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Entanglement Characterization of B Gate

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Abstract: -Quantum computer is shown to be faster than classical computer in certain problems. The weird properties of quantum mechanics such as quantum superposition, quantum entanglement, quantum tunneling make quantum computer so fast and secure. Quantum gates and quantum circuits are basic parts for quantum computer. Quantum information are carried out and manipulated through quantum gates. While Hadamard, Quantum Not gate, Phase gate are single qubit gates, Controlled-Not gate or CNOT gate is two qubit gates. CNOT gate is basically two qubit functional gates, in which first qubit acts as target and second qubit acts as control. It is universal gate and any multiple qubit logic gates can be made from CNOT gate.In this work, we present the review on basic fundamental elements for quantum computer, namely quantum logic gates.Finding universal quantum circuit comprising of basic gates is an important task in the research area of quantum computation. One of the universal circuits is constructed using CNOT. In the recent time, B gate is used for the optimal implementation of gate in quantum computing in which switching time is minimum than CNOT gate. Aim of the present work is to analyze the characteristics of B gate.

Keywords – Quantum computer, Quantum entanglement, CNOT gate, B logic gate

I. INTRODUCTION

Quantum computers are more powerful and capable than ordinary computer [1].Quantum entanglement is the heart of quantum computing. It is play an important role in quantum information and quantum computing [2][1305.4454]. Quantum entanglement is making the quantum computer so fast and speed up. Although there are many difficulties to achieve this type of mysterious quantum properties at atomic level, main challenge is to combat quantum decoherence as a consequence of coupling with external environment [3]

Logic gates and logic circuits took part major role in the theory of computation. Quantum computers require quantum logic, something fundamentally different to classical Boolean logic. This difference leads to a greater efficiency of quantum computation over its classical counterpart Quantum gates and quantum circuits are somewhat different in mechanism and basic operation [4] .CNOT is universally gate and we can make any Qubit using CNOT gate. Unless in the CNOT gate has decoherence problem and switching time maximum that is problem for computation [5]. B gate can be use place of CNOT gate and it has advantage over the CNOT gate [6].in this paper we simplify matrix and unitary operation for B gate.

II. QUANTUM SUPERPOSITION

The fundamental principle behind quantum mechanics is the 'superposition of quantum states' [7] called as Quantum Superposition. According to this phenomenon, two or more states may superimpose and give a new state [8]. It can be also define as the quantum superposition of a quantum system as a linear superposition of those two and more states.Superposition can be mathematically represented as—

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (1)$$

Where, for example, if $\acute{\alpha}$ is amplitude of state $|0\rangle$, $\acute{\beta}$ is amplitude of state $|1\rangle$then, α and β are complex number and $|\alpha|^2+|\beta|^2=1$(State of Normalization)

If it could be assume asa k-level quantum system then its superposition state defined as –

$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \ldots\ldots\ldots \alpha_{k-1}|k-1\rangle$,(2)

Where,

$\sum_{j=0}^{k-1} |\alpha_j|^2=1$ (State of Normalization)

Thebasic unit of information in Quantum computing is referred to as Quantum Bit (Qubit). Unlike classical state, which can be either $|0\rangle$or$|1\rangle$, Qubit can take state which is complex superposition of $|0\rangle$and$|1\rangle$. The classical states $|0\rangle$ and $|1\rangle$ are referred to as 'computational basis'. If it is measured, Qubit in terms of computational basis we can see its states after measurement will be $|0\rangle$ with probability$|\acute{\alpha}|^2$, and $|1\rangle$ with probability$|\acute{\beta}|^2$ [9].

III. QUANTUM ENTANGLEMENT

The multiple particles are linked together in a way when I has been measured of one particle's quantum state and then it could be determine the possible quantum state of the other wave functions. This phenomenon is called quantum entanglement. The most basic entangled quantum system is a pair of Qubits. Suppose there are two Qubits they are given by,$\alpha_0|0\rangle + \alpha_1|1\rangle$, and$\beta_0|0\rangle + \beta_1|1\rangle$. Then joint state of two Qubits is given by the tensor product and is represented as-

$$\alpha_0\beta_0|00\rangle + \alpha_0\beta_1|01\rangle + \alpha_1\beta_0|10\rangle + \alpha_1\beta_1|11\rangle$$

Generally two Qubits are entangled and they cannot be decomposed into individual Qubits. Quantum entanglement state is generally represented by [10] –

$|\psi(x_1,x_2\rangle = \int_{i=1}^{\infty} \alpha_i\, |\psi(x_i)\rangle_i |\emptyset(x_2)\rangle_i (3)$

IV. QUANTUM LOGIC GATES

Quantum computation falls under the category of unitary transformation. A unitary transformation isn't irreversible; because, there is need reversible gates in order to be able to implement quantum gates. A unitary transformation can be operated on a single qubit or multiple qubit. These quantum gates are the basic building blocks of quantum computers. A quantum logic gate works based on unitary operator. It acts on the states of a specific set of Qubits. If the number of Qubits is n, the quantum gate is given by $2^n \times 2^n$matrix in the unitary group U (2^n). It's a reversible gate: we can recover the initial quantum state from the final state [11].

A. Quantum NOT gate

Quantum NOT gate is a one quantum bit gate. It is change the role of qubit according linear manner. Quantum not operation on qubit defines as unitary basis [12]-

$X=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$(4)Quantum not gate is change the role of $|0\rangle$ and$|1\rangle$. It takes the input as $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and corresponding out of not gate is $|\psi\rangle = \alpha|1\rangle + \beta|0\rangle$.

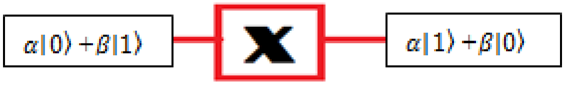

Figure – function of NOT gate

B. Hadamard Gate

Hadamard gate is the most common gate in quantum computing for creating a uniformly distribution super position state [13]

Hadamard matrix is written for one qubit as –

$H = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$(7)

$|x\rangle \rightarrow \mathcal{H} \rightarrow (-1)^x|x\rangle + |1-x\rangle$(8)

Hadamard matrix is the ($|0\rangle$&$|1\rangle$) comutatinal basis .

$|0\rangle \rightarrow \mathcal{H} \rightarrow \frac{|0\rangle+|1\rangle}{\sqrt{2}}$(9)

$|1\rangle \rightarrow \mathcal{H} \rightarrow \frac{|0\rangle-|1\rangle}{\sqrt{2}}$(10)

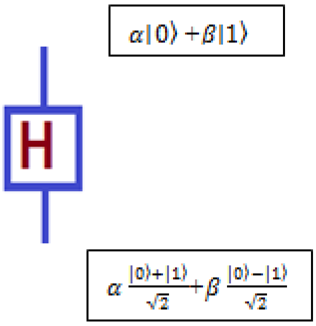

Figure – function of Hadamard gate

C. CNOT Gate

CNOT gate is the two-qubit controlled -NOT or CNOT-gate. It takes as input a control and a target qubit and executes a NOT operation on the target only if the control is in the state$|1\rangle$. If the control is set to$|0\rangle$, nothing happens to the target [14].

CNOT gate is the computation basis (when first qubit as control and second is target)-

$|00\rangle \rightarrow |00\rangle$,

$|01\rangle \rightarrow |01\rangle$,

$|10\rangle \rightarrow |11\rangle$,

$|11\rangle \rightarrow |10\rangle$,

CNOT gate can be considered as a generalized XOR gate since the action of the gate can be considered as $|A, B\rangle = |A, B \oplus A\rangle$where $\oplus$ is represented 2-module addition.

CNOT gate operation defines on qubit by unitary operation –

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} (10)$$

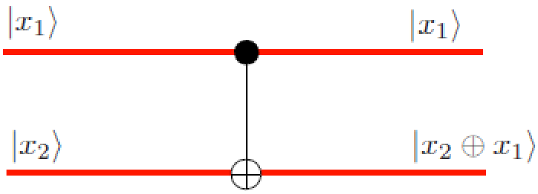

Figure – function of CNOT gate

D. *SWAP gate*

SWAP gate, a simple quantum circuit,made up of three CNOT gates, reads from left to right wherein each line represents a quantum passage.

SWAP Gate is represented by a unitary matrix as—

$$U_{SWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} (11)$$

E. *Controlled-Z gate*

It is another example of a controlled gate, i.e. gates in which the operation is of the kind "if A is not true then do B".

$$\text{Z gate} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} (12)$$

F. *B Gate*

Quantum circuit is a combination of quantum logic gates that requires minimum switching time and a minimum effect of decoherence which arises from Qubits and external environment. B gate is more efficient than CNOT gate.Itovercomes all the shortcomings in CNOT gate, like reduced decoherence affect and improved switching speed and time of operations[6].

Unitary transformation for B gate is given by[15] –

$$H_u = C_1\sigma_x \oplus \sigma_x + C_2\, \sigma_y \oplus \sigma_y + C_3\, \sigma_z \oplus \sigma_z \qquad (13)$$

Where, σ_x ,σ_y and σ_z are the Pauli matrices.

$C_1=\frac{\pi}{2}$, $C_2=\frac{\pi}{4}$ and $C_3= 0$

B gate operation defines on qubit by unitary matrix –

$$U_B = \begin{bmatrix} 0 & 0 & 0 & \frac{\pi}{4} \\ 0 & 0 & \frac{3\pi}{4} & 0 \\ 0 & \frac{3\pi}{4} & 0 & 0 \\ \frac{\pi}{4} & 0 & 0 & 0 \end{bmatrix} (14)$$

V. ENTANGLEMENT CHARACTERISTICS OF CNOT AND B GATES.

Let two qubit for input for CNOT and B Gate as $|\psi_1\rangle$ and $|\psi_2\rangle$.

$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$(15)

$|\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$(16)

Total input for gates is in the tensor product of these two Qubits and it is given as-

$|\varphi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$(17)

$$|\varphi\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2\Big|01\rangle + \beta_1\alpha_2\Big|10\rangle + \beta_1\beta_2|11\rangle \qquad (18)$$

CNOT gate operation for $|\varphi\rangle$–

$$\text{CNOT}|\varphi\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1\alpha_2 \\ \alpha_1\beta_2 \\ \beta_1\alpha_2 \\ \beta_1\beta_2 \end{bmatrix} [|00\rangle \quad |01\rangle \quad |10\rangle \quad |11\rangle]$$

Output of CNOT gate –

$|\varphi'\rangle = |\alpha_1\alpha_2|00\rangle + \alpha_1\beta_2\Big|01\rangle + \beta_1\beta_2\Big|10\rangle + \beta_1\alpha_2|11\rangle$

Entanglement in qubit [16] when $|\alpha_1\rangle$=0, 1 or $|\beta_1\rangle$=1, 0. Otherwise output of CNOT gate is in non-entanglement.

B gate operation for $|\varphi\rangle$-

$$B|\varphi\rangle=\begin{bmatrix} 0 & 0 & 0 & \frac{\pi}{4} \\ 0 & 0 & \frac{3\pi}{4} & 0 \\ 0 & \frac{3\pi}{4} & 0 & 0 \\ \frac{\pi}{4} & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \alpha_1\alpha_2 \\ \alpha_1\beta_2 \\ \beta_1\alpha_2 \\ \beta_1\beta_2 \end{bmatrix}[|00\rangle \quad |01\rangle \quad |10\rangle \quad |11\rangle]$$

Output of B gate $|\varphi''\rangle$ written as

$|\varphi''\rangle=\frac{\pi}{4}(\beta_1\beta_2|00\rangle + 3\beta_1\alpha_2|01\rangle + |3\alpha_1\beta_2 10\rangle + \alpha_1\alpha_2|11\rangle)$

Entanglement in qubit when, $|\alpha_1\rangle$=0, 1 or $|\beta_1\rangle$=1, 0 and $|\alpha_2\rangle$= 0, 1 or $|\beta_2\rangle$=1, 0

VI. CONCLUSION

In this work, it is studied about single and two qubit gates. In particular, matrix form of B gate is introduced and entanglement producing ability of B gate is analyzed. It is found we required more entanglement condition on the input product state upon which B gate can produce maximally entanglement state.

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