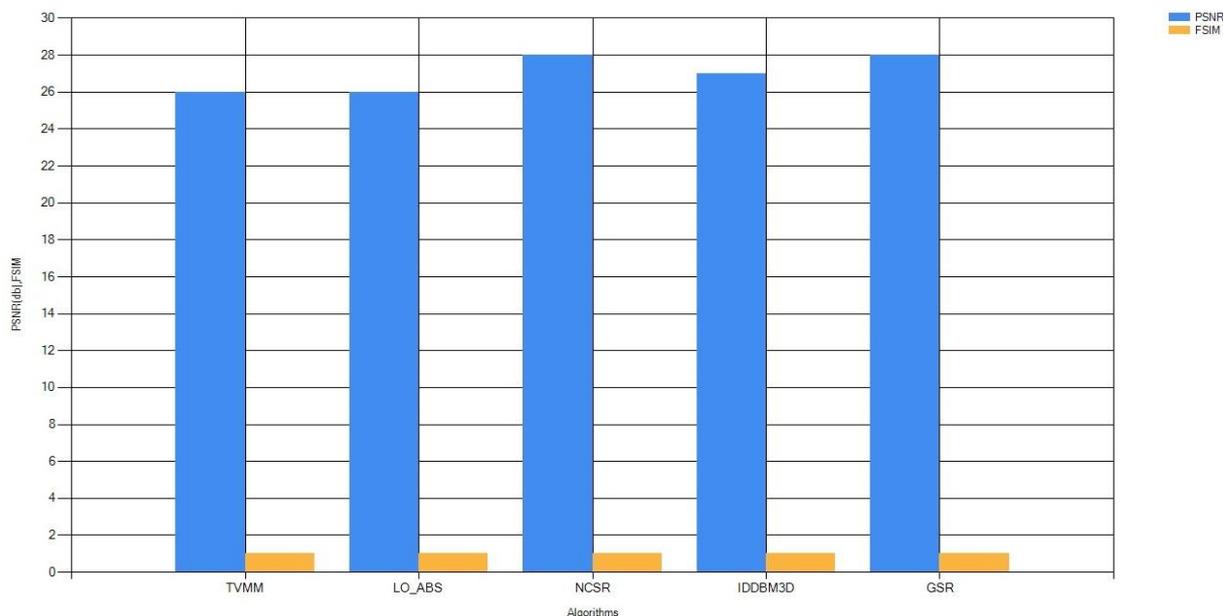


Fig. 6 Comparison of performance of GSR for gray image Barbara in case of image deblurring

C. Image Compressive Sensing Recovery

In this subsection, the CS measurements are obtained by applying a Gaussian random projection matrix to the original image signal at block level. GSR is compared with four representative CS recovery methods in literature, i.e., wavelet method (DWT), total variation (TV) method [36], multi-hypothesis (MH) method [37], collaborative sparsity (CoS) method [6], which deal with image signals in the wavelet domain, the gradient domain, the random projection residual domain, and the hybrid space-transform domain, respectively. MH and CoS are known as the current state-of-the-art algorithms for image CS recovery.

Following bar graph shows the comparison of performance of GSR with four representative CS recovery methods.

1) Gray image Barbara in the case of ratio = 20%

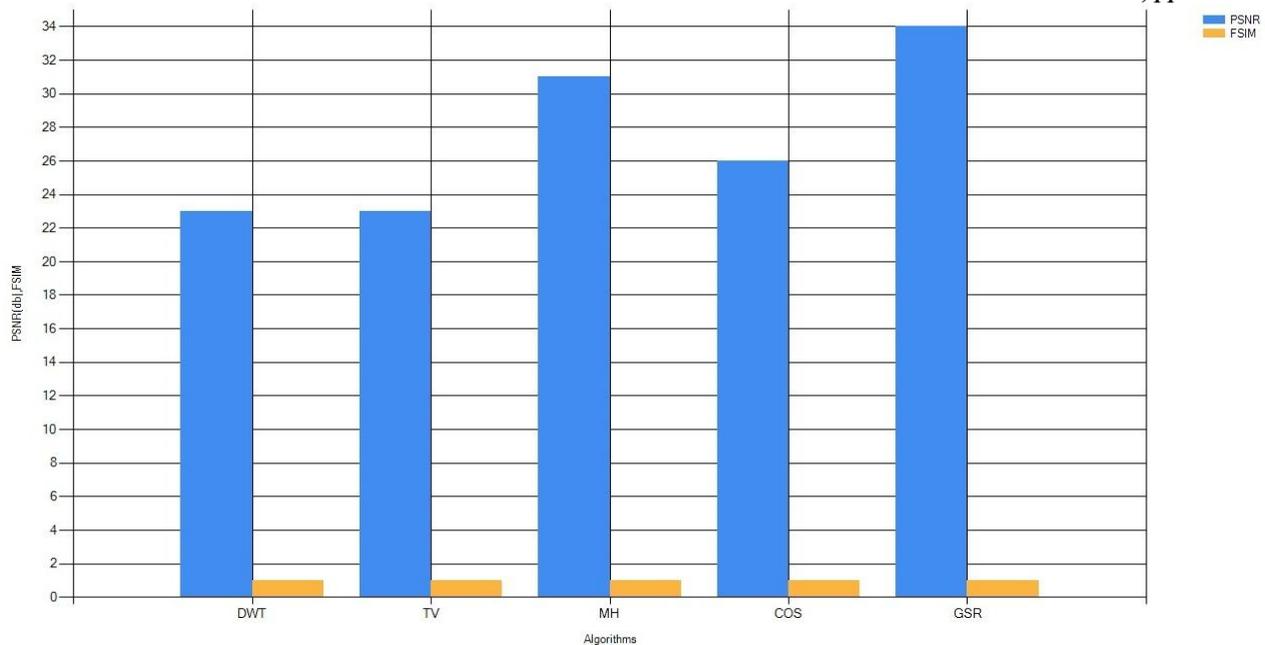


Fig. 7 Comparison of performance of GSR for gray image Barbara in case of image compressive sensing recovery

VI. CONCLUSION

We have presented an approach for image restoration technique which uses group as the fundamental unit of sparse representation. Ignorance of the relationships between similar patches, such as self-similarity which is one of the problems associated with the previous patch-based sparse representation modelling is overcome by using this concept. Large scale optimization which is another problem associated with previous patch-based sparse representation modelling is also overcome by using a GSR-driven ℓ_0 -minimization approach which makes use of Split Bregman Iteration (SBI) algorithm. SBI algorithm makes GSR robust and tractable. Also our GSR is able to remove mixed Gaussian and impulse noise from the blurred images. On the whole, the GSR makes image restoration task easier by overcoming the problems associated with the previous approach. Further research work can include video restoration and other possible applications.

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