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The Comparison of Bondage Number and the Diameter of an Interval Graph

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Abstract: Among the various applications of the theory of domination and the distance, the most often discussed is a communication network. This network consists of communication links all distance between affixed set of sites. Interval graphs are rich in combinatorial structures and have found applications in several disciplines such as Biology, Ecology, Psychology, Traffic control, Genetics, Computer sciences and particularly useful in cyclic scheduling and computer storage allocation problems etc. The problem is to select the smallest set of sites at which transmitters are placed so that every site in the network that does not have a transmitters is joined by a direct communication link to the site, which has a transmitter then this problem reduces to that of finding a minimum dominating set in the graph corresponding to this network. Suppose communication network does not work due to link failure. Then the problem is what is the fewest number of communication links such that at least one additional transmitter would be required in order that communication with all sites as possible. This leads to the introducing of the concept of the bondage number and the average distance. In this paper the comparison of the bondage number and the diameter of an interval graph.

Key Words: Interval family, Interval graph, dominating set, Bondage Number, Distance, Diameter.

I. INTRODUCTION

The research of the domination in graphs has been an evergreen of the graph theory. Its basic concept is the dominating set and the domination number. The theory of domination in graphs was introduced by Ore [1] and Berge [2]. A survey on results and applications of dominating sets was presented by E.J.Cockayane and S.T.Hedetniemi [3]. In 1997 Kulli et.al introduced the concept of Non-split domination [4] and studied these parameters for various standard graphs and obtained the bounds for these parameters.

In general an undirected graph $G=(V,E)$ is an interval graph (IG), if the vertex set V can be put into one-to-one correspondence with a set of intervals I on the real line R, such that two vertices are adjacent in G, if and only if their corresponding intervals have non-empty intersection. The set I is called an interval representation of G and G is referred to as the intersection graph I. Let $I=\{I_1,I_2,I_3,I_4,.........I_n\}$ be any interval family where, each I_i is an interval on the real line and $I_i=[a_i,b_i]$ for $i=1,2,3,4,..........n$. Here a_i is called the left end point labeling and b_i is the right end point labeling of I_i.Without loss of generality we assume that all end points of the intervals in I are distinct numbers between 1 and $2n$.

Two intervals i and j are said to be intersect each other if they have non empty intersection. Also we say that the intervals contain both its end points and that no two intervals share a common end point. The intervals and vertices of an interval graph are one and the same thing. The graph G is connected, and the list of sorted end point is given and the intervals in I are indexed by increasing right end points, that is $b_1<b_2<b_3<........<b_n$.

Let $G=(V,E)$ be a graph. A set $D\subseteq V(G)$ is a dominating set of G if every vertex in V/D is adjacent to some vertex in D. A subset D of v is said to be a dominating set of G if every vertex not in D is adjacent to vertex in D. The domination number γ(G) of a graph G is the minimum cardinality of a dominating set in G[5].

The bondage number b(G) of a non empty graph G is the minimum cardinality[6] among all sets of edges E1 for which γ(G-E1) > γ(G). The diameter and radius are two of the most basic graph parameters. The diameter of a graph is the largest distance between its vertices.

he distance orientations of graphs by V.Chvatal [7]. The diameter of a graph G is the maximum of eccentricity of all its vertices and is denoted by Diam (G) that is [8] Diam (G) = max {e (v): v ∈ V (G)}, where the maximum distance from vertex u to any vertex of G is called eccentricity of the vertex v and is denoted by ecc (V) that is ecc (v) = max {d (u, v): u ЄV (G)},where as the distance between two vertices u and v of a graph is the length of the shortest path between them and is denoted by d (u, v) or d (v, u).

II. MAIN THEOREMS

Theorem1: Let I = {i, j, k,………n} be an interval family and let G be an interval graph corresponding to the interval family I. Let i, j € I and suppose j is contained in i and i ≠ 1 and there is no other interval that intersects j, other than i. Then the bondage number b(G) is less than the diameter of G.

Proof: Let G be the interval graph corresponding to the given interval family I = {i, j, k,……….n}. Let i, j be any two intervals in I, which satisfying the hypothesis of the theorem. Then clearly I ∈ D from I , where D is a minimum dominating set of G. because there is no other interval in I, other than I, that dominates j. Consider the edge e = (i, j) in G. If we remove this edge from G, then j becomes an isolated vertex in G-e, as there is no other vertex in G, other than i, that adjacent with j. Hence the dominating set D_1 = D U {j} becomes a dominating set of G-e and since D is a minimum dominating set of G it follows that D_1 also a minimum dominating set of G-e.

Therefore γ (G-e) = | D_1 | = |D| +1 > |D| = γ (G), thus the bondage number b(G) = 1.

Next we will prove that the diameter of G. If we will arise two cases

Case(a) : The distance of G

Case(b) : The eccentricity of G

Case(a) : Let I = {i, j, k,………n} be an interval family. Let G be an interval graph corresponding to the interval family I. For any two vertices i, j in an interval graph G, corresponding to an interval family I. The distance from i to j is denoted by d(i, j) and defined as the length of a shortest (i, j)-path in an interval graph G. the term distance we just defined satisfies all four of the following axioms.

1. d (i , j) ≥ 0, for all i, j ∈ V(G).
2. d (i, j) = 0, if and only if i = j .
3. d (i, j) = d (j, i), for all i, j_j ∈ V(G).
4. d (i, k) ≤ d (i, j) + d (j, k), for all i, j, k ∈ V(G).

If G is not a connected and suppose G_1 and G_2 are two graphs of G such that G_1 = G_1 U G_2 and E(G_1) U E(G_2) and G_1 ∩ G_2= Ø.
That is V(G_1) ∩ V(G_2)= Ø
and E(G_1)∩ E(G_2)= Ø.
Then d(i, j) = ∞ for i € V(G_1)
and j ∈ V(G_2).
Since G must be connected V(G_1) ∩ V(G_2) and
E(G_1)∩ E(G_2)= Ø.

Case(b) : In this case we have to find the eccentricity of G. The eccentricity e(i) of a vertex i to vertex farthest from i. Vertex j is said to be a farthest neighbor of the vertex i of δ(i, j) =e(i),
Where e(i) = max{ δ(i, j): j ∈ V }.From above said cases we have to find the diameter of a graph G is the maximum among all eccentricities. Therefore diam (G) = max {δ(i, j): i ∈ V } Now we have to comparison of the bondage number b(G). Then we got b(G)<diam(G).

Hence the theorem is proved.

III. EXPERIMENTAL PROBLEM OF THEOREM.1

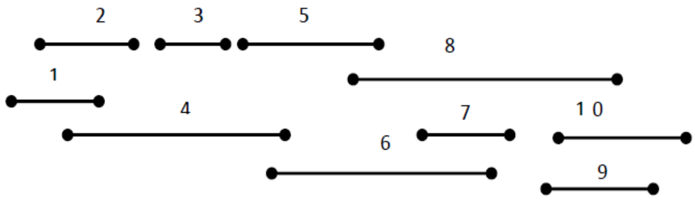

Fig.1. Interval family

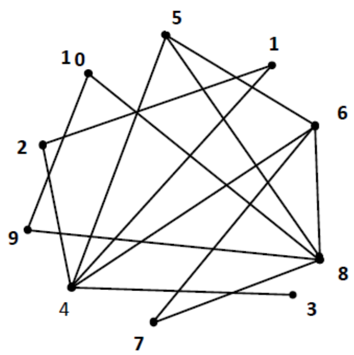

Fig.2. Interval graph G

DS = {4, 8} and γ (G) = 2

Remove the edge e = (8, 10).

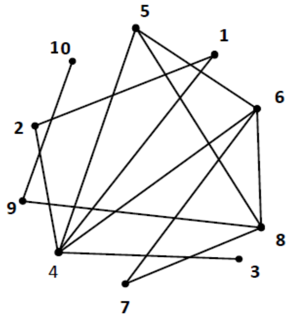

Fig.3. Interval graph G-e

Since γ (G-e) = 3.
Therefore γ (G-e) > γ (G).
Therefore the bondage number b (G) = 1.

IV. TO FIND THE DISTANCES FROM G

d(1,1)=0	d(2,1)=1	d(3,1)=2	d(4,1)=1	d(5,1)=2
d(1,2)=1	d(2,2)=0	d(3,2)=2	d(4,2)=1	d(5,2)=2
d(1,3)=2	d(2,3)=2	d(3,3)=0	d(4,3)=1	d(5,3)=2
d(1,4)=1	d(2,4)=1	d(3,4)=1	d(4,4)=0	d(5,4)=1
d(1,5)=2	d(2,5)=2	d(3,5)=2	d(4,5)=1	d(5,5)=0
d(1,6)=2	d(2,6)=2	d(3,6)=2	d(4,6)=1	d(5,6)=1
d(1,7)=3	d(2,7)=3	d(3,7)=3	d(4,7)=2	d(5,7)=2
d(1,8)=3	d(2,8)=3	d(3,8)=3	d(4,8)=2	d(5,8)=1
d(1,9)=4	d(2,9)=4	d(3,9)=4	d(4,9)=3	d(5,9)=2
d(1,10)=4	d(2,10)=4	d(3,10)=4	d(4,10)=3	d(5,10)=2

d(6,1)=2	d(7,1)=3	d(8,1)=3	d(9,1)=4	d(10,1)=4
d(6,2)=2	d(7,2)=3	d(8,2)=3	d(9,2)=4	d(10,2)=4
d(6,3)=2	d(7,3)=3	d(8,3)=3	d(9,3)=4	d(10,3)=4
d(6,4)=1	d(7,4)=2	d(8,4)=2	d(9,4)=3	d(10,4)=3
d(6,5)=1	d(7,5)=2	d(8,5)=1	d(9,5)=2	d(10,5)=2
d(6,6)=0	d(7,6)=1	d(8,6)=1	d(9,6)=2	d(10,6)=2
d(6,7)=1	d(7,7)=0	d(8,7)=1	d(9,7)=2	d(10,7)=2
d(6,8)=1	d(7,8)=1	d(8,8)=0	d(9,8)=1	d(10,8)=1
d(6,9)=2	d(7,9)=2	d(8,9)=1	d(9,9)=0	d(10,9)=1
d(6,10)=2	d(7,10)=2	d(8,10)=1	d(9,10)=1	d(10,10)=0

V. TO FIND AN ECCENTRICITY

e (i) = max {δ(i, j) : j ∈ V}
e(1) = max {0, 1, 2, 1, 2, 2, 3, 3, 4, 4}=4, e(2) = max {1, 0, 2, 1, 2, 2, 3, 3, 4, 4}=4
e(3) = max {2, 2, 0, 1, 2, 2, 3, 3, 4, 4}=4, e(4) = max {1, 1, 1, 0, 1, 1, 2, 2, 3, 3}=3
e(5) = max {2, 2, 2, 1, 0, 1, 2, 1, 2, 2}=2, e(6) = max {2, 2, 2, 1, 1, 0, 1, 1, 2, 2}=2
e(7) = max {3, 3, 3, 2, 2, 1, 0, 1, 2, 2}=3, e(8) = max {3, 3, 3, 2, 1, 1, 1, 0, 1, 1}=3
e(9) = max {4, 4, 4, 3, 2, 2, 2, 1, 0, 1}=4, e(10) = max {4, 4, 4, 3,2, 2, 2, 1, 1, 0}=4
e (i) = max {4, 4, 4, 3, 2, 2, 3, 3, 4, 4}=4

VI. TO FIND THE DIAMETER

diam (G) = max { δ(i, j): i ∈ V }
Diam (G) = max {4, 4, 4, 3, 2, 2, 3, 3, 4, 4}=4

Therefore Diam (G) = 4

Therefore b(G) < diam(G).

Theorem2 : Let the dominating set D(G) consists of two vertices only, say x and y. Suppose x dominates the vertex set S_1 = {1,2,…….i} and y dominates the vertex set S_2 = { i+1, i+2,……..n}. Suppose there is no vertex in S_1 other than x that dominates S_1 and no vertex in S_2 other than y that dominates S_2 then the bondage number b(G) < the diameter of G.

Proof: Let I = {i, j,………n} be an interval family and G be an interval graph corresponding to I. Let D = (x, y). Suppose x and y satisfies the hypothesis of the theorem. Since x alone dominates S_1, there is no vertex in S_3 = {1,2,………i}-{x} that can dominates S_1. Let j be any vertex in S_3 and edge e = (x, j). Consider the graph G-e. In this graph, x dominates every vertex in S_1 except j.

Now consider a vertex in S_1 which is adjacent with j, say k. Then clearly the set {x, k} dominates the set S_1 that is adjacent with j , then clearly the graph G becomes disconnected. So, there is at least one vertex in S_1 that is adjacent with j. Let us assume that there is a single vertex say z, z ≠x such that z dominates the set S_1 in G-e. this implies that z also dominates the set S_1 in G, a contradiction, because by hypothesis x is the only vertex that dominates the set S_1 in G. hence a single vertex cannot dominate S_1 in G-e.

Thus D_1 = D U {k} becomes a dominating set of G-e. Since D is a minimum in G, D_1 is also minimum in G-e, So that γ(G-e) > γ(G) . Hence the bondage number b(G) = 1. Similarly argument with vertex y also gives the bondage number b(G) = 1.

Next we have to prove that the diameter of G.

Since diam (G) = max{ e(i) : i ∈ V}, where e(i) = an eccentricity (G).

Since e(i) = max {δ(i, j): j∈V}. And also we know that δ(i, j) is the distance of G. This proof already proved in theorem1.

Therefore the bondage number is less than the diameter of G. Its symbolic form is b(G) < diam (G).

Hence the theorem is proved.

VII. EXPERIMENTAL PROBLEM OF THEOREM.2

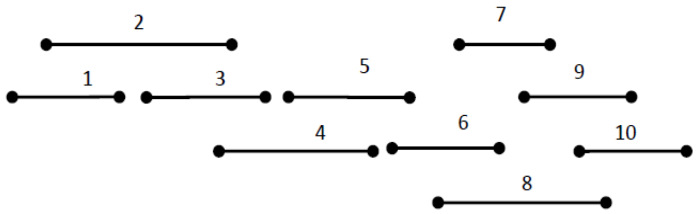

Fig. 4. Interval family

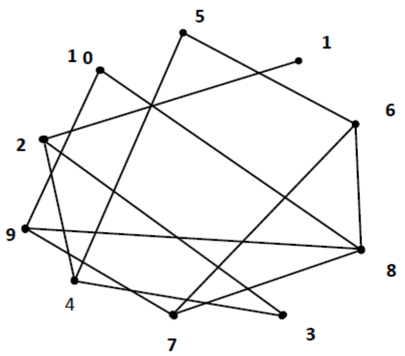

Fig. 5. Interval Graph G.

DS = {2, 5, 8} and γ(G) = 3

Remove the edge e = (8, 10), Since γ(G-e) = 4

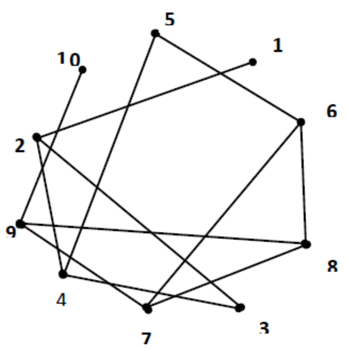

Fig. 6. Interval Graph G-e

Since γ (G-e) = 4

Theref.ore γ (G-e) > γ (G).

Therefore the bondage number b(G :1.

VIII. TO FIND THE DISTANCES FROM G

d(1,1)=0	d(2,1)=1	d(3,1)=2	d(4,1)=2	d(5,1)=3
d(1,2)=1	d(2,2)=0	d(3,2)=1	d(4,2)=1	d(5,2)=2
d(1,3)=2	d(2,3)=1	d(3,3)=0	d(4,3)=1	d(5,3)=2
d(1,4)=2	d(2,4)=1	d(3,4)=1	d(4,4)=0	d(5,4)=1
d(1,5)=3	d(2,5)=2	d(3,5)=2	d(4,5)=1	d(5,5)=0
d(1,6)=4	d(2,6)=3	d(3,6)=3	d(4,6)=2	d(5,6)=1
d(1,7)=5	d(2,7)=4	d(3,7)=4	d(4,7)=3	d(5,7)=2
d(1,8)=5	d(2,8)=4	d(3,8)=4	d(4, =3	d(5,8)=2
d(1,9)=6	d(2,9)=5	d(3,9)=5	d(4,9)=4	d(5,9)=3
d(1,10)=6	d(2,10)=5	d(3,10)=5	d(4,10)=4	d(5,10)=3
d(6,1)=4	d(7,1)=5	d(8,1)=5	d(9,1)=6	d(10,1)=6
d(6,2)=3	d(7,2)=4	d(8,2)=4	d(9,2)=5	d(10,2)=5
d(6,3)=3	d(7,3)=4	d(8,3)=4	d(9,3)=5	d(10,3)=5
d(6,4)=2	d(7,4)=3	d(8,4)=3	d(9,4)=4	d(10,4)=4
d(6,5)=1	d(7,5)=2	d(8,5)=2	d(9,5)=3	d(10,5)=3
d(6,6)=0	d(7,6)=1	d(8,6)=1	d(9,6)=2	d(10,6)=2
d(6,7)=1	d(7,7)=0	d(8,7)=1	d(9,7)=1	d(10,7)=2
d(6,8)=1	d(7,8)=1	d(8,8)=0	d(9,8)=1	d(10,8)=1
d(6,9)=2	d(7,9)=1	d(8,9)=1	d(9,9)=0	d(10,9)=1
d(6,10)=2	d(7,10)=2	d(8,10)=1	d(9,10)=1	d(10,10)=0

IX. TO FIND AN ECCENTRICITY

e (i) = max {δ(i, j) : j ∈ V}

e(1) = max {0, 1, 2, 2, 3, 4, 5, 5, 6, 6}=6, e(2) = max {1, 0, 1, 1, 2, 3, 4, 4, 5, 5}=5

e(3) = max {2, 1, 0, 1, 2, 3, 4, 4, 5, 5}=5, e(4) = max {2, 1, 1, 0, 1, 2, 3, 3, 4, 4}=4

e(5) = max {3, 2, 2, 1, 0, 1, 2, 2, 3, 3}=3, e(6) = max {4, 3, 3, 2, 1, 0, 1, 1, 2, 2}=4

e(7) = max {5, 4, 4, 3, 2, 1, 0, 1, 1, 2}=5, e(8) = max {5, 4, 4, 3, 2, 1, 1, 0, 1, 1}=5

e(9) = max {6, 5, 5, 4, 3, 2, 1, 1, 0, 1}=6, e(10) = max {6, 5, 5, 4, 3, 2, 2, 1, 1, 0}=6

e(i) = max {6, 5, 5, 4, 3, 4, 5, 5, 6, 6}=6

X. TO FIND THE DIAMETER

diam (G) = max { δ(i, j): i ∈ V }

Diam(G) = max {6, 5, 5, 4, 3, 4, 5, 5, 6, 6}=6

Therefore Diam(G) = 6

Therefore b(G) < diam(G).

Theorem3: Let I = {i, j, ………..n} be an interval family and let G be an interval graph of I. Let the dominating set D(G) consists of two vertices only, say x and y. suppose x dominates the vertex set S_1 = {1,2,…….i} and y dominates the vertex set S_2 = { i+1, i+2,…..n }. Suppose there is one more vertex z ∈ S_1 or S_2 respectively then bondage number b(G) is less than the diameter of G.

Proof: Let the dominating set D = {x, y} and x dominates S_1 and y dominates S_2. Let z ∈ S_1 such that z also dominates S_1. Let an edge e = (x, z). Consider the graph G-e. In this graph the vertices x and z are not adjacent. Hence x alone cannot dominate the set S_1 in G-e. We require at least two vertices in S_1,which dominate S_1 in G-e. therefore the donating set of G-e contains more than two vertices. Thus γ (G-e) > γ (G). Hence the bondage number b(G) = 1. Similar is the case if z ∈ S_2. Again we have to prove that the diameter of G. In this we have already proved in theorem1. Therefore b(G) < diam(G).

XI. EXPERIMENTAL PROBLEM OF THEOREM.3

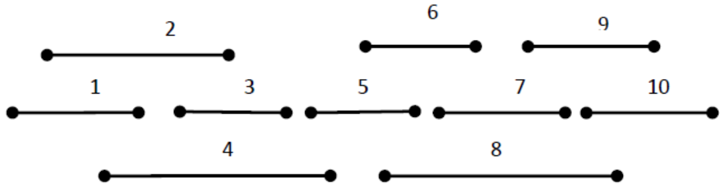

Fig.7. Interval family

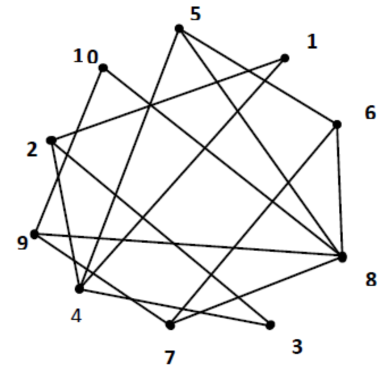

Fig. 8. Interval Graph G

DS = { 4, 8} and γ (G) = 2

Remove the edge e = (1, 4).

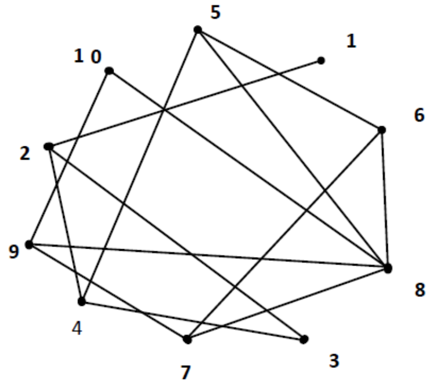

Fig.9. Interval graph G

Since γ (G-e) = 3

Theref.ore γ (G-e) > γ (G).

Therefore the bondage number b(G) = 1.

XII. TO FIND THE DISTANCES FROM G

d(1,1)=0	d(2,1)=1	d(3,1)=2	d(4,1)=1	d(5,1)=2
d(1,2)=1	d(2,2)=0	**d(3,2)=1**	d(4,2)=1	d(5,2)=2
d(1,3)=2	d(2,3)=1	d(3,3)=0	d(4,3)=1	d(5,3)=2
d(1,4)=1	d(2,4)=1	d(3,4)=1	d(4,4)=0	d(5,4)=1
d(1,5)=2	d(2,5)=2	d(3,5)=2	d(4,5)=1	d(5,5)=0
d(1,6)=3	d(2,6)=3	d(3,6)=3	d(4,6)=2	d(5,6)=1
d(1,7)=4	d(2,7)=4	d(3,7)=4	d(4,7)=3	d(5,7)=2
d(1,8)=3	d(2,8)=3	d(3,8)=3	d(4,8)=2	d(5,8)=1
d(1,9)=4	d(2,9)=4	d(3,9)=4	d(4,9)=3	d(5,9)=2
d(1,10)=4	d(2,10)=4	d(3,10)=4	d(4,10)=3	d(5,10)=2

d(6,1)=3	d(7,1)=4	d(8,1)=3	d(9,1)=4	d(10,1)=4
d(6,2)=3	d(7,2)=4	d(8,2)=3	d(9,2)=4	d(10,2)=4
d(6,3)=3	d(7,3)=4	d(8,3)=3	d(9,3)=4	d(10,3)=4
d(6,4)=2	d(7,4)=3	d(8,4)=2	d(9,4)=3	d(10,4)=3
d(6,5)=1	d(7,5)=2	d(8,5)=1	d(9,5)=2	d(10,5)=2
d(6,6)=0	d(7,6)=1	d(8,6)=1	d(9,6)=2	d(10,6)=2
d(6,7)=1	d(7,7)=0	d(8,7)=1	d(9,7)=2	d(10,7)=2
d(6,8)=1	d(7,8)=1	d(8,8)=0	d(9,8)=1	d(10,8)=1
d(6,9)=2	d(7,9)=2	d(8,9)=1	d(9,9)=0	d(10,9)=1
d(6,10)=2	d(7,10)=2	d(8,10)=1	d(9,10)=1	d(10,10)=0

XIII. TO FIND AN ECCENTRICITY

$e(i) = \max\{\delta(i, j) : j \in V\}$

$e(1) = \max\{0, 1, 2, 1, 2, 3, 4, 3, 4, 4\}=4$, $e(2) = \max\{1, 0, 1, 1, 2, 3, 4, 3, 4, 4\}=4$

$e(3) = \max\{2, 1, 0, 1, 2, 3, 4, 3, 4, 4\}=4$, $e(4) = \max\{1, 1, 1, 0, 1, 2, 3, 2, 3, 3\}=3$

$e(5) = \max\{2, 2, 2, 1, 0, 1, 2, 1, 2, 2\}=2$, $e(6) = \max\{3, 3, 3, 2, 1, 0, 1, 1, 2, 2\}=3$

$e(7) = \max\{4, 4, 4, 3, 2, 1, 0, 1, 2, 2\}=4$, $e(8) = \max\{3, 3, 3, 2, 1, 0, 1, 1, 2, 2\}=3$

$e(9) = \max\{4, 4, 4, 3, 2, 2, 2, 1, 0, 1\}=4$, $e(10) = \max\{4, 4, 4, 3, 2, 2, 2, 1, 1, 0\}=4$

$e(i) = \max\{4, 4, 4, 3, 2, 3, 4, 3, 4, 4\}=4$

XIV. TO FIND THE DIAMETER

$diam(G) = \max\{\delta(i, j): i \in V\}$

$Diam(G) = \max\{4, 4, 4, 3, 2, 3, 4, 3, 4, 4\}=4$

Therefore $Diam(G) = 4$

Therefore $b(G) < diam(G)$.

XV. CONCLUSION

Resolving the comparison of the bondage number and the diameter of some special classes of interval graphs has been the main focus of the paper. Especially, the nature of the intervals played a major role in determining the bondage number and diameter of the interval graphs with amazing ease. Some categorized graphs have been chosen in the process of exploration. In future, efforts will be put to identify the interval graphs with the comparison of the bondage number and the diameter of an interval graph.

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