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A Computerized Computationally Faster Uniformly Convergent Difference Schemes using Fitted operator and Fitted mesh for Singularly Perturbed Problems in Industry

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Abstract— ***This paper presents a uniformly convergent difference scheme for singularly perturbed problem with a left boundary layer in industry using fitted operator and fitted mesh. A new scheme is designed using fitted operator method and then fitted mesh is applied to view the left boundary layer. Experimental results are provided to show the performance of the scheme with respect to the singularly perturbed parameter, computational time and storage space in modern digital computer.***

Keywords— ***fitted operator method, fitted mesh method, numerical methods, finite difference schemes, singular perturbation problems, uniform convergence, order of convergence.***

I. INTRODUCTION

Singular perturbation problems(SPP) occurring in chemical reactor theory, control system electro magnetic wave propagation, semi-conductor device, fluid dynamics, etc[1-19]. The traditional standard numerical methods will not solve the SPPs due to instability of the numerical solution. And so, explicit exponential fitted schemes have been designed based on fitted operator methods.[1,3-18] To view the initial/ boundary/ interior layers computational methods have been designed using uniform and variable meshes[7-11, 13-15]. In particular, in [7], a two point boundary value problem have been solved using a computational method, in which at the terminal point the solution of the SPP is approximated by the solution of the reduced problem. The region of domain is partitioned into the smooth and transient region. Both the regions are solved by a single exponential fitted operator method with one mesh in the smooth region and another mesh in the transient region. In the transient region an iterative procedure is applied. After the introduction of Shishkin fitted mesh, lot of changes[1,19] in the field of SPPs. Few draw backs are there in Shishkin fitted mesh methods and in fitted operator method, in the sense that, a method designed for a linear SPP can not be directly extended to non-linear. Similarly can not be extended from one-space dimension to higher dimensions[1,19]. In [1], a direction is given to select either fitted operator or fitted mesh methods for a SPP with respect to the real time situation. Both the fitted operator and fitted mesh methods have to be further developed[1]. In [2] a modified form of Shishkin fitted mesh is presented but require more time and space during computation.

In [3], using fitted operator, explicit exponentially fitted operator schemes have been designed for linear and non-linear ordinary and partial SPPs. In [4] fitted operator higher order(two) explicit, uniform and optimal methods for first order linear SPPs are designed. In [5], fitted operator method of order one is designed for nonlinear SPP. In [6], using fitted operator method and shooting method a computational procedure is given for second order SPPs with mixed boundary conditions and with left boundary layer. In [7], using fitted operator method and boundary value technique using two different meshes, one mesh for smooth and another mesh for left boundary layer a computational procedure is given. In [8], using fitted operator method a chemical reactor problem is solved. In [9], using fitted operator method and boundary value technique using two different meshes, one mesh(h_1) for smooth and another mesh but same mesh(h_2). for both left and right boundary layers a computational procedure is given., In [10], using fitted operator method one-space dimensional heat equation is solved. In [11], using fitted operator method and initial value technique a computational procedure is given. In [12], using fitted operator method and shooting method a computational procedure is given for SPPs with Dirichlet's conditions with left boundary layer. In [13], a fitted operator method is presented for a non-linear SPP with initial layer. In [14], using fitted operator method and boundary value technique a computational procedure is given as in [9], but with a change in evaluation of solution at terminal points. In [15], using fitted operator method and boundary value technique a computational procedure is given for linear first order SPPs. In [16], using fitted operator method an uniform and optimal method is designed for non-linear SPPs. In [17], a finite difference scheme is presented for non-liner problems. In [18], using fitted operator method an uniform and optimal scheme is given for first order linear SPPs. In [19], a stable numerical method is designed for ball bearing problems using fitted operator methods in [3]. In [3, 20], a full literature survey is given and the 30 years of war in designing numerical methods for SPPs is narrated.

In this paper, a new uniformly convergent numerical method for a SPP is designed using fitted operator and fitted mesh methods. This method do not involve asymptotic expansion for fixing the terminal point . In [7,9,11,14,15], asymptotic expansion is used to fix the solution value at terminal points, this labour is reduced in this work. In [7,9,11,14,15], terminal point is fixed iteratively, it is not so in this paper, this labour is also reduced. The step size taken in [7,9,11,14,15] itself determine the step size in the transient regions but require asymptotic expansion for the solution. In the present work no need to know the asymptotic expansion in advance. And so, both computation time and storage space are reduced enormously in the present work.

Mathematical modelling of a chemical flow reactor problem is given in section II. In Section III a fitted operator method is presented for the SPP (1a,b). Fitted mes h method for (1a,b) is given in section IV. An algorithm is given in section V. Final section VI gives the experimental results using modern digital computer.

Throughout this paper, $\rho = h/\varepsilon$ and C will be used to denote a generic constant independent of i, h and ε. Error stands for absolute error.

II. MATHEMATICAL MODELLING

The mathematical modelling for the problem of flow through reactor and reactant supply in Industrial Organic Chemistry[3], an industrial related problem is provided. If u(t) is the displacement of a chemical

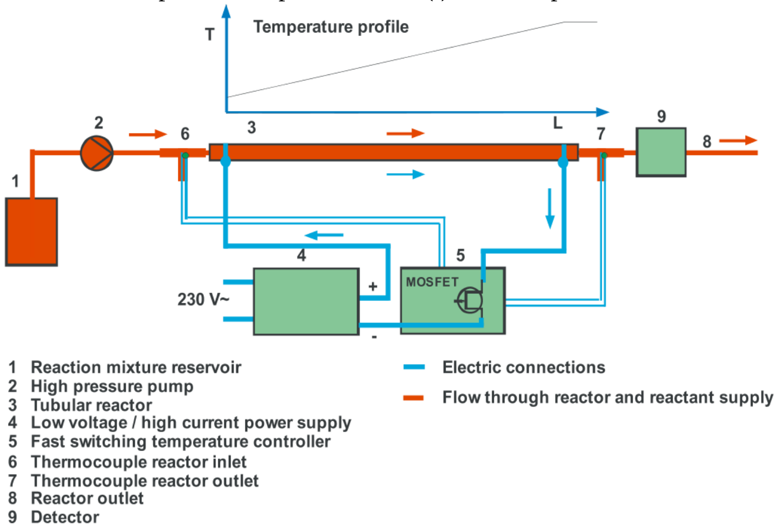

Fig. 1 Chemical Flow through reactor and reactant supply in Industrial Organic Chemistry

Flow of the reactant in time t, u'(t) stands for velocity of the flow, u''(t) stands for rate of change of velocity, Lu(t) standa for input reactant and f(t) stands for product, ε, a(t) and b(t) stands for the respective raise/lower of acceleration, velocity and displacement of flow and ϕ_1 and ϕ_2 refers to the product at time t=0 and t=1repectively then the one dimensional flow is modelled by a singularly perturbed differential equation:

$$Lu(t) \equiv \varepsilon u''(t) + a(t)u'(t) - b(t)u(t) = f(t), \quad 0 < t < 1, \qquad (1a)$$

$$B_0 u(0) \equiv -u'(0) = \phi_1, \quad B_1 u(1) \equiv u(1) + \varepsilon u'(1) = \phi_2 \qquad (1b)$$

where $\varepsilon > 0$ is a small parameter,, ϕ_1 and ϕ_2 are constants, a, b and f are smooth functions satisfying

$A \geq a(t) \geq \alpha > 0$ and $b(t) \geq 0$ for all t Є [0, 1]

ε is a singular perturbation problem(SPP). The main aim is to solve (1.1a,b for u(t) numerically. On using classical scheme the numerical solution u ot the problem (1a,b) is stable but it yield unstable numerical solution to ifs derivative u'.

The solution of the problem u converges uniformly to the solution $u_0(t)$ of the reduced problem of (1a,b)

$$a(t)\, u_0'(t) - b(t)\, u_0(t) = f(t), \quad 0 < t < 1, \qquad (2a)$$

$$u_0(1) = \phi_2 \qquad (2b)$$

as ε goes to zero at t = 0. But its derivative u', in general, does not converge uniformly as ε goes to zero at t=0. Because of this property, the problem (1a,b) is a singular perturbation problem. This property of the problem (1a,b) is so important. If he derivative u' of the problem (1a,b) is not considered and the solution u alone is considered then the problem (1a,b) is not a singular perturbation problem, because for small values of ε the solution will not form boundary layer in the interval [0, 1]. The derivative of the solution alone form boundary layer for small values of ε at the left end of the interval [0, 1]. And so finding the numerical solution with respect to the derivative of the solution of the problem (1a,b) is so important.

And because of this property, classical schemes may not produce good approximations to the derivative of the solution u(t) especially when ε is small but it solves the solution u numerically. It is an issue in this problem. The tri-diagonal scheme proposed in this paper solves this issue.

The tri-diagonal scheme solves both the solution and derivative of the solution numerically. The tri-diagonal scheme reduces the computation time and storage space by changing it into a one-step method instead of applying traditional LU decomposition method which involves matrix inversion. The scheme is exponentially fitted and uniform. The method of proof for convergence is the boot-strapping technique [1-20]. To prove convergence the solution the following theorem is presented without proof [3]

Theorm 1.. *Let* u *satisfy the SPP*(1a,b) *Then,*

$$u(t) = \gamma\, \varepsilon\, exp\,(- a(0)\, t / \varepsilon) + z(t), \quad (3a)$$

$$u'(t) = \beta\, exp\,(- a(0)\, t / \varepsilon) + z'(t) \quad (3b)$$

where

$$|\gamma| \le C_1 ,\ |\beta| \le C_3$$

$$|z^{(i)}(t)| \le C_2\, [\, 1 + \varepsilon^{-i+2}\, exp(- \alpha\, t / \varepsilon)\,],\ i = 2(1)j+2 \quad (3c)$$

where $C_j > 0$, j=1,2,3 *independent* of ε.

It is noted that

1. the solution is expressed as a sum of two smooth functions z(t) and $\gamma\, \varepsilon\, exp\,(- a(0)\, t / \varepsilon)$,
2. the derivative of the solution is expressed as a sum of a smooth function z'(t) and a boundary layer function $\beta\, exp\,(- a(0)\, t / \varepsilon)$,

III. FITTED OPERATOR METHOD

Using fitted operator method a tri-diagonal difference schemes is presented for the BVP (1a,b). The open interval (0, 1) is divided into N uniformly spaced mesh intervals, with the mesh spacing h = 1/N and mesh points $t_i = ih$, i=0(1)N. Using the usual notations for divided differences ,

$$D_+ u_i = (u_{i+1} - u_i)/h ,\ D_- u_i = (u_i - u_{i-1})/h \quad \text{and} \quad D_+ D_- u_i = (u_{i+1} - 2u_i + u_{i-1}) /h^2 ,$$

the following difference scheme is presented for the SPP (1a,b)

$$L_h\, u_i \equiv \varepsilon\, \sigma_i(\rho)\, D_+ D_- u_i + a(t_i) D_+ u_i - b(t_i)\, u_i = f(t_i),\ i=1(1)N-1, \quad (4a)$$

$$B_0\, u_0 \equiv - D_+\, u_0 = \phi_1 , \quad (4b)$$

$$B_1\, u_N \equiv u_N + \varepsilon\, D_-\, u_N = \phi_2 \quad (4c)$$

where

$$\sigma_i(\rho) = \sigma(\rho\, a(t_i)) = \rho\, a(t_i) / [\, exp(\rho\, a(t_i)) - 1\,] ,\ h = t_{i+1} - t_i ,\ \rho = h/\varepsilon,$$

The above scheme Is consistent with the SPP(1a,b). To get numerical solution of the derivative of u,
set ~~v~~$_i$ = $[(u_{i+1} - u_i)/h]$ in the scheme(4a-c), it is transformed into an initial value problem

$$L^*_h\, v_i \equiv \varepsilon\, \sigma_i(\rho)\, D_- u_i + a(t_i) v_i = b(t_i)\, u_i + f(t_i),\ i=1(1)N,\ v_0 = - \phi_1 ,$$

satisfying the relation

$u_N + \varepsilon\, v_N = \phi_2$.

A single scheme(4a-c) itself provide numerical solution to both u and u'.

In the following a necessary and sufficient conditions for the uniform convergence of the solution of the scheme (4a-c) are given.

A. Necessary Condition for Uniform Convergence

Theorem 2. *Assume that the solution of* (4a-c) *converges uniformly in* ε *to the solution of* the *SPP* (1a,b). *Let* ρ = h/ε *and* i, *a non negative integer be fixed. Then,*

$$lim\ \sigma_i(\rho) = \sigma_0(\rho)\ as\ h \to 0 . \quad (5)$$

Proof. See [3].

A difference inequality theorem and a stability theorem for the scheme (4a-c) is provided for stability.

Theorem 3. *If there exist real numbers* s_i , i=0(1)N *such that*

$$L_h\, s_i < 0 ,\ i=1(1)N-1,\ B_0\, s_0 > 0,\ B_1\, s_N > 0$$

then

$$L_h\, w_i < L_h\, v_i ,\ B_0\, v_0 \le B_0\, w_0 ,\ B_1\, v_N \le B_1\, w_N \Rightarrow v_i \le w_i \quad i=0(1)N ,$$

where v_i *and* w_i *are any two mesh functions*.

Proof.**: See[3].**

Corollary 3.1. $B_0\, u_0 \ge 0,\ B_1\, u_N \ge 0,\ L_h\, u_i \le 0$, i=1(1)N *implies* $u_i \ge 0$ *for all* i=1(1)N.

Theorem 4. *Let* v_i *be any mesh function. Then, for all* i=1(1)N,

$$|\, v_i\, || \le C\, [\, |B_0\, v_0| + |B_1\, v_N| + max\, |L_h\, v_j|\,],\ j=1(1)N,$$

Proof. See[3]

B. Sufficient Condition for Uniform Convergence

Theorem 5. *Let* u(t) *and* u_i *be the solutions of the SPP*(1a,b) *and* (4a-c) *respectively*. *Then, for all* i=0(1)N,

$$\left| u(t_i) - u_i \right| \le C\,h \qquad (6)$$

Proof. The truncation error T_i for the scheme (4a-c) with respect to the SPP(1a,b) is

For i=0, $T_0 = B_0\,[u(0) - u_0] = B_0\,u(0) - B_0\,u_0$,

For i-= N , $T_N = B_1\,[u(1) - u_N] = B_1\,u(1) - B_1\,u_N$

and for all i=1(1)N-1

$T_i = L_h\,[u(t_i) - u_i] = L_h\,u(t_i) - L_h\,u_i = L_h\,u(t_i) - f(t_i) = L_h\,u(t_i) - L\,u(t_i)$.

Using Theorem 1,we have,

$$\left| T_0 \right| \le \left| u'(0) - D_+ u(0) \right| \le C \int_0^h \left| u^{(2)}(t) \right| dt$$

$$\le C \int_0^h [1 + \varepsilon^{-1} \exp(-\alpha t/\varepsilon)]\,dt \le C\,h, \qquad (7)$$

$$\left| T_N \right| = \varepsilon \left| D_- u(1) - u'(1) \right| \le C \int_{1-h}^{1} \varepsilon \left| u^{(2)}(t) \right| dt$$

$$\le C \int_{1-h}^{1} [1 + \exp(-\alpha t/\varepsilon)]\,dt \le C\,h. \qquad (8)$$

Hence,

$$\left| T_i \right| \le C\,h \text{ for i=0 and N.} \qquad (9)$$

Using Theorem 4, for i=1(1)N-1,

$$T_i = \gamma\,[L_h\,v(t_i) - L\,v(t_i)] + [L_h\,z(t_i) - L\,z(t_i)]. \qquad (10)$$

Using the fundamental theorem of calculus and Lemma 4.5of [3],

$$\left| L_h\,z(t_i) - L\,z(t_i) \right| \le C\,h \qquad (11)$$

and from Lemma 3.4 of [3]

$$\left| L_h\,v(t_i) - L\,v(t_i) \right| \le C\,h. \qquad (12)$$

From (3.6)-(3.9) ,

$$\left| T_i \right| \le C\,h \text{ for i=0 ()1)N.} \qquad (13)$$

Now the desired estimate (6) follows from (13) and the discrete stability result.

The error estimate for the absolute error with respect to the derivative u' of the solution u of the problem (1a,b) using the scheme (4a-c) is given In the following theorem.

Theorem 6. Let u(t) and u'(t) be the solution and the derivative of the solution of the SPP(1a,b) and u_i and ~~v~~$_i$ =[(u_{i+1} -u_i)/h] are the numerical solutions of solution u and derivative u' of solution u of the SPP(1a,b) using the scheme (4a-c). Then, for all i=0(1)N,

$$\left| u'(t_i) - v_i \right| \le C\,h \qquad (14)$$

Proof. Set u' = v in the SPP(1a,b), it turns into an initial value problem which is a singularly perturbed problem with a initial layer at the left end of the interval [0, 1]

$$L^* v(t) \equiv \varepsilon v' + a(t)\,v = b(t)u + f(t), \; 0< t <1, \qquad (15)$$

$$v(0) = -\phi_1 \qquad (16)$$

satisfying the relation

$$u(1) + \varepsilon\,v(1) = \phi_2. \qquad (17)$$

L^* admits maximum principle [3] and so the solution v is bounded and stable subject to the condition $a(t) \ge \alpha > 0$ for all t in [0, 1].

And set ~~v~~$_i$ = [(u_{i+1} -u_i)/h] in the scheme(4a-c), it is transformed into

$$L^*_h\,v_i \equiv \varepsilon\,\sigma_i(\rho)\,D_-\,v_i + a(t_i)v_i = b(t_i)\,u_i + f(t_i), \; i=1(1)N, \qquad (18a)$$

$$v_0 = -\phi_1, \qquad (18b)$$

satisfying the relation

$$u_N + \varepsilon\,v_N = \phi_2 \qquad (18c)$$

L^*_h admits discrete maximum principle [3] and so the solution v_i is bounded and subject to the condition $a(t) \ge \alpha > 0$ for all t in [0, 1].

The truncation error T^*_i for the scheme (18a-c) with respect to the derivative u' of the solution u of tbe SPP(1a,b) is

For i=0, $T^*_0 = v(0) - v_0 = -\phi_1 - (-\phi_1) = 0,$ (19)

For all i=1(1)N,

$$T^*_i = L^*_h [v(t_i) - v_i] = L^*_h v(t_i) - L^*_h v_i = L^*_h v(t_i) - f(t_i) - b(t_i) u_i$$

$$= L^*_h u(t_i) - [f(t_i) + b(t_i) u(t_i)] - b(t_i) u_i + b(t_i) u(t_i)$$

$$= [L^*_h u(t_i) - L^* u(t_i)] + b(t_i) [u(t_i) - u_i]$$

From [3, 6, 7], $| L^*_h u(t_i) - L^* u(t_i) | \leq C h$.

From Theorem 3.6 $| u(t_i) - u_i | \leq C h$.

And so,

$$| T^*_i | \leq | L^*_h u(t_i) - L^* u(t_i) | + | b(t_i) | | u(t_i) - u_i | \leq C h. \quad (20)$$

From [3], using stability result, we have,

$$| T^*_i | \leq C h \text{ for } i=0 (1)N. \quad (21)$$

Now the desired estimate (14) follows from (21) and the discrete stability result.

Note: From the above results following observations have been noted, namely,

1. Theorem 5 and Theorem 6 gives error estimates b for the absolute errors for the numerical solution of the solution and its first derivative of the solution of the SPP (1a,b) using the scheme (4a-c).
2. A single scheme itself enough to solve both solution and its first derivative.
3. On using single scheme, it reduce both storage space and computation time on implementing the scheme in modern computers.
4. To view the boundary layer one can use variable mesh in case of derivative of u.
5. At t = 1, ($t = 1 = t_N$) , the scheme satisfies the relation (18c) $u_N + \varepsilon v_N = \phi_2$ as h goes to zero.
6. The relation (18c) have a bound ,

$$| u_N + \varepsilon v_N - \phi_2 | \leq | u(1) - u_N | + \varepsilon | v(1) - v_N | \leq C h .$$

Since, $u_N + \varepsilon v_N - \phi_2 = u_N + \varepsilon v_N - [u(1) + \varepsilon v(1)] = -[u(1) - u_N] - \varepsilon [v(1) - v_N]$

and from (6) and (14), $| u(1) - u_N | \leq C h$ and $| v(1) - v_N | \leq C h$.

IV. FITTED MESH METHOD

In this section the mashes are no longer uniform it is necessary to extend the fitted operator method from the uniform meshes in section III to non-uniform meshes[1]. To introduce the method of fitted mesh the problems discussed in the previous section is is considered again here. In all cases a a piecewise uniform mesh turns out to be sufficient for the construction of ε- uniform method. Of course more complicated meshes may also be used but the simplicity of the piecewise uniform meshes is considered. Further more piecewise uniform meshes turns out to be adequate for handling a surprisingly a wide variety of singularly perturbed problems,.

Perhaps a simplest example of a piecewise uniform mesh is constructed on the interval Ω=(0,1) as follows. Choose a point τ satisfying $0<\tau \leq 1/2$and assume that $N=2^r$, for some $r\geq 2$. The point τ is called a transition point and it divides Ω into the two intervals (0,τ) and (τ,1). The corresponding piecewise uniform mesh is constructed by dividing both (0,τ) and (τ,1) into N/2 equal subintervals. Piecewise uniform meshes with N subintervals and a single parameter ι are denoted by Ω^r_N .

Fig 2. The piecewise uniform mesh Ω^r_N condensing at the point x=0

The piecewise uniform mesh Ω^r_N is used with the following location of the transition point

$$\tau=\min\{1/2, 2\varepsilon\, n(N)\} \quad (22)$$

depend on ε and N. This means location of the mesh points changes whenever ε or N changes.The transition point ï takes the value ½ if N is exponentially large and so Ω^r_N will be a uniform mesh with N subintervals. This will happen rarely in practice. We are interested in real time situation in which for all other values of ï, 0<ï<1/2. the subinterval (0,τ) is smaller than the subinterval and (τ,1). The fitted operator method in the previous section can be applied to the piecewise uniform mesh Ω^r_N . which leads to the following fitted mesh method

Find $\{u_\varepsilon\}_0^N \in R^{N+1}$, defined on Ω^r_N , such that $-D_+ u_0 = \phi_1$, $u_N + \varepsilon D_- u_N = \phi_2$ and for all 1≤i≤N-1,

$$\varepsilon \sigma_i(\rho) D_+ D_- u_i + a(t_i) D_+ u_i - b(t_i) u_i = f(t_i).$$

and

Find $\{v_\varepsilon\}_0^N \in R^{N+1}$, defined on Ω^r_N , such that $v_0 = -\phi_1$, and for all 1≤i≤N,

$\sigma_i(\rho)\, D_-\, v_i + a(t_i)v_i = b(t_i)\, u_i + f(t_i)$ where u'=v.

The fitted mesh method discussed above is ε-uniform and the solution satisfies the ε-uniform error estimate, for all $N \geq N_0$, $0<\varepsilon\leq 1$, $\sup \| u_{\varepsilon,N} - u_\varepsilon \|_\omega \leq C\, N^{-1} \ln(N)$, where $N_=$ and C are independent of ε.

V. ALGORITHM

An algorithm is presented so that an user can perform experiment without any difficulty in steps.

Step 1. Subdivide the interval (0,1) into N intervals and generate a sequence $x_0, x_1,...,x_N$.
Step 2. Rewrite the scheme(4a-c) in tri-diagonal form
Step 3. Using sweep method [21], rewrite the tri-diagonal form into a single step equation and solve for u_i.
Step 4. Rewrite the scheme (18a-b) as a single step form and solve for u'_i. And chck the condition(18c)
Step 5. Find ï from (22)
Step 6. Subdivide the subinterval (0,1) into (0,τ) and (τ,1).
Step 7. Subdivide both the subintervals into N/2 intervals.
Step 8. Repeat the steps2-3 and find the solution u_i in both the intervals (0,τ) and (τ,1).
Step 9. Repeat the step 4 and find u'_i.

Using the steps 1-9, both the numerical solution of u and u' can be evaluated. The scheme is solved as a single step method. So the method is computationally faster.

VI. EXPERIMENTAL RESULTS

To show the performance of the fitted operator and mesh method and to view the uniform convergence experiments were performed using modern digital computers. To test the method in the problem(1a,b), choose a=1, b=1, f=0, $\phi_1 = 0$ aad $\phi_2 = 1$. Experimental orders of uniform convergence and classical convergence are obtained following the procedure given in [1-20]. we define , for $N = 2^3, 2^4, 2^5, \ldots, 2^9$,

$$EMAX = \max \left| u^{N}_{J} - u^{2N}_{J} \right|, \quad j = 0(10N ,$$

$$RATE = [\log(E^1) - \log(E^2)]$$

and

$$ORDER1 = \text{average}\ (RATE)_h$$

where E^1 and E^2 correspond to EMAX for h=1/N and h=/2N respectively The experimental order of uniform convergence is taken to be the minimum value of ORDER1 overall ε considered . And the experimental order of classical convergence is taken as the value of ORDER1 for the largest ε considered.

Table 1 Numerical Solution Of U and U' Using Classical Scheme, **$h = 2^{-4}$, $\varepsilon = 10^{-2}$**

t_i	u_i	$u(t_i)$	$u(t_i) - u_i$	u'_i	$u'(t_i)$	$u'(t_i) - u'_i$
0.00000E+00	4.05826E-01	3.71469E-01	3.71469E-01	0.00000E+00	0.00000E+00	0.00000E+00
6.25000E-02	4.05286E-01	3.91354E-01	3.91354E-01	2.53641E+00	3.86849E-01	2.14956E+00
1.25000E-01	4.27691E-01	4.16332E-01	4.16332E-01	1.07797E+01	4.12248E-01	1.11920E+01
1.87500E-01	4.53751E-01	4.42911E-01	4.42911E-01	5.92668E+01	4.38568E-01	5.88282E+01
2.50000E-01	4.81793E-01	4.71188E-01	4.71188E-01	-3.08315E+02	4.66567E-01	3.08781E+02
3.12500E-01	5.11620E-01	5.01269E-01	5.01269E-01	1.62166E+03	4.96354E-01	1.62117E+03
3.75000E-01	5.43300E-01	5.33271E-01	5.33271E-01	-8.51053E+03	5.28042E-01	8.51106E+03
4.37500E-01	5.76942E-01	5.67317E-01	5.67317E-01	4.46837E+04	5.16754E-01	4.46831E+04
5.00000E-01	6.12668E-01	6.03536E-01	6.03536E-01	-2.34586E+05	5.97617E-01	2.34586E+05
5.62500E-01	6.50605E-01	6.42067E-01	6.42067E-01	1.23158E+06	6.35771E-01	1.23158E+06
6.25000E-01	6.90892E-01	6.83058E-01	6.83058E-01	-6.46579E+06	6.76360E-01	6.46579E+06
6.87500E-01	7.33674E-01	7.26666E-01	7.26666E-01	3.39454E+07	7.19540E-01	3.39454E+07
7.50000E-01	7.79105E-01	7.73058E-01	7.73058E-01	-1.78213E+08	7.65477E-01	1.78213E+08
8.12500E-01	8.27349E-01	8.22411E-01	8.22411E-01	9.35619E+08	8.14347E-01	9.35619E+08
8.75000E-01	8.78580E-01	8.74916E-01	8.74916E-01	-4.91200E+09	8.66337E-01	4.91200E+09
9.37500E-01	9.32984E-01	9.03772E-01	9.03772E-01	2.57880E+10	9.21645E-01	.57880E+10
1.00000*E*+00	9.90756E-01	9.90195E-01	9.90195E-01	9.243611e+01	9.80469E-01	5.61357E-02

Tables 1 gives the numerical solutions of u and its first derivative u′ using the classical scheme. It is observed that

1. he classical method solves the solution u but not the derivative u' .
2. the relation $u_N + \varepsilon\, v_N = \phi_2$ is not satisfied since $u_N + \varepsilon\, v_N = 1.921117 \neq 1 = \phi_2$.

It shows the issue.

Table. 2 gives the solution u and its first derivative u′ using the non-standard finite difference scheme(3.1a-c). It is observed that

1. the scheme solves the solution u and its derivative u'.
2. The scheme satisfies the relation $u_N + \varepsilon v_N = 1.00000006 \equiv 1 = \phi_2$.

Table 2 Numerical Solution Of U and U' Using Non-Standard Scheme(4a-c), **$h = 2^{-4}$, $\varepsilon = 10^{-2}$**

t_i	u_i	$u(t_i)$	$u(t_i) - u_i$	u'_i	$u'(t_i)$	$u'(t_i) - u'_i$
0.00000E+00	3.99105E-01	3.71469E-01	2.76366E-02	0.00000E+00	0.00000E+00	0.00000E+00
6.25000E-02	3.99105E-01	3.91354E-01	7.75170E-03	3.99105E-01	3.86849E-01	1.14863E-02
1.25000E-01	4.24001E-01	4.16332E-01	7.66972E-03	4.24001E-01	4.12248E-01	1.31440E-02
1.87500E-01	4.50498E-01	4.42911E-01	7.58716E-03	4.50498E-01	4.38568E-01	1.46148E-02
2.50000E-01	4.78651E-01	4.71188E-01	7.46369E-03	4.78651E-01	4.66567E-01	1.61201E-0
3.12500E-01	5.08564E-01	5.01269E-01	7.29436E-03	5.08564E-01	4.96354E-01	1.77571E-02
3.75000E-01	5.40345E-01	5.33271E-01	7.07388E-03	5.40345E-01	5.28042E-01	1.95364E-02
4.37500E-01	5.74113E-01	5.67317E-01	6.79636E-03	5.74113E-01	5.16754E-01	2.14699E-02
5.00000E-01	6.09991E-01	6.03536E-01	6.45566E-03	6.09991E-01	5.97617E-01	2.35695E-02
5.62500E-01	6.48111E-01	6.42067E-01	6.04469E-03	6.48111E-01	6.35771E-01	2.58488E-02
6.25000E-01	6.88614E-01	6.83058E-01	5.55611E-03	6.88614E-01	6.76360E-01	2.83220E-02
6.87500E-01	7.31647E-01	7.26666E-01	4.98170E-03	7.31647E-01	7.19540E-01	3.10046E-02
7.50000E-01	7.77370E-01	7.73058E-01	4.31246E-03	7.77370E-01	7.65477E-01	3.39131E-02
8.12500E-01	8.25950E-01	8.22411E-01	3.53897E-03	8.25950E-01	8.14347E-01	3.70651E-02
8.75000E-01	8.77566E-01	8.74916E-01	2.65044E-03	8.77566E-01	8.66337E-01	4.04803E-02
9.37500E-01	9.32408E-01	9.03772E-01	1.63549E-03	9.32408E-01	9.21645E-01	4.41790E-02
1.00000E+00	9.90677E-01	9.90195E-01	4.81904E-04	9.32306E-01	9.80469E-01	4.81904E-02

Experimental orders of numerical convergence are given in Table 3 for the scheme (4a-c). For $h > 2^{-8}$, it is noted that $u_i^{\,h} - u_{2i}^{\,h/2} = 0$ and so results are given for $h \leq 2^{-7}$.

Table 3 Experimental Orders of Numerical Convergence

ε h	2^{-3}	2^{-4}	2^{-5}	2^{-6}	2^{-7}	ORDER1
2^{-1}	1.87591E+00	4.78644E+00	1.03490E+00	2.36199E+00	5.79842E+03	1.15145E+00
2^{-2}	1.94338E+00	2.32870E+00	3.53306E-01	3.00379E+00	1.97522E+00	1.92088E+00
2^{-3}	1.93002E+00	1.96190E+00	9.33423E-01	9.44182E-01	1.96159E+00	1.54622E+00
2^{-4}	1.86918E+00	1.94350E+00	1.51876E+00	3.78081E-01	1.90919E+00	1.52374E+00
2^{-5}	1.65916E+00	1.89893E+00	1.75123E+00	1.98057E+00	8.34788E-01	1.62493E+00
2^{-5}	1.27042E+00	1.69416E+00	1.91112E+00	1.61135E+00	4.84394E-01	1.39429E+00
2^{-6}	9.82209E-01	1.30991E+00	1.69617E+00	2.08046E+00	8.05037E-01	1.37276E+00
2^{-7}	9.32144E-01	1.01993E+00	1.32069E+00	1.69565E+00	1.44032E+00	1.28175E+00

From Table 3,

Classical order of convergence = 1.15145

Uniform order of convergence = 1.15145

To view numerical data of uniform convergence of the solution u and its derivative u' in the following figures time versus absolute error(Error) is plotted. In figure 3, error stands for absolute error with respect to the solution u and in figure 4 absolute error refers to the absolute error with respect to he drivative of the solution u'.

It is observed from Figure 3, the numerical solution u of the test problem using non-standard method converges uniformly. Similarly numerical solution u of the test problem using standard classical scheme converges uniformly.

In figure 4, it can be viewed that the curve with respect to absolute error of u' using the standard classical method grow and decay at the right hand side of the interval [0,1], it shows the non-uniform convergence of numerical solution of u'. The curve with respect to absolute error of u' using the non- standard method is a constant curve, it shows the uniform convergence of u' at all nodal points of [0,1].

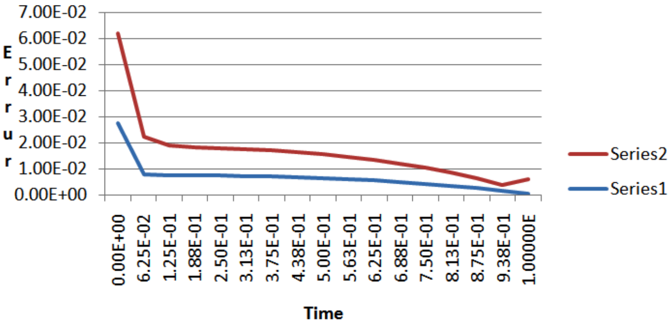

Fig 3. Uniform convergence of u

Series 1- absolute error curve using non-standard method, **Series 2-** absolute error curve using classical method

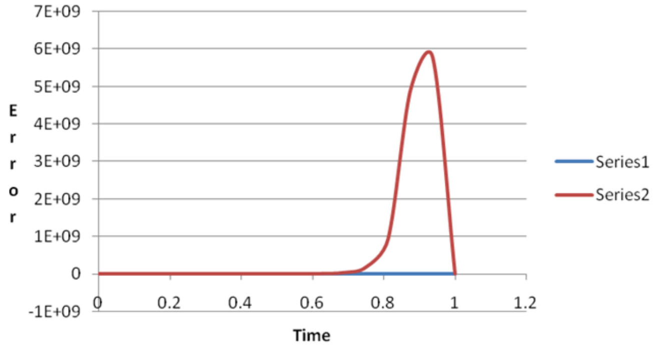

Fig 4. Uniform convergence of u'\

Series 1- absolute error curve using non-standard method, **Series 2**- absolute error curve using classical method

Again on using fitted mesh method to the above problem we have the transition point τ as

$$\tau=\min\{1/2, 2\varepsilon\, n(N)\} = \min\{1/2, 0.055451774\}= \min\{0.5, 0.06)= 0.06$$

since N=16 and $\varepsilon = 10^{-2}$. The subintervals (0, 0.06) and (0.06, 1) are subdivided into N/2=8 intervals each and so the interval (0, 0.06) with step 0.075 and the interval (0.06, 1) with step size 0.1175. In the subinterval (0. 0.06) more number of points can be achieved using fitted mesh method which can not be done by fitted operator method directly.

Using the sweep method,[21] the scheme is converted into a single step method. Because of it the computation time and storage space for the execution of the computer program get reduced considerably. No need for inversion of matrix for the evaluation of the numerical solution. From the above numerical experiment it is observed that the fitted operator and fitted mesh method proposed in this paper solves numerically both the solution u and its derivative u' and the respective numerical solutions converges uniformly.

VII. CONCLUSIONS

A difference scheme for a singularly perturbed problem in industry is designed which is uniform of order one. The numerical method is designed using fitted operator and fitted mesh methods. This method do not involve asymptotic expansion for fixing the transition point. Without using asymptotic expansion the solution value at terminal point is fixed, the labour is reduced in this work. The terminal point is fixed without iteration in this paper, this labour is reduced. And so automatically, both computation time and storage space are reduced enormously in the present work. This method is faster than other methods since we use sweep method replacing matrix inversion of tri-diagonal form of the fitted operator and fitted mesh method. One can apply this method in future to the problems in real time situations. Many research results are available, but present world really need a method which can be directly extended to non-linear and partial differential equations with a small parameter, this can be a future extension.

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