



Strong Conflict-Free Coloring with Respect to Intervals and Circular Intervals

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Abstract -- Given a hypergraph $H = (V, E)$, a coloring of its vertices is said to be conflict-free if for every hyperedge $S \in E$ there is at least one vertex in S whose color is distinct from the colors of all other vertices in S . More generally if every hyperedge has k -distinct color then the coloring is called k -strong conflict-free coloring.

In this paper, we present a polynomial k -strong conflict-free coloring algorithm for set of discrete intervals (referred as hyperedge denoted by H_n) and also present a polynomial algorithm for a special case where we take only some of the subsets of H_n such that each subset have at least k unique color. Our algorithm uses at most $2k(\log n^2) + 1$ color in worst condition.

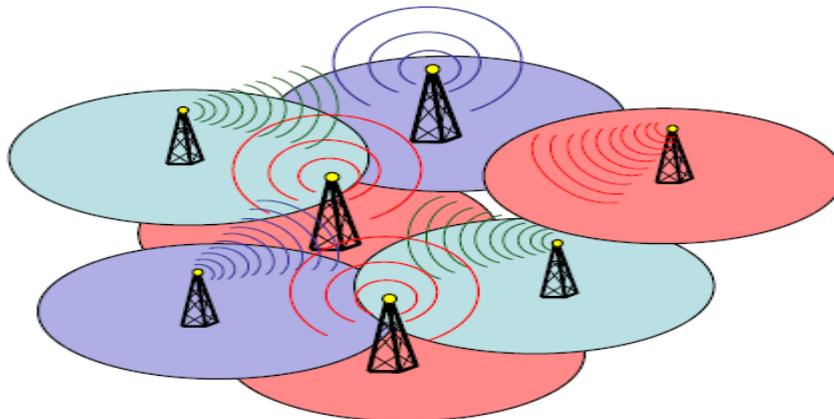
Keywords -- Frequency-assignment problem, Cellular network, Strong Conflict-free coloring, Intervals, Capacity of base stations.

I. INTRODUCTION AND PRELIMINARIES

A hypergraph is a pair (V, E) where V is a set and E is a collection of subsets of V . The elements of V are called vertices and the elements of E are called hyperedges. If for any $e \in E$, $|e| = 2$, then the pair (V, E) is a simple graph. For a subset $V' \subset V$, we call the hypergraph $H(V') = (V', \{S \cap V' \mid S \in E\})$ the sub-hypergraph induced by V' . An m -coloring for some $m \in \mathbb{N}$ of (the vertices of) H is a function $\phi: V \rightarrow \{1, \dots, m\}$. Let ϕ be an m -coloring of H , if for any $e \in E$ with $|e| \geq 2$, there exists at least two vertices $x, y \in e$ such that $\phi(x) \neq \phi(y)$, we call ϕ proper or non-monochromatic. Let $\chi(H)$ denote the least integer m for which H admits a proper coloring with m colors. The following coloring is more restrictive than non-monochromatic coloring.

Definition 1 (CF Coloring): A conflict free vertex coloring of a hypergraph $H = (V, E)$ is a function $C: V \rightarrow \mathbb{N}$ such that for each $e \in E$ there exists a vertex $v \in e$ such that $C(u) \neq C(v)$ for any $u \in e$ with $u \neq v$.

The Conflict-Free coloring problem was introduced by [14] for modeling the frequency allocation problem that arises in wireless communication. In the frequency allocation problem, servers (base stations) and clients (mobile devices) are connected by radio links. To establish a communication between them, a mobile continuously scans the available frequencies in search for a base station with good reception. To avoid any interference, the mobile device needs to choose a specific frequency among all available frequencies such that only one of the reachable base stations is assigned with that frequency. A better solution is to assign each base station a different frequency so that there is no conflict in frequencies. Since the spectrum is limited and costly, this is not a solution. The target is to minimize the total number of frequencies assigned to the base stations. So that for any mobile device, among the frequencies assigned to all its reachable base stations, there is always a frequency assigned to exactly one of these stations.



Traditionally, this type of problem was considered as an application of traditional graph coloring problem. We can consider the base stations as the nodes of a graph and two nodes are connected by an edge if the ranges of their associated base stations intersect. If each node (base-station) is assigned a color (frequency) such that no two adjacent nodes are

colored with the same color, certainly every client (mobile device) can be served by some base station without interference. However in [14], it was shown that this is not the right way to tackle the problem since even an optimal coloring may use large number of colors. Instead of this, we have to color the base station's regions in such a way that each point in the plane is covered only once by at least one color. If the sending areas of all base stations (servers) are disks of the same radius, it was shown by Even that $\log(n)$ colors (where n is the number of base stations) always suffice to find **conflict-free** coloring. However, there are configurations where the algorithm of [14] uses $\log(n)$ colors although a small constant of colors would be enough.

The notion of k -Strong CF coloring (k -SCF coloring), first introduced in [1], extends that of CF-coloring. A k -SCF coloring is a coloring that remains conflict-free after an arbitrary collection of $k - 1$ vertex is deleted from the set. Thus a k -SCF coloring for $k = 1$ is simply a standard CF coloring.

Definition 2 (k-SCF Coloring): Let $H = (V, E)$ be a hypergraph and k be a positive integer. A coloring $C: V \rightarrow N$ is called a k -Strong Conflict-Free Coloring if for any $e \in E$

- if $|e| \leq k$ then $C(u) \neq C(v)$ for each $u, v \in e$ with $u \neq v$;
- if $|e| > k$ then at least k colors are unique in e , namely there exists $c_1, c_2, \dots, c_k \in N$ such that $|\{v \in e, C(v) = c_i\}| = 1$, for $i = 1, \dots, k$.

The goal is to minimize the size of the range of the k -SCF coloring function C . We denote by $\chi_k(H)$ the smallest number of colors in any possible k -SCF coloring of H .

In the context of cellular networks, this can be viewed as ensuring that for any client in an area covered by at least k base stations, there always exists at least k different frequencies with which the client can communicate without interference. This can be used both to serve up to k clients at the same location as well as to deal with malfunctioning of some base stations.

In section 2, we describe more general definitions of conflict-free colorings and understand the concept of strong conflict free coloring of points for intervals. In section 3, we present some previous work on this field by many researchers specially Luisa Gargano and Adele A. Rescigno. In section 4, we present an algorithm that color the discrete interval hypergraph H_n and show that the algorithm uses at most $2k(\log n^2) + 1$ color in worst case. Then in section 5, we work on the concept of conflict-free coloring with respect to a subset of intervals i.e. subhypergraph of the discrete interval hypergraph and present an algorithm that color the subhypergraph of the discrete interval hypergraph and show that the analysis is tight, i.e., there are subhypergraphs of H_n for which the algorithm computes a conflict-free coloring with twice the optimal (minimum) number of colors. In section 6, we show an example to implement the above algorithms and prove that the algorithm works well & produce valid conflict-free coloring for a given complete hypergraph H_n and subhypergraph of H_n . Finally in section 7, we present some open problems for future work.

II. STRONG CONFLICT-FREE COLORING OF POINTS WITH RESPECT TO INTERVALS

Let P be a set of n points in the plane and let R be a family of regions in the plane (e.g., all closed discs). We denote by $H = H_R(P)$ the hypergraph on the set P whose hyperedges are all subsets P' that can be cut off from P by a region in R . That is, all subsets P' such that there exists some region $r \in R$ with $r \cap P = P'$. We refer to such a hypergraph as the hypergraph induced by P with respect to R .

Now, consider the hypergraph induced by a set of n collinear points with respect to the family of closed disks in the plane. It is not difficult to see that this hypergraph is isomorphic to the hypergraph induced by a set of n real numbers with respect to the family of closed intervals, which is also isomorphic to the following discrete interval hypergraph.

Definition 3: Let $[n] = \{1, \dots, n\}$. For $s \leq t, s, t \in [n]$, we define the (discrete) interval $[s, t] = \{i \in [n] \mid s \leq i \leq t\}$. The discrete interval hypergraph H_n has vertex set $[n]$ and hyperedge set $I_n = \{[s, t] \mid s \leq t, s, t \in [n]\}$.

Several authors focused on this special case of CF colorings of the family of all intervals on a line with n points; here an interval is intended as intersecting at least one point on the line and two intervals are considered equivalent if they contain the same points. Hence, the problem can be modeled as the CF coloring of the hypergraph

$H_n = (V, I)$ with $V = \{1, \dots, n\}$ and $I = \{\{i, i + 1, \dots, j\} \mid 1 \leq i \leq j \leq n\}$, where each interval is represented by the set of consecutive points it contains.

Conflict-free coloring for intervals models the assignment of frequencies in a chain of unit disks; this arises in approximately unidimensional networks as in the case of agents moving along a road. Moreover, it is important because it plays a role in the study of conflict-free coloring for more complicated cases, as for example in the general case of CF coloring of unit disks [14, 19]. An example of conflict free coloring for intervals is given in Figure 1.

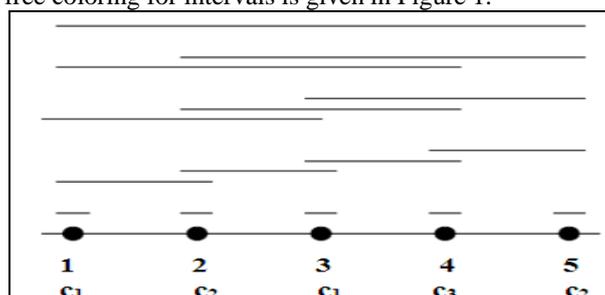


Figure 1: The hypergraph H_5 representing all the intervals on a line with 5 points.

A 1-SCF coloring for H_5 is: $C(1) = C(3) = c_1, C(2) = C(5) = c_2, C(4) = c_3$.

While some papers require the conflict-free property for all possible intervals on the line, in many applications good reception is needed only in some locations, in such a case it is necessary to supply only a given subset of the cells of the arrangement of the disks [17]. Indeed, in the context of channel assignment for broadcasting in wireless mesh network, it can occur that, at some step of the broadcasting process, sparse receivers of the broadcasted message are within the transmission range of a linear sequence of transmitters. In this case only part of the cells of the linear arrangements of disks representing the transmitters are involved [20, 24].

In this paper we consider the k-strong conflict-free coloring of points with respect to an arbitrary family of intervals. Hence, throughout the rest of the paper, we consider hypergraphs

$H = (V, I)$ with $V = \{1, \dots, n\}$ and $I \subseteq \{\{i, i+1, \dots, j\} \mid 1 \leq i \leq j \leq n\}$.

We shall refer to the above as interval hypergraphs and to H_n as the complete interval hypergraph.

Figure 2 shows a 1-SCF coloring of the interval hypergraphs H and H' on 6 points. It is not hard to see that any 1-SCF coloring for H needs at least 3 colors, while 2 colors suffice for H'

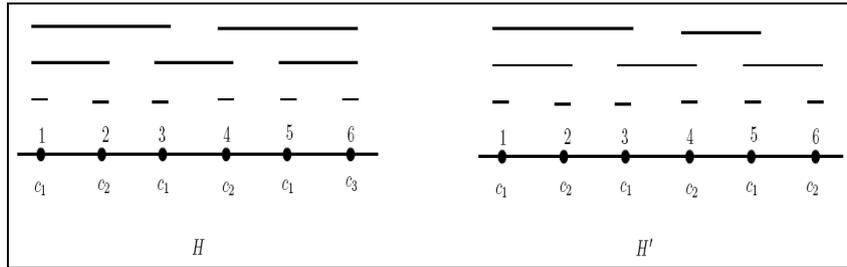


Figure 2: The hypergraphs H and H' on $V = \{1, \dots, 6\}$ with their CF-colorings.

III. PREVIOUS RELATED WORK

Conflict-free colorings have attracted many researchers from the computer science community as well as from the mathematics community [2], [3], [4], [12], [14], [15], [16], [19], [21], [22] due to both its practical motivations and its theoretical interest; a survey of results in the area is given in [23].

Conflict-free coloring in the case of the complete interval hypergraph has been first studied by Even et al. in [14]; they gave an algorithm showing that the problem can be optimally solved by using $\lceil \log n \rceil + 1$ colors.

The problem of CF-coloring the points of a line with respect to an arbitrary family of intervals is studied in [17], [18], [8], [9]. In particular, [8] and [9] prove a 2-approximation algorithm for the problem. It is not known whether the CF-coloring problem is NP-hard for interval hypergraphs.

The k-SCF Coloring problem was first considered in [1] and has since then been studied in various papers under different scenarios. Recently, the minimum number of colors needed for k-SCF coloring the complete interval hypergraph H_n has been studied in [13], where the exact number of needed colors for $k = 2$ and $k = 3$ has been determined. Moreover, Horev. et al. shows that H_n admits a k-SCF coloring with $k \log n$ colors, for any k ; this results follows as a specific case of a more general framework [16].

Luisa Gargano and Adele A. Rescigno present a polynomial algorithm for the general problem; the algorithm has an approximation factor $5 - \frac{2}{k}$ when $k \geq 2$ and approximation factor 2 for $k = 1$. In the special case the family contains all the possible intervals on the given set of points; they show that a 2 approximation algorithm exists, for any $k \geq 1$.

They present the following algorithm to find conflict-free coloring with respect to a subset of Intervals:

```

k-COLOR( $\mathcal{I}$ )
Set  $t = 1$ .
 $\mathcal{I}_1 = \mathcal{I}$ . [ $\mathcal{I}_t$  represents the set of intervals still to be k-colored at the beginning of step  $t$ ]
 $\mathcal{X}_1 = \emptyset$ . [ $\mathcal{X}_t \subset \mathcal{I}_t$  contains the intervals that become k-colored during step  $t$ ]
while  $\mathcal{I}_t \neq \emptyset$ 
  Execute the following step  $t$ 
  1. Let  $P_t = \{p_0, p_1, \dots, p_{n_t}\}$  be the set of points returned by SELECT( $\mathcal{I}_t$ )
  2. for  $i = 0$  to  $n_t$ 
     Assign to  $p_i$  color  $c_i = (t - 1)c(k) + (i \bmod c(k)) + 1$ 
  3. for each  $I \in \mathcal{I}_t$ 
     if  $I$  is k-colored then  $\mathcal{X}_t = \mathcal{X}_t \cup \{I\}$ 
  4.  $\mathcal{I}_{t+1} = \mathcal{I}_t \setminus \mathcal{X}_t$ 
  5.  $t = t + 1$ 

SELECT( $\mathcal{I}_t$ )
Set  $P_t = \emptyset$ . [ $P_t$  represents the set of selected points at step  $t$ ]
for each  $I \in \mathcal{I}_t$  by increasing order according to relation  $\prec$ 
  if  $|I \cap P_t| < \min\{|I|, k\}$  then
  1. Let  $P_t(I)$  be the set containing the largest  $\min\{|I|, k\} - |I \cap P_t|$  points of  $I \setminus P_t$ 
  2.  $P_t = P_t \cup P_t(I)$ 
Return  $P_t$ 
    
```

Figure 3: The k-SCF coloring algorithm for H .

Example. Consider $H = (V, I)$, where $V = \{1, 2, \dots, 23\}$ and I is the set of 13 intervals given in Fig. 4. Run k -COLOR(I) with $k = 2$; hence $c(2) = 4$ colors are used at each iteration. Initially, $I_1 = I$ and SELECT(I_1) returns $P_1 = \{3, 4, 7, 8, 9, 11, 12, 14, 15, 17, 18, 19, 20, 22, 23\}$ whose points are colored with c_1, c_2, c_3, c_4 in cyclic sequence. Only 3 intervals remain in I_2 ; all the others are in X_1 , being 2-colored at the end of step 1. SELECT(I_2) returns $P_2 = \{14, 15, 23\}$ and these points are colored with c_5, c_6, c_7 . Now $I_3 = I_2 \setminus X_2 = \emptyset$ and the algorithm ends.

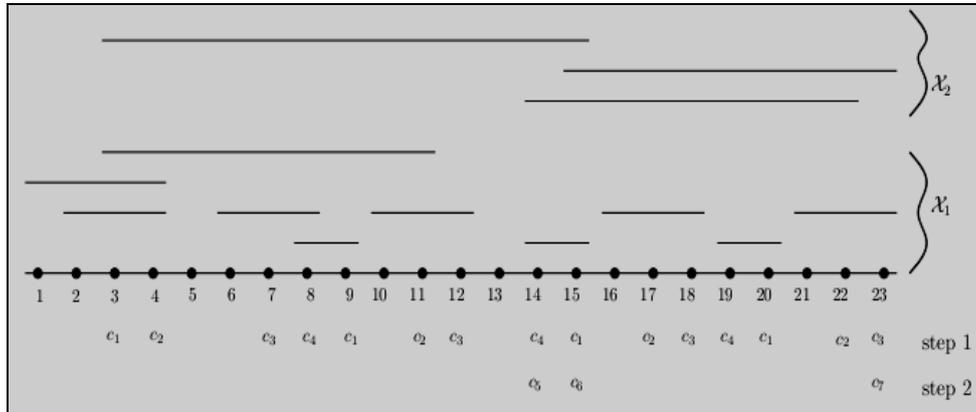


Figure 4: Colors assigned by k -COLOR when $k = 2$.

IV. K -STRONG CONFLICT-FREE COLORING ALGORITHM FOR INTERVALS:

Strong Conflict-free coloring is very important because

1. There may be a situation when there are large number of people connect to just one tower (i.e. Uniquely Colored Base-Station).
2. Capacity of a tower is low i.e. maximum number of users connected to that base-station is very few.
3. Uninterrupted service i.e. once a user is connected to some base station; it uses uninterrupted service until the user not moves anywhere.

In this section, we present an algorithm for conflict-free coloring a hypergraph. This algorithm colors the vertices in various levels depends on set of all intervals I . In first level, we start coloring of those intervals which contains maximum vertices. Then in next level, we color those intervals which contain some fewer vertexes than in previous level. We repeat these steps until all the intervals are considered. It is based on repeatedly computing a fully covered interval set in hypergraphs.

Definition 4:

We can define I as a subsets of intervals i.e. $I = \{I_2, I_3, I_4, I_5, \dots, I_n\}$

where

I_2 contains all set of two vertices only

I_3 contains all set of three vertices only

.

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.

I_n contains all set of n (maximum) vertices.

Definition 5 (Previously colored vertices): Previously colored vertices are those vertices which got color by the algorithm some time back. We denote the color of that vertex PC_c by c .

In the below algorithm we have to type of vertices (i) Current vertex yet to be colored (ii) Current visited vertex which was colored previously

Algorithm for Strong Conflict-free coloring with respect to intervals

```

Algorithm: StrongCFCforInterval
Ci = color of current vertex with i color
PCi = color of previously visited (i.e. colored) vertex with i color
Next-j-PCi = color of next jth vertex from the current visit vertex i color
Step 1: Initialize
    n = Total number of vertices to be colored
    k = Number of unique colors required in each intervals
    j = 1, x = k + 1, z = 1;
Step 2: Color first k vertices with color C1 to Ck;
Step 3: Color next vertex with color Cx, x++;
Step 4: If Next-j-PCz = 1 then j = j + 1
        Color next j number of vertices with Color PCz, PCz++;
Step 5: Repeat steps 3 & 4 until all n vertices are colored
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```

V. K-STRONG CONFLICT-FREE COLORING ALGORITHM FOR SUBSETS OF INTERVALS

In the literature, a conflict-free coloring is an assignment of colors (positive integers) to the vertices of the hypergraph. In this work, we introduce and consider a slight variation of conflict-free coloring, in which we allow some vertices to not be assigned colors, as long as in every hyperedge, there exists a vertex with assigned color that is uniquely occurring in the hyperedge. We can use a special color '0' given to vertices that are not assigned any positive color.

Remark 1: We claim that this variation of conflict-free coloring, with the partial coloring function or the placeholder color '0', is interesting from the point of view of applications. As mentioned in section 1, vertices model base stations in a cellular network. A vertex with no positive color assigned to it can model a situation where a base station is not activated at all, and therefore the base station does not consume energy. One can also think of a bi-criteria optimization problem where a conflict-free assignment of frequencies has to be found with small number of frequencies (in order to conserve the frequency spectrum) and few activated base stations (in order to conserve energy).

Studying conflict-free coloring for subhypergraphs of geometric hypergraphs can be justified by applications where only a given subset of the hyperedge set is required to have the conflict-free property. It is also helpful to save energy as well as money.

Algorithm: CFCforSubInterval

```

Step 1: Initialize
    I ← Ii = {In, In-1, ..., I3, I2}
    k = number of unique color required in any interval;
    n = number of vertices in largest interval;

Step 2: Take first interval from In;
    Check number of uncolored vertices in In, let z;
    If z > 0
    {
    Color that interval using Algorithm StrongCFCforInterval
    with colors not used in In in such a way that a color (if
    required) is not repeated on at least k + 1 position;
    }
    else
    {
    goto step 3;
    }

Step 3: If In = φ
    {
    n = n - 1;
    goto step 4;
    }
    Else
    {
    Take another interval from In;
    Check number of uncolored vertices in In, let z;
    If z > 0
    {
    Color that interval using Algorithm StrongCFCforInterval
    with colors not used in In in such a way that a color is
    not repeated (if required) on at least k + 1 position
    }
    Else
    {
    goto step 3;
    }
    }
    Remove the visited interval from In;
    n = n - 1;

Step 4: If I is not empty
    {
    goto step 2;
    }
    Else
    {
    Color remaining uncolored vertices (if any) with color 0;
    }
    END;

```

Lemma 1: Algorithm CFCforSubInterval terminates.

Proof: At every iteration of the loop, there is some hyperedge $e \in E^\ell$ for which $|e \cap S^\ell| = 1$. This follows from the minimality of S^ℓ . Thus, $|E^\ell| > |E^{\ell+1}|$. Therefore, the number of hyperedges decreases at every iteration of the loop, and necessarily reaches zero after a finite number of iterations of the loop.

Lemma 2: Algorithm 1 produces a conflict-free coloring.

Proof: We first show that for every hyperedge $e \in E$, there is some ℓ for which $|e \cap S^\ell| = 1$. Notice that for every iteration $i > 0$, we have $S^{i-1} \supseteq S^i$. If $|e \cap S^0| > 1$, consider the maximum i for which $|e \cap S^i| > 1$. Then, hyperedge $e \cap S^i = e \cap V^{i+1}$ belongs to E^{i+1} and has to be hit by S^{i+1} , i.e., $(e \cap S^i) \cap S^{i+1} = e \cap S^{i+1}$ is non-empty and thus $|e \cap S^{i+1}| = 1$, because of the maximality of i .

Let v be the one element of $e \cap S^l$. Vertex v is colored with some color greater than l by the algorithm and all other vertices of e are colored with colors which are at most of value l . Thus, e has the conflict-free property.

VI. AN ILLUSTRATED EXAMPLE

1. Example of Strong Conflict-Free Coloring for Intervals

Step 1: Let $k = 2$ and $n = 10, j = 1, x = 3, z = 1$;
Step 2: Color first 2 vertices with color 1 to 2, we get 12
Step 3: Color 3rd vertex with color 3, $x = 4$ and we get 123
Step 4: $\text{next-1-PC}_z = 2 \neq 1$, Color next 1 number of vertices with color 1,
 $\text{PC}_z = 2$ and we get 1231
Step 5: Repeat step 3 & 4

Step 3: Color 5th vertex with color 4, $x = 5$ and we get 12314
Step 4: $\text{next-1-PC}_z = 3 \neq 1$, Color next 1 number of vertices with color 2,
 $\text{PC}_z = 3$ and we get 123142
Step 5: Repeat step 3 & 4

Step 3: Color 7th vertex with color 5, $x = 6$ and we get 1231425
Step 4: $\text{next-1-PC}_z = 1$, Color next 2 numbers of vertices with color 31,
 $\text{PC}_z = 4$ and we get 123142531
Step 5: Repeat step 3 & 4

Step 3: Color 10th vertex with color 6, $x = 7$ and we get 1231425316

Step 6: END;

Another Example:

Step 1: Let $k = 3$ and $n = 17, j = 1, x = k + 1 = 4, z = 1$;
Step 2: Color first 3 vertices with color 1 to 3, we get 123
Step 3: Color 4th vertex with color 4, $x = 5$ and we get 1234
Step 4: $\text{next-1-PC}_z = 2 \neq 1$, Color next 1 number of vertices with color 1,
 $\text{PC}_z = 2$ and we get 12341
Step 5: Repeat step 3 & 4

Step 3: Color 6th vertex with color 5, $x = 6$ and we get 123415
Step 4: $\text{next-1-PC}_z = 3 \neq 1$, Color next 1 number of vertices with color 2,
 $\text{PC}_z = 3$ and we get 1234152
Step 5: Repeat step 3 & 4

Step 3: Color 8th vertex with color 6, $x = 7$ and we get 12341526
Step 4: $\text{next-1-PC}_z = 4 \neq 1$, Color next 1 number of vertices with color 3,
 $\text{PC}_z = 4$ and we get 123415263
Step 5: Repeat step 3 & 4

Step 3: Color 10th vertex with color 7, $x = 8$ and we get 1234152637
Step 4: $\text{next-2-PC}_z = 1$, Color next 2 numbers of vertices with color 41,
 $\text{PC}_z = 5$ and we get 123415263741
Step 5: Repeat step 3 & 4

Step 3: Color 13th vertex with color 8, $x = 9$ and we get 1234152637418
Step 4: $\text{next-2-PC}_z = 6$, Color next 2 numbers of vertices with color 52,
 $\text{PC}_z = 6$ and we get 123415263741852
Step 5: Repeat step 3 & 4

Step 3: Color 16th vertex with color 9, $x = 10$ and we get 1234152637418529
Step 4: $\text{next-2-PC}_z = 7$, Color next 1 number of vertices with color 6,
 $\text{PC}_z = 7$ and we get 12341526374185297

Step 6: END;

Our algorithm give color to the new vertex based on the previously (last) colored vertex.

2. Example of Strong Conflict-Free Coloring for Subset of Intervals

We describe algorithm for conflict-free coloring any hypergraph $H = (V, E)$ that have various level.

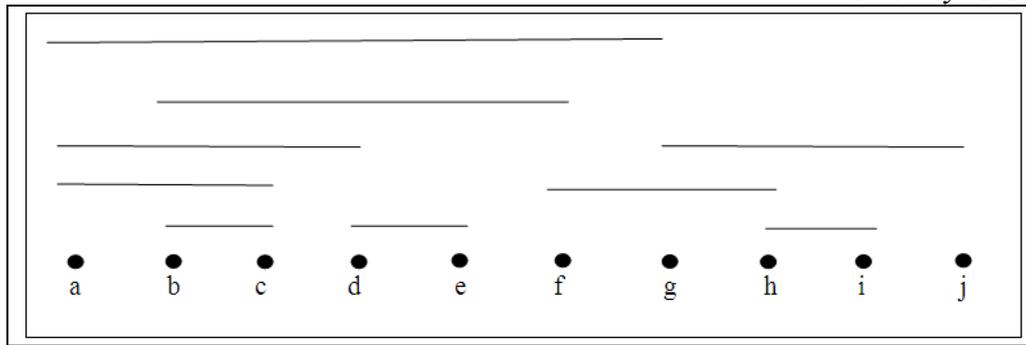


Figure 5: An input hypergraph having various subset of interval

Initially all vertices are colored with “0” color

Step 1:

$I_1 \sqsubseteq \{I_2, I_3, I_4, I_5, I_6, I_7\}$, $\max \sqsubseteq 2$

$I_7 = \{a, b, c, d, e, f, g\}$

$I_6 = \{\emptyset\}$

$I_5 = \{(b, c, d, e, f)\}$

$I_4 = \{(a, b, c, d) (g, h, i, j)\}$

$I_3 = \{(a, b, c) (f, g, h)\}$

$I_2 = \{(b, c) (d, e) (h, i)\}$

$k = 2$

Step 2:

Take I_7

Here uncolored vertices $z = 7$

$a \sqsubseteq 1, b \sqsubseteq 2, c \sqsubseteq 3, d \sqsubseteq 1, e \sqsubseteq 4, f \sqsubseteq 2, g \sqsubseteq 5$

i.e. $\chi = 1231425$

Step 2:

Take $I_6 = \emptyset$;

Step 2:

Take I_5

Here uncolored vertices $z = 0$;

Step 2:

Take I_4

Here uncolored vertices $z = 0$;

Step 3:

Take another I_4

Here uncolored vertices $z = 3$;

unused colors are = 1, 2, 3, 4

$h \sqsubseteq 1, i \sqsubseteq 3, j \sqsubseteq 2$

Step 2:

Take I_3

Here uncolored vertices $z = 0$;

Step 3:

Take another I_3

Here uncolored vertices $z = 0$;

Step 2:

Take I_3

Here uncolored vertices $z = 0$;

Step 3:

Take another I_3

Here uncolored vertices $z = 0$;

Step 3:

Take another I_3
Here uncolored vertices $z = 0$;

Step 4:

Here I_i is empty
End;

Final Coloring of the vertices is:

$a \square 1, b \square 2, c \square 3, d \square 1, e \square 4, f \square 2, g \square 5, h \square 1, i \square 3, j \square 2$

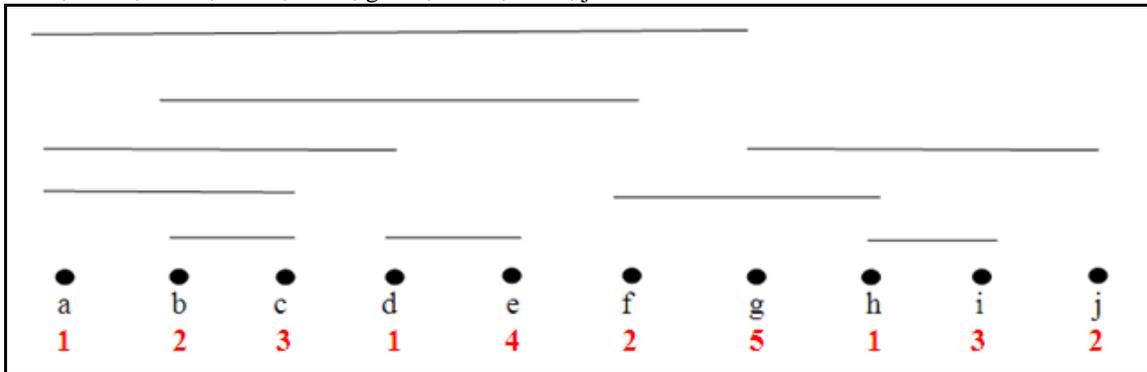


Figure 6: An output hypergraph having various subset of Strong CF-Colored interval

VII. DISCUSSION AND OPEN PROBLEMS

Several problems remain open in the area of SCF coloring of interval hypergraphs. Mainly the complexity of the CF-coloring problem has not been assessed. Is it possible to improve the approximation factor for SCF-coloring algorithms? In case of the complete interval hypergraph H_n , it would be interesting to close the gap between the upper and lower bounds. The exact complexity of computing an optimal cf-coloring for a subhypergraph of the discrete interval hypergraph remains an open problem. One can try to improve the approximation ratio, find a polynomial time approximation scheme, or even find a polynomial time exact algorithm. It would also be interesting to study the complexity of computing optimal conflict-free colorings for subhypergraphs of other geometric hypergraphs, like the hypergraph induced by a set of n points in the plane with respect to a given set of closed disks in the plane.

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