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Design of PI/PID Controller for FOPDT System

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Abstract— This paper presents a simple but effective method for designing robust PI or PID controller. The robust PI/PID controller designproblem is solved by the maximization, on a finite interval, of the shortest distance from the Nyquist curve of the open loop transferfunction to the critical point -1 ***i.e. from the knowledge of maximum sensitivity*** M_s ***. Simple formulae are derived to tune/design PI/PID controllers to achieve the improved performance for the given process or system. From control theory we know that all the real time processes have inherent time delays and time constants associated with it. The PI/PID tuning method elaborated in this paper is found to be superior as compared with basic PI/PID tuning methods based on first-order plus delay-time (FOPDT) model of process. Two simulation examples are demonstrated to show the applicability and effectiveness of the given method***

Keywords— Numerical Optimization, Time-Delay, Sensitivity, PID controller, Process Models.

I. INTRODUCTION

PID controller is a name commonly given to three-term controller. The mnemonic*PID* refers to the First letters of the names of the individual terms that make up thestandard three-term controller. These are *P* for the proportional term, *I* for the integralterm and *D* for the derivative term in the controller. *PID* controllers are probably themost widely used industrial controller. Even complex industrial control systems maycomprise a control network whose main control building block is a *PID* control module. The three-term *PID* controller had a long history of use and has survived the changesof technology from the analogue era into the digital computer control system age.Many thousands of instrumentation and control engineers worldwide are using thesecontrollers in day-to-day work [1]. The *PID* algorithm can be approached in many different directions. It can be viewed as a device that can be operated with a fewrules of thumb, but it can also be approached analytically. Applying a *PID* controllaw consists of applying properly the sum of three types of control actions namely aproportional, an integral and a derivative.

The proportional-integral (*PI*) and proportional-integral-derivative (*PID*) controllers are widely used in many industrial control systems for several decades since Ziegler and Nichols proposed their first PID tuning method. This is because the *PID* controller structure is simple and its principle is easier to understand than most other advanced controllers ([1] [14] [16]). On the other hand, the general performance of *PID* controller is satisfactory in many applications. For these reasons, the majority of the controllers used in industry are of *PI/PID* type.

Over the past years, a number of *PID* design and tuning methods for first-order-plus-delay-time have been reported in the literature. An earlier of them is Ziegler- Nichols method [2], Cohen- Coon method [3], Kappa Tau, constant open loop transfer function method [4], synthesis method [5], internal model controller [6], and so on. However, these tuning methods have certain limitations, and often do not provide good tuning parameterizations of the PID controllers for highorder- plus-delay-time *(HOPDT)* processes. The gain and phase margin *(GPM)* specifications methods have been used in many applications to design the *PI/PD/PID* type controllers for Time Delay and Integral Plus Time Delayprocesses (e. g. see [7]-[8]-[21]). In this damping factor of the system is related to phase margin of the systems and served as a measure of robustness. In GPM the solutions are normally obtained by numerically or graphically by means of trial-and-error, generally using the Bode plots. This methods are certainly not suitable for systems having infinite phase crossover frequencies (e.g. systems without time delays and having number of zeros less than the number of poles by one i.e. system is not perfect). The main drawback of the *GPM* method is that the transfer function of the controlled systems is restricted to the *FOPDT* or *SOPDT*. An auto tuning of *PID* is also is one of the method of controller tuning in which tuning of the parameters of controller is doneautomatically [23]. The fuzzy logic based *PID* controller design is also possible [24].

In this paper novel approach of *PI/PID*controller tuning is elaborated. The paper is organized as follows; Section 2 represents the general process models. Section 3 includes *PI/PID* controller tuning/design. Section 4 represents numerical optimization approach based on *FOPDT* model of process. The simulation results are incorporated in Section 5. Finally the paper is summarized in Section 6 with conclusion.

II. GENERAL PROCESS MODELS

According to the control theory and literature every control engineer is know that there are wide range of linear self-regulating processes with various dynamics including those with low and high-order, small and large dead time and monotonic or oscillatory responses [11]. All these processes are expressed in control theory with the help of transfer function or the dynamic model of the process.

As the industrial processes are of different dynamics a very broad class is characterized by aperiodic response. This important class of the process dynamics can be represented by the first-order plus delay-time model as given by Eq. (1)

$$G(s) = \frac{k}{1+\tau s} exp^{(-t_0 s)} \tag{1}$$

Note that process model expressed by Eq. (1) is only used for the purpose of simplified analysis. The actual process may have multiple lags, non-minimum phase zero, etc. In Eq. (1) k, τ and t_0 represents the steady state gain, time constant and time delay of the actual process. Another important class of the industrial process is expressed by non-aperiodic response. This category of processescan be expressed by a second-order plus delay-time model as given by Eq. (2)

$$G(s) = \frac{k\,exp^{\,(-t_0 s)}}{s^2+a_1 s+a_0} \tag{2}$$

Many identification techniques can be used to obtain the first-order plus delay-time or second-order plus delay- time model for *PI/PID* control ([1], [21],[22]). A simple method isbased on the analysis of the open-loop step response. The first-order plus dead-time model in Eq. (1) is obtained as follows:

$$k = y_\infty$$
$$t_0 = 2.8t_1 - 1.8t_2$$
$$\tau = 5.5(t_2 - t_1) \tag{3}$$

Where $y_\infty$the final value of the step response of the process is, t_1 is the time where the output attains 28% of its final value and t_2 is the time where the output attains 40% of its final value.

The way by which the first-order plus delay-time model of process can be obtained as follows:

$$k = \frac{\Delta y(t)}{\Delta u(t)}$$
$$\tau = \frac{3}{2}(t_{63} - t_{28})$$
$$t_0 = t_{63} - \tau \tag{4}$$

Where t_{63} and t_{28} is the time at which process output reaches 63.2% and 28.3% of its final value.

For second-order plus delay-time model given by Eq. (2), the parameters are obtained as

$$k = y_\infty$$
$$t_0 = is\ the\ apparent\ time\ delay$$
$$a_1 = \frac{2|ln(D_1)|}{\pi t_p},\ a_0 = \frac{\pi^2 + ln(D_1)^2}{\pi^2 t_p^2} \tag{5}$$

Where D_1 is the first overshoot for the unit step response of the process and $t_p$is the corresponding time. These models can also derive from relay feedback method ([1], [12]).

III.The PI/PID controller tuning

In this section, the PI/PID controller design theory and considerations are explained. Consider general unity feedback control system as shown in Fig 1.

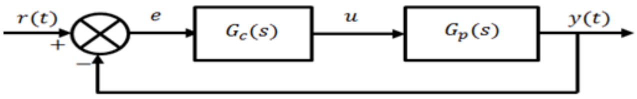

Fig. 1 A general unity feedback control system with controller

Where $G_p(s)$ is the process model either given by Eq. (1) or Eq. (2) and $G_c(s)$ the transfer function of the standard *PI/PID*controller as given by Eq. (6)

$$G_c(s) = K_p + {}^{K_i}/_{s}\ for\ PI$$
$$G_c(s) = K_p + {}^{K_i}/_{s} + K_d s\ \ for\ PID \tag{6}$$

For this control system, the sensitivity function $S(s)$ and complementary sensitivity function $T(s)$ which is the transfer function of the closed loop system, are respectively, defined by

$$S(s) = \frac{1}{1+G_c(s)G_p(s)} = \frac{1}{1+L(s)} \tag{7}$$

Where $L(s) = G_c(s)G_p(s)$ is the open loop transfer function and

$$T(s) = 1 - S(s) = \frac{L(s)}{1+L(s)} \tag{8}$$

The quantity $|T(j\omega)|$ represents the input output gain at the frequency${}^{2\pi}/_{\omega}$, for a *PI/PID* controller this gain is equal to one in the low frequency domain that is the steady-state error is equal to zero.The quantity $M_p = max_\omega |T(j\omega)|$ is the peak magnitude of the frequency response of the closed-loop system. It is well known that M_p is related to the overshoot for the step response of the closed-loop system. In order to impose good transient response it is necessary to have

$$M_p \leq M_p^+ \tag{9}$$

Where $M_p^+ > 1$ is the upper bound of the maximum of the complementary sensitivity function. In an equivalent manner the following constraint is required:

$$D_1 \leq D_1^+ \quad (10)$$

Where D_1 the first overshoot of the step response and D_1^+ is the upper bound value of this overshoot. It is then possible to introduce a lower bound pseudo-damping factor ξ_m, which is related to the upper bound of the firs overshoot by the relation:

$$\xi_m = \frac{|ln(D_1^+)|}{\sqrt{\pi^2 + ln(D_1^+)^2}} \quad (11)$$

The relation between M_p^+ and the lower bound pseudo damping factor ξ_m is given by

$$M_p^+ = \frac{1}{2\xi_m\sqrt{1-(\xi_m)^2}} \quad (12)$$

For good transient response it is required that

$$\xi \geq \xi_m \quad (13)$$

Where ξ is the pseudo-damping factor of the closed-loop system. The quantity $1/|S(j\omega)|$ represents the distance between the Nyquist curve of the open-loop transfer function $L(s)$ and the critical point-1 at the frequency $2\pi/\omega$. The minimum of this distance represents then a good measure of the stability margin.Consider an additive error model of the open loop transfer function $\Delta L(s)$, the influence of this error on the closed-loop transfer function can be deduced from the first order Taylor series expansion as

$$T\big(L(s) + \Delta L(s)\big) = T\big(L(s)\big) + \frac{\partial T(s)}{\partial L(s)}\Delta L(s) \quad (14)$$

This gives the well-known results

$$\frac{\Delta T(s)}{T(s)} = S(s)\frac{\Delta L(s)}{L(s)} \quad (15)$$

The quantity $max_\omega|S(j\omega)|$ represents then a good evaluation of the robustness in the presence of model uncertainties. The sensitivity function $S(s)$ appears also in the transfer function of the input disturbance $D(s)$ to the output $Y(s)$

$$Y(s) = G_p(s)S(s)D(s) \quad (16)$$

The quantity $max_\omega|S(j\omega)|$ represents good evaluation of the performance rejection of the load disturbance. Finally, in order to achieve good transient response, good stability margin, good robustness in the presence of model uncertainties and good rejection of the load disturbance, it is necessary to determine the parameters K_p, K_i and K_d such that

$$\xi \geq \xi_m$$

$$max_{K_pK_iK_d}\left\{\left|\frac{1}{S(j\omega, K_p, K_i, K_d)}\right|\right\} \quad (17)$$

There is not a known analytical solution of this optimization problem. A way to solve this problem is the numerical optimization.

IV. THE NUMERICAL OPTIMIZATION

4.1 Numerical Optimization ofthe *PI* controller with the first-order plus delay-time process model:

Consider the standard *PI*controller given by Eq. (6) and the process model Eq. (1), the open-loop transfer function is given by

$$L(s) = \frac{k(1+K_pT_is)exp^{-t_0s}}{T_is(1+\tau s)} \quad (18)$$

With $T_i = 1/K_i$. Using the approximation $exp^{-t_0s} \approx 1/(1+t_0s)$, the polynomial characteristic of the closed-loop system is given by

$$p(s) = s^3 + \frac{t_0+\tau}{t_0\tau}s^2 + \frac{1+K_pk}{t_0\tau}s + \frac{k}{T_it_0\tau} \quad (19)$$

This is of the form of

$$p(s) = (s+a)(s^2 + 2\xi\omega_0 s + \omega_0^2)$$

With

$$a = \frac{t_0+\tau}{t_0\tau} - 2\xi\omega_0$$

$$K_p = \frac{(\omega_0 + 2a\xi)\omega_0t_0\tau - 1}{k}$$

$$K_i = \frac{a\omega_0^2t_0\tau}{k} \quad (20)$$

The closed-loop stability impose $a > 0$ which is verified if

$$\frac{t_0+2}{\xi\omega_0t_0\tau} > 2 \quad (21)$$

The above inequality is satisfied for

$$\frac{t_0+2}{\xi\omega_0t_0\tau} = b$$

With $b > 2$. Taking into consideration of $\xi = \xi_m$ one can have

$$\omega_0 = \frac{t_0+\tau}{b\xi_mt_0\tau}$$

$$a = \frac{t_0+\tau}{t_0\tau} - 2\xi_m\omega_0 \quad (22)$$

The optimization problem is then written as follows

$$max_{b>2}\{min_\omega|1+L(j\omega,b)|\}$$
$$L(s)=\frac{k(1+K_pT_is)exp^{-t_0s}}{T_is(1+\tau s)}$$
$$\omega_0=\frac{t_0+\tau}{b\xi_m t_0\tau}$$
$$a=\frac{t_0+\tau}{t_0\tau}-2\xi_m\omega_0$$
$$K_p=\frac{(\omega_0+2a\xi_m)\omega_0t_0\tau-1}{k}$$
$$K_i=\frac{a\omega_0^2t_0\tau}{k} \quad (23)$$

which is numerically easy to solve. The *PI* controller is sufficient when the process dynamics is essentially first order. For higher-order processes the *PI* controller is not performing well, in this case the PID controller will be used.

4.2 Numerical Optimization of the *PID* controller with the first-order plus delay-time process model:

The dynamic performance obtained with *PI* controller can be improved by the use of *PID* controller. Consider the standard *PID* controller given by Eq. (6) and the process model Eq. (1), the open-loop transfer function is given by

$$L(s)=\frac{k(1+K_pT_is+K_dT_is^2)exp^{-t_0s}}{T_is(1+\tau s)} \quad (24)$$

with $T_i=1/K_i$. Using the approximation $exp^{-t_0s}\approx 1/(1+t_0/2\,s)^2$, the polynomial characteristic of the closed-loop system is given by

$$p(s)=s^4+\frac{t_0+4\tau}{t_0\tau}s^3+\frac{4(t_0+\tau+K_dk)}{t_0{}^2\tau}s^2+\frac{4(1+K_pk)}{t_0^2\tau}s+\frac{4k}{T_it_0{}^2\tau} \quad (25)$$

This is of the form of

$$p(s)=(s+a)^2(s^2+2\xi\omega_0s+\omega_0^2)$$

With

$$a=\frac{t_0+4\tau}{2t_0\tau}-\xi\omega_0$$
$$K_p=\frac{(\omega_0+a\xi)a\omega_0t_0{}^2\tau-2}{2k}$$
$$K_i=\frac{a^2\omega_0^2t_0{}^2\tau}{4k}$$
$$K_d=\frac{(a^2+4a\xi\omega_0+\omega_0^2)t_0^2\tau-4(t_0+\tau)}{4k} \quad (26)$$

The closed-loop stability impose $a>0$ which is verified if

$$\frac{t_0+4\tau}{2\xi\omega_0t_0\tau}>1 \quad (27)$$

The above inequality is satisfied for

$$\frac{t_0+4\tau}{2\xi\omega_0t_0\tau}=b$$

With $b>1$. Taking into consideration of $\xi=\xi_m$ one can have

$$\omega_0=\frac{t_0+4\tau}{2b\xi_mt_0\tau}$$
$$a=\frac{t_0+4\tau}{2t_0\tau}-\xi_m\omega_0 \quad (28)$$

The optimization problem is then written as follows

$$max_{b>1}\{min_\omega|1+L(j\omega,b)|\}$$
$$L(s)=\frac{k(1+K_pT_is+K_dT_is^2)exp^{-t_0s}}{T_is(1+\tau s)}$$
$$\omega_0=\frac{t_0+4\tau}{2b\xi_mt_0\tau}$$
$$a=\frac{t_0+4\tau}{2t_0\tau}-\xi_m\omega_0$$
$$K_p=\frac{(\omega_0+a\xi_m)a\omega_0t_0{}^2\tau-2}{2k}$$
$$K_i=\frac{a^2\omega_0^2t_0{}^2\tau}{4k}$$
$$K_d=\frac{(a^2+4a\xi_m\omega_0+\omega_0^2)t_0^2\tau-4(t_0+\tau)}{4k} \quad (29)$$

which is numerically easy to solve.

V. SIMULATION RESULTS

In this Section two examples are simulated to show the applicability and effectiveness of the Numerical Optimization approach.

Example 1:

Consider the process having first-order plus delay-time model as given by Eq. (1) with parameters in Eq. (30)

$$G_p(s) = \frac{2.24*exp^{(-0.95*s)}}{1+7.67*s} \tag{30}$$

For this process model the *PID* controller is design by using the basic *PID*tuning methods such as Ziegler-Nichols (Z-N) method, Cohen-Coon (C-C) method and Kappa-Tau (K-T) method. These basic *PID*controllers' parameters are obtained from the knowledge of the process parameters such as k, t_0 and τ.

As mentioned in Section 4.1 and 4.2 the *PI* and *PID* controller is design by using the Numerical Optimization approach with the damping and tuning factor ($\xi_m = 0.65, b = 3.8$)and ($\xi_m = 0.6,\ b = 4.8$) respectively. The closed-loop step response of the process is obtained with *PI/PID* controller in MATLAB 7.11.0,as shown in Fig. 2 and various performance specifications are found as given in the Table 1.

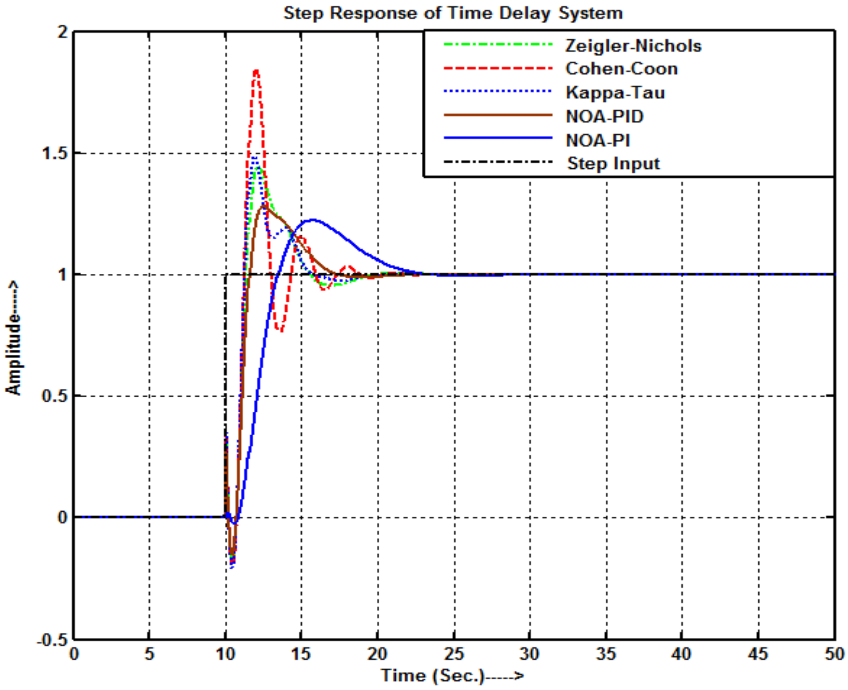

Fig. 2 Closed-loop unit step response of Example 1 with various controllers.

Table I Simulation Result for Example 1

(OS=Overshoot in %, ST=Settling Time, ISE=Integral Square Error and ITAE=Integral Time Absolute Error)

Method	**Controller Parameter**			**Specifications**			
	K_p	K_i	K_d	**OS**	**ST**	**ISE**	**ITAE**
Ziegler-Nichols	3.59	2.02	1.62	44.42	8.41	13.64	262.50
Cohen-Coon	5.02	2.24	1.71	84.83	8.35	17.04	293.81
Kappa-Tau	3.86	1.99	1.88	48.81	7.98	12.86	237.16
NOA-PID	3.15	1.32	1.27	28.19	6.59	12.46	246.47
NOA-PI	1.45	0.42	-	22.24	11.51	19.19	422.99

Example 2:

Consider the process having first-order plus delay-time model as given by Eq. (1) with parameters in Eq. (31)

$$G_p(s) = \frac{0.5*exp^{(-2*s)}}{1+14*s} \tag{31}$$

For this process model the *PID* controller is design by using the basic *PID*tuning methods such as Ziegler-Nichols (Z-N) method, Cohen-Coon (C-C) method and Kappa-Tau (K-T) method. These basic *PID*controllers' parameters are obtained from the knowledge of the process parameters such as k, t_0 and τ.

As mentioned in Section 4.1 and 4.2 the *PI* and *PID* controller is design by using the Numerical Optimization approach with the damping and tuning factor ($\xi_m = 0.75, b = 2.3$)and ($\xi_m = 0.6,\ b = 5.9$) respectively. The closed-loop step response of the process is obtained with *PI/PID* controller in MATLAB 7.11.0, as shown in Fig. 3 and various performance specifications are found as given in the Table 2.

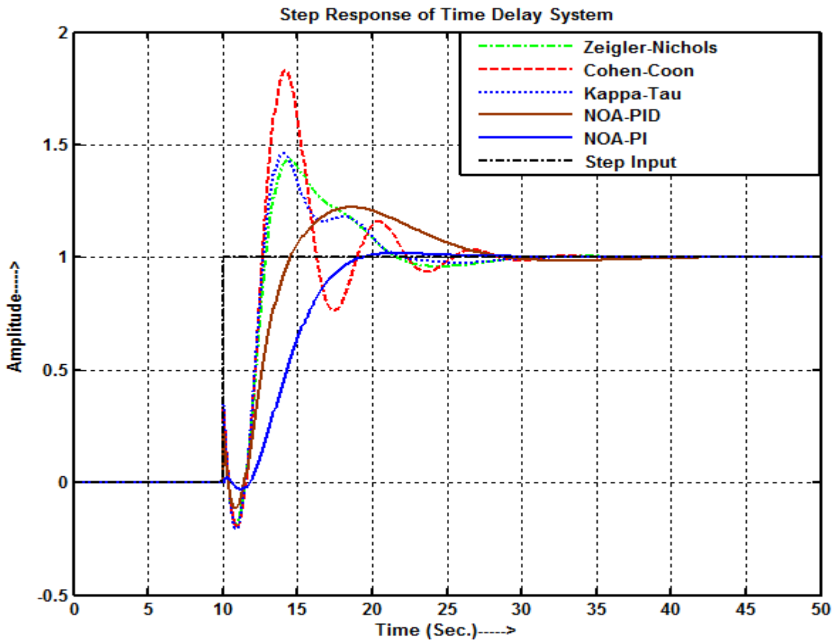

Fig. 3 Closed-loop unit step response of Example 2 with various controllers.

Table II Simulation Result for Example 2
(OS=Overshoot in %, ST=Settling Time, ISE=Integral Square Error and ITAE=Integral Time Absolute Error)

Method	**Controller Parameter**			**Specifications**			
	K_p	K_i	K_d	**OS**	**ST**	**ISE**	**ITAE**
Ziegler-Nichols	14.08	3.75	13.24	43.33	17.41	27.38	626.76
Cohen-Coon	19.56	4.17	13.96	83.09	17.49	35.02	717.88
Kappa-Tau	14.88	3.73	16.43	46.37	16.43	26.55	571.38
NOA-PID	9.39	1.77	8.02	22.26	17.12	26.79	707.72
NOA-PI	6.22	0.45	-	1.82	8.52	36.10	617.53

VI. CONCLUSIONS

In this paper the *PID*controller is designed by various basis methods such as Ziegler-Nichols, Cohen-Coon and Kappa-Tau for two different first-order plus delay-time process model. For the same processes the *PI* and *PID*controller is designed by Numerical Optimization approach. From simulation result it is found that *NOA-PI* and *NOA-PID* controller gives the monotonic (less oscillatory) response with less settling time and minimum Overshoot which avoids the excessive oscillation of the final control element as compared with other basic controllers.Therefore the Numerical Optimization approach for tuning of *PI* and *PID* controller is found to be more effective and applicable for the processes.

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