Volume 5, Issue 10, October-2015 ISSN: 2277 128X
International Journal of Advanced Research in Computer Science and Software Engineering
Research Paper
Available online at: www.ijarcsse.com

Mathematical Modeling of Democratic Relationship in Social Networks– Implementation of Game Theory

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Abstract: Social networks always maintain the relationship between the nodes of Social networks. The members in social networks may express their views independently when they want. The similar things happen in a democratic society where a member of society hold some powers and authority to express his/her view in the same society where he/she belongs. In the present paper the authors tried to establish the democratic relationship among nodes in Social Networks. The authors also tried to implement the use of Game theory model in Social Networks for the establishment of Democratic relationship between the nodes of Social networks.

Keywords: Democracy, Game theory, Social Networks, nodes, society;

I. INTRODUCTION

The word Democracy is an important term for formation of a government in a Society. The name comes from Greek for 'rule by the people'. At the most basic level, it is a type of government or political system ruled by citizens, people who are members of a society. In a democracy, citizens hold some level of power and authority, and they participate actively in the political, or decision-making, process of their government. Democracy is a form government. In the Social Networks it is observed that, nodes of social networks follow some features of democracy. Democratic relationship is essentially a relationship in a social networking , in which nodes have an opportunity to share /express/comments related to any topics [17][18].

A game theory problem can also be applicable in the some Social networking relation. Game theory deals with decision made by the members of social networks (or nodes) in the favor of two or more conflicts or competitive nodes of social networks. Game theory is a branch of applied mathematics that provides tools for analyzing situations in which parties; called players, make decisions that are interdependent. This interdependence causes each player to consider the other player's possible decisions, or strategies, in formulating his own strategy. A solution to a game describes the optimal decisions of the players, who may have similar, opposed, or mixed interests, and the outcomes that may result from these decisions. [19].

A matrix representation can be used to represents the democracy in social networking.

II. MATHEMATICAL DESCRIPTION OF CONNECTIVITY OF NODES IN SOCIAL NETWORK

A. *Social network* [1,2]

A social network is a social structure made up of individuals (or organizations) called "nodes", which are tied (connected) by one or more specific types of interdependency, such as friendship, kinship, common interest, financial exchange, likings or disliking, or relationships of beliefs, knowledge or prestige. Social network analysis views social relationships in terms of network theory consisting of nodes and ties (also called edges, links, or connections). Nodes are the individual actors within the networks, and ties are the relationships between the actors.

Hence, mathematically , social networking can be defined as the collection of socially connected elements/objects. i.e, set

$$S = \{ \text{social elements} : \text{social elements are connected} \}$$

In a social network S, consider two nodes (i)node1 represented by N_1 and(ii)node2 represented by N_2 and consider a function f defined between node N_1 and node N_2

$$\text{i.e., } f : N_1 \rightarrow N_2$$

such that message send by node N_1 is received by node N_2 .i.e, if $m_1,m_2,m_3\ldots\ldots..m_n$ be the message send by the node N_1 to N_2,then $f(m_1)$, $f(m_2)$,$f(m_3)$,……..$f(m_n)$ will be message received by N2.

$$\text{i.e., } \{m_1,m_2,m_3\ldots\ldots..m_n\} \rightarrow \{ f(m_1), f(m_2) ,f(m_3),\ldots\ldots..f(m_n)\}$$

i.e, set of send message by node N_1 → set of received message by node N_2.

B. *Definition1* [3]

A social network is modeled as a graph G(V, E), where V represents a set of users embedded in a social context, and the edge set $E=\{(x, y)| x, y \in V\}$ represents friendship among users . An edge e = (x, y) is added to E when a friend request from x to y or from y to x is accepted. In SNSs(social networking sites), such as Facebook and LinkedIn, edges are

usually undirected. For each user u (u ∈V), the set F(u)={ x | x ∈V, (x,u)∈E}represents the friend list of u. Note, for each edge e = (x, y), x ∈ F(y) and y ∈F(x).

The mutual friends between two users can be defined as follows:

C. Definition 2 [3][4]

Given two users x and y (x, y ∈V), we define the set of the mutual friends, MF(x, y),between them as MF(x, y) = F(x) ∩ F(y). Here, MF(x, y) stands for mutual friendship between x and y,

Intuitively, the definition of the mutual friend has two properties:

Given x, y, z ∈ V, y ∈ MF(x, z) ⇔ y ∈ F(x) or y ∈F(z).

Given x, z, MF(x, z) = MF(z, x); i.e., mutual friendship is irreflexive and symmetric.

III. FEATURES OF DEMOCRACY AND ITS CO-RELATION WITH SOCIAL NETWORKS

Following features of democracy are correlated with social networks[20].

A. In the Majority Rule– the system of government is based on parliamentary majorities.

In the social networking, let a Social Networking S have n numbers of nodes which may be either directly, indirectly or virtually Connected.

Let S= {n_1,n_2,n_3,…….,n_n}. Let n_i and n_j ($i \neq j$) be two nodes of the set S and their views are differ on any topics and let m(less than n) numbers of nodes follows nodes n_i and p(less than n) numbers of nodes follows nodes n_j and if m > p then nodes n_i has majority of followers with respect to nodes n_j. Here,i=1,2,……,n and j=1,2,……,n.

B. Representative Elections -the people are allowed to elect representatives to speak for their views and interests.

Figure1(a): Examples to show democracy for expressing views and interest[21]

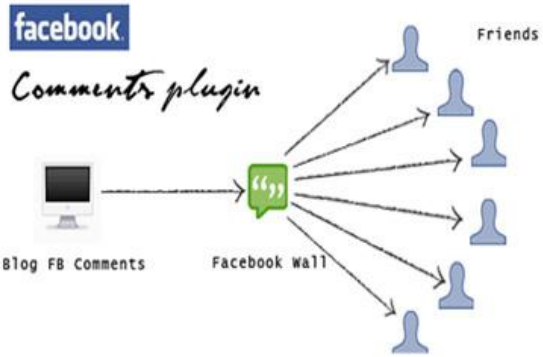

Figure1(b):Example of comment of a message [36]

In the social networking, Let a Social Networking S have countable numbers of nodes which may be either directly, indirectly (mutually) Connected. If the relations between nodes are publically related then the message send by one node can be seen by nodes which are publically connected in direct or mutually related in social networks.

C. Multi party system-voters have the opportunity to choose from a variety of political parties, representing a wide range of political opinion.

In the social networking, it is possible for the nodes to create a group of nodes in which it is possible to express their views .Hence, a multiparty system can be generated in a social networks.

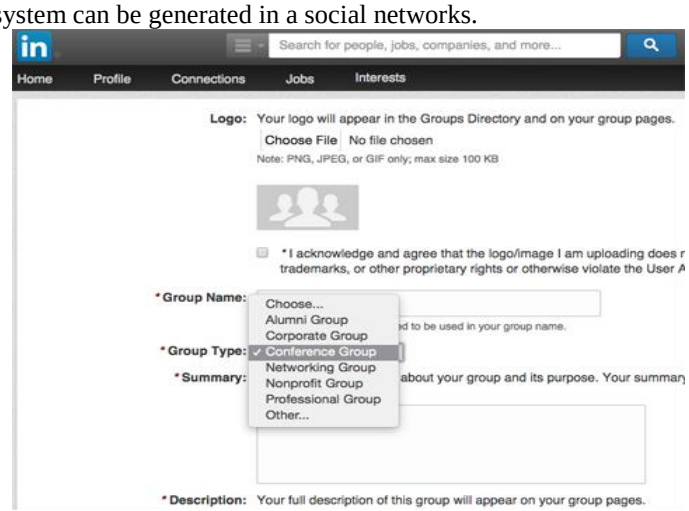

Figure2 :Examples to show to form different types of a groups in a social networks[22]

D. Freedom of speech-no restriction is placed on the right to opinions and express then openly.

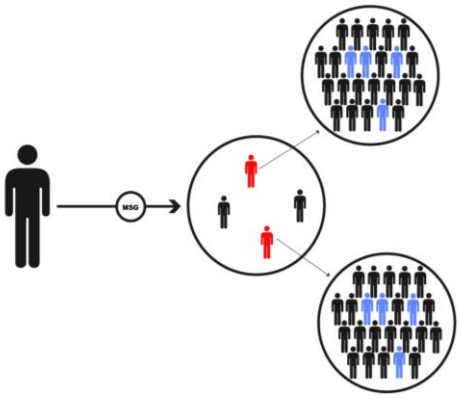

Figure3: Examples to show freedom of speech of nodes in a social networks[23]

A nodes of social networks have freedom to express their views to the members of social networks .Nodes of a social network can express their view to their direct connected nodes publically or privately and indirect(mutual) or virtually connected nodes publically.

E. Freedom of association-no restriction is placed on people organizing political parties to take part in democratic life.

Figure4: Example of making friends in a social networks[24]

In the social networking it is possible for nodes to make connection with the other members (nodes) of social networks. A friend request may be send by nodes to the members of Social networks and members of Social Networks have option to either accept or reject their friend request. If the friend requested is accepted then the connected nodes are treated as direct connected (or associated) nodes. A node can send the friend request to any members of social networks.

F. Freedom of Assembly-no restriction is placed on the right to hold meetings or organize demonstrations provided those do not violate the rights of others.

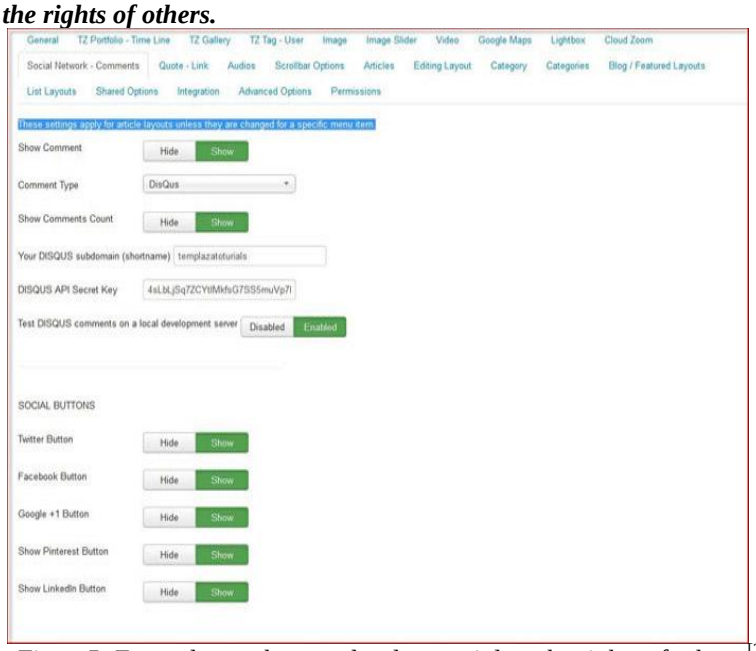

Figure5: Examples to show nodes do not violate the rights of others [24]

Members of social networks also can contact or send message to each other at any time without violating the rules of relationship between the nodes of social networks.

G. Respect the individual rights-the state protects individuals whose rights are threatened by the actions of others.

Figure6 : Examples to show a setting for security of individaual nodes[27]

In the social networking, it is possible to receive the message or related notification in the secure manners.

I. Respect for Minority rights-minorities should not have their basic rights violated by the majority.

Figure7: Example to different types of social networks with countable comments[27]

The different types of Social Networking does not violate the rules and regulations of other social networkings.Hence, there is democratic rules follows for different types of social networks.

J. Respect for the Law-citizens who are given democratic rights should obey the laws that provide these rights.

Figure8 : Example to show for nodes to follow the different types of option in social networks[28]

Members of Social Networks have option to take decision for making either public or private friends/groups. And therefore, these members may have option to follow the different types of law.

K. Respect for Democratic Procedures-individuals or groups who have grievances against the system should operate within it, seeking to change the law through legal means.

Figure9: Example for showing different types of settings in the social networks [30]

Figure10: Example for showing different types of nodes to view message in the social networks [29]

In the social networks it is possible to make different types settings to tackle different types of changes for friend's circles.

L. Channels of Influence-individuals and groups have channels of access to decision makers at every level.

Figure11(a) : Examples to show decision making situation in social networks[31]

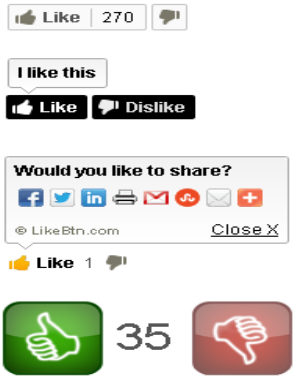

Figure11(b) Example of countable like and dislike message[37]

In the social networking there is also a decision making situation works for a nodes of Social networks to make the relatioship with the nodes of social networks.

IV. TYPES OF DEMOCRACY

The basic needs of a node in social area networks are to maintain the relationship among all other nodes. The relationship between two nodes may be directly or indirectly connected[34][35].

A. Direct democracy: Direct democracy is related with the direct relation between nodes of social networks.

Direct relation is a relationship between two nodes in which relationship are established using direct connection by using edges. Direct relation may be one-to-one or one - to – many. Therefore, these nodes may be considered as direct connected nodes. Mathematically, these types of relation are defined as follows:

Set R = {nodes: nodes are members of social networking and there exists an edge between two connected nodes}

Hence, nodes can establish direct democracy in these types of relationship.

B. Indirect democracy: Indirect democracy is related with the indirect relation between nodes of social networks.

Indirect relation is that type of relation in which relationship are not directly established. Mutual relationship or virtual relationship is an example of indirect relationship. Indirect relationship is an ordered pair which may be defined as follows in the following cases:

Case1: Combination of Direct and Indirect Connected nodes along mutual relationship

The relationship between nodes of social networking can be enlarged if the mutual relationship among nodes is public. There exists an option for both direct connected nodes to enlarge the relationship among their mutual nodes. i.e., these

types of relationship depend on direct connected nodes. There is need to establish an edge (i.e., method of sending friend request and accepting or rejecting this) between any two nodes in this case.

Mathematically, these types of relationship are defined as follows:

Set R1 = {(direct connected nodes, mutual nodes): nodes are members of social networking and there may exist an edge to establish relationship between nodes}

Note: It must be noted here that the relationship is working only on the connected set. So therefore, the democratic relationship can be established between the indirectly connected nodes also.

Case2: Combination of Direct and Indirect Connected nodes along virtual relationship

In the social networking there are some nodes which remain as disconnected set but there may exist a relation and this type of relation is called as Virtual relation. There may not exist edges between two nodes but a virtual relation may exist between them. These types of relation are an ordered pair. Mathematically, these types of relation can be defined as follows:

Set R_2 = {(direct +indirect connected nodes, virtual related nodes): nodes are members of social networking and there may or may not exist an edge to establish relationship between nodes}

Note: This relationship is working on the connected and disconnected set. Also this is an extension of set of Combination of Direct and Indirect Connected nodes along mutual relationship. So in these types of relationship democratic relationship can be established between the connected and disconnected nodes of social networks.

V. IMPLEMENTATION OF GAME THEORY ALGORITHMS IN SOCIAL NETWORKS

Following mathematical terms are used in a game theory[32].

A. Game

A game is a formal description of a strategic situation.

B. Game theory

Game theory is the formal study of decision-making where several players must make choices that potentially affect the interests of the other players.

C. Mixed strategy

A mixed strategy is an active randomization, with given probabilities, that determines the player's decision. As a special case, a mixed strategy can be the deterministic choice of one of the given pure strategies.

D. Nash equilibrium

A Nash equilibrium, also called strategic equilibrium, is a list of strategies, one for each player, which has the property that no player can unilaterally change his strategy and get a better payoff.

E. Payoff

A payoff is a number, also called utility, that reflects the desirability of an outcome to a player, for whatever reason. When the outcome is random, payoffs are usually weighted with their probabilities. The expected payoff incorporates the player's attitude towards risk.

F. Perfect information

A game has perfect information when at any point in time only one player makes a move, and knows all the actions that have been made until then.

G. Player

A player is an agent who makes decisions in a game.

H. Rationality

A player is said to be rational if he seeks to play in a manner which maximizes his own payoff. It is often assumed that the rationality of all players is common knowledge.

I. Strategic form

A game in strategic form, also called normal form, is a compact representation of a game in which players simultaneously choose their strategies. The resulting payoffs are presented in a table with a cell for each strategy combination.

J. Strategy

In a game in strategic form, a strategy is one of the given possible actions of a player. In an extensive game, a strategy is a complete plan of choices, one for each decision point of the player.

K. Zero-sum game

A game is said to be zero-sum if for any outcome, the sum of the payoffs to all players is zero. In a two-player zero-sum game, one player's gain is the other player's loss, so their interests are diametrically opposed.

Example: Let A and B plays a game in which each has social networks S_1, S_2,S_3,.....S_n. Let players A views is opposed by players B.

Let number of casting votes in favor for player A is N_1, N2, and N3.....Nn. From the social networks S_1, S_2, S3, .S_n respectfully.

and Let number of casting votes for the same topics (not favors for players A) for player B (favoring B) is M_1, M_2,M_3,…….M_n from the social networks S_1, S_2,S_3,.....S_n respectfully.

Let each player select the number of votes of a social network without the knowledge of other choice.

If the difference of numbers of casts votes(likes) in social network (Si, Sj) for i=1,2,…..,n and j=1,2,….,m is positive then players A will win and players B will loss.

If the difference of numbers of casts votes(likes) in social network (Si, Sj) for i=1,2,…..,n and j=1,2,….,m

is negative then players B will win and players A will loss.

Player A

Social Network	**Number of favourable nodes(votes) in each social networks for player A**
S_1	N_1
S_2	N_2
S_3	N_3
……	……
S_n	N_n

Table1

Player B

Social Network	**Number of favourable nodes(votes) in each social networks for player B**
S_1	M_1
S_2	M_2
S_3	M_3
…..	…..
S_n	M_m

Table2

Pay-off matrix for Player A

Table3

A \ B	S_1	S_2	……	S_n
S_1	N_1-M_1	N_2-M_2		N_n-M_n
S_2	N_1-M_2	N_2-M_2		N_n-M_2
…….				
S_n	N_1-M_m	N_2-M_m		N_n-M_m

These types of matrix representation in game theory problem can be solved from using following method as per requirement of the problems.

(i) Game with saddle point

(ii) Game without saddle point

(iii)Solution of 2x2 games (Mixed Strategies)

(iv)Solution of rectangular games with mixed strategy.

(v)Solution using Dominance rule.

(vi)Graphical method of solution for game theory problem

(vii) Solution of a game with linear programming problem.

So therefore, one can conclude that by using game theory results/methods it is possible to establish democratic relationship in social networks.

VI. CONCLUSION AND FUTURE SCOPE

The last few years the impact of social area networks in our day to day life is unbelievable. Srivastav et al[1-3] already made extensive studies on mathematical modeling of social area networks. In the present work the authors have shown that there is a scope to establish the democratic relationship between nodes of Social networks. The authors also explored the Game theory problem and its application in democratic relationship in social area networks. However, there is lot of scope to explore the game theory problem in social are networks.

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