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# Mathematics Education: Tragedy or Comedy?

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***Abstract—The aim of the present paper is to show how to use situational games to improve mathematics education. Many students consider mathematics as a tragedy when it is time to learn it as they find it too abstract and difficult. However, if the learning of mathematics is communicated in a non-traditional way like through a game or through theatre or competitions students can find it enjoyable and laugh a lot like in a comedy. The focus is on identifying key elements in developing teaching and learning mathematics concepts while solving problems in theatrical circumstances. In particular we discuss a teaching method to make mathematics more attractive to young people with interesting activities. The method includes guidelines for teachers on how to design problem solving activities in a more challenging atmosphere. On the other hand automatic computer generated tests may help not only students to use it in the learning procedure but teachers to minimize subjective factors in their decisions when giving grades.***

***Keywords—didactical situation, situational games, didactics, computer-assisted learning, computer generated tests.***

## I. INTRODUCTION

This special issue reflects a concern about the needs to improve mathematical competences, a better understanding of concepts and relate didactical research to practices of teaching and learning. There is a huge accumulation of how to teach and learn mathematics. Today children are born into a complex and changing world. Human knowledge in mathematics is growing rapidly. Mathematics knowledge and skills empower a person. However, the complexity and diversity of this knowledge leaves some uncertainty about its successful implementation in the mathematics classroom. Throughout the $19^{th}$ century mathematics became increasingly abstract. Mathematicians before the $19^{th}$ century met real problems to solve and they discovered solutions, definitions, theorems and proofs. They were interested in mathematics by relating mathematical concepts for their applications in real life. Students are expected to acquire a considerable amount of knowledge in mathematics and other subjects, however this knowledge is new to them and they are not machines to pour it into their heads. Some of them are even expected to further develop mathematical knowledge and to solve new or unsolved problems. Therefore teachers must remember through what stages certain mathematics concepts went through while they were discovered. Students need to be engaged in independent research and in activities encouraging experimental learning and investigation. Carefully designed didactical situations provide opportunities for students to understand why mathematical concepts are useful for them and how to apply them to real-life situations. However, a very influential model is the theory of didactical situations, founded by Brousseau [2]. In French, the word *connaissance* refers to the personal knowledge of an individual developed and mobilised in concrete situations. It is to some extent linked to situational approaches. Didactic perspectives of integrating games and activities into mathematics lessons were described by Gyöngyösi [3].

Most of the personal knowledge of an individual is used in certain situations of practice, but need not be explicit or shared with others. On the contrary, the French word *savoir* implies socially shared and acknowledged knowledge called public knowledge (or simply knowledge). Particularly, public knowledge includes scholarly knowledge found explicitly in textbooks, scientific papers and so on. Hence provided that learning through situational games in didactical situations is successful then computer generated questions may help students to learn concepts making their individual knowledge more explicit. All public knowledge has its origin in personal knowledge. While personal knowledge is extremely contextual, public knowledge – particularly scientific knowledge – tends to be decontextualized, which means freed from a particular situation. The process of decontextualisation is, as pointed out by Brousseau [2], crucial for the recognition, sharing and use of scientific knowledge.

"The teacher's work is to some extent the opposite of the researcher's; she must produce a recontextualisation and repersonalisation of the knowledge. It must become the student's knowledge, that is to say, a fairly natural response to relatively particular conditions, conditions that are essential if she is to make sense of this knowledge." Brousseau[2].

However, the teacher must also ensure a final movement from personal knowledge to public knowledge – as the researcher must decontextualise her findings. In this work computers can help as students like using these modern tools and they can even study concepts of public knowledge at home without teacher's help providing the teacher and students with more time to play in the classroom in order to understand and enjoy mathematical activities.

"The students, in their return, must decontextualise and repersonalise their knowledge in such a way as to identify what they produce with the knowledge which is in current use in the scientific and cultural community of their time." Brousseau[2]. After acquiring public knowledge students can test it filling in computer generated tests. The software designed by Bednarik [1] outputs questions in electronic or printable format. In case of multiple-choice questions in an electronic version both generated questions and possible answers appear in the computer environment in front of the user. In this version, users attending the test can have a local menu on the screen offering all the possible answers to questions by clicking on the blank part of the sentence containing the question to be answered. After giving the answer the chosen alternative is automatically substituted in the sentence. In order to make the test more difficult it is possible not to give any alternatives but students have to find out one or more missing words to complete a definition or a theorem. After finishing the test, the filled test sheet can be saved in a file format and can be forwarded to the reviewer of the test. Golden hours may be saved in this way as students can learn concepts and test their knowledge at home and they can experiment and play more mathematics during lessons.

Brousseau's model for teaching situations can be called a double game[11]. The central part of it is the adidactial game in which the pupils interact with the tasks and objects proposed by the teacher – the adidactical milieu.But in order to for this game to happen, the teacher must construct, introduce and regulate it – this interaction between the teacher and the pupils' adidactical game is called the didactical game by Brousseau [2].Such a double game is called a didactical situation, with the two games being 'played' in different phases: the teacher's devolution of the adidactical game and the pupils' adidactical game, and various forms of regulation of the game that the teacher exercises. Hence the teacher as a director organises games in didactical situations and students play as actors and actresses while discovering and learning mathematics.The use of the term 'game' is not just metaphorical. Unfortunately here it is not possible to go in detail Hersant and Perrin-Glorian[5]situational games through which students learn mathematical knowledge involve teachers and pupils with their intentions, roles and skills and their interactions follow certain formal and informal rules. The purpose of situational games can thus be considered as a didactical'conquest' of mathematical territories, explored by mathematicians in the past but still not widely accessible to average students in average classes. A similar idea for the didactics of science was formulated by Lijnse [7].In French teacher education, the professional memoir [10] is a written report on teaching practice carried out at the end of teacher education, which could provide student teachers with a chance to see connections between didactical theory and practice, as a sort of mini research project.

Approaches to the didactics of mathematics seem to be particularly well developed in France therefore French didacticians strongly influenced this research of the authors. This paper mainly focuses on the context of mathematics teaching and learning, however, several of the points made seem to have a wider scope.

## II. DESIGNING SITUATIONAL GAMES

Any teaching starts from an intention that somebody (students) acquire some particular knowledge. Before planning a didactical situation from situational game perspective, steps of situational game design needed to be considered. The first step is to determine the target group, topic and focus within the broader topic. The second step is to set up the situational game context, to choose and reveal time, location, people and relationships, problems. The final step is to find out the framework of the situational game context. In particular, the form of the situational game context is determined. Moreover, from what point of view the situational game context is examined. The third element is to create a problem situation and a motivating force to solve the problem [8]. Focusing on one particular field of mathematics such as geometry we demonstrate how to approach it in an interesting and challenging way such that students can play roles while solving problems using their acting skills which may give them a feeling of success even if they do not find the solution for a mathematics problem. However, this partially successful feeling can be completed if they are stimulated to go on playing and help each other to find the solution for the mathematics problem together.

## III. MOTIVES OF THE RESEARCH

According to the 2012 PISA survey the Hungarian students do not excel in creative problem solving. At the beginning of the 80s the Hungarian students were still a world leader in mathematics and scientific knowledge in the field, a decade later they were all in the middle. This problem is likely to mirror the situation in many other countries.

Our research took place at Comenius Faculty of Eszterházy Károly College and students were prospective primary school teachers. Most teacher training colleges are aware of the difficulties students experience during their first months of study and therefore apply a variety of measures to ease the institutional transition. Advance mathematics programmes in colleges adapt the first courses to the students' background from secondary school. In Hungary this means for instance that these first courses focus on students' practice with concrete calculations, while the theoretical part (precise definitions, theorems and proofs) is left in the background or it is entirely omitted. Topics like thinking methods, algebra and number theory, functions, geometry and statistics and probability are revisited and extended progressively together with teaching methods to enable students to be able to transfer this knowledge to pupils of age between 6-10. However, students will encounter the problem of how to teach concepts precisely but not too abstract and easy to understand.

In order to improve students' creative problem solving competences we were experimenting with teaching a geometry course differently in our College where primary school teachers are also trained.

Hence our hypothesis is that didactically organised situational games can create enjoyable learning situations that can arouse students' interest to take part in solving problems actively while their understanding of concepts is improved. The idea came from a European funded Commission Funded Project "Learning mathematics through new communication factors". The aim of this project is the development of methodology in teaching and learning mathematics with the creation of two main tools that can be used by teachers. The tools will be created in such a way in order to be offered as in-service training courses to teachers who teach mathematics to pupils of age 9-18. We wanted to target pupils of age 6-10 and their future teachers. "Pupils will be able to be taught and trained by their teachers on how to explain a math theorem, or a mathematical method or a mathematics application in a way that can be understood, appreciated and enjoyed by non-experts" Le-Math [6].

On the other hand, there seems to be little research done on how to support teachers in objective decision making when they give grades to students. This motivated us to explore the potentials of situational games through the design of tasks and to compare the results obtained by computer tests and by the teacher examining students. The aim was to realise learning public knowledge and raise potentials through the facilitation of students' teamwork with active learning in situational games, moreover to use computer generated tests contributing to the examiner's objective decision.

## IV. DESIGNING TASKS FOR SITUATIONAL GAMES IN GEOMETRY

After discussing theoretical preliminaries, it is now time to consider some concrete examples.The example is a treasure hunt competition during a lesson of 45 minutes, the target group in our research was prospective primary school teachers, and the topic was geometry. Students were split into groups of 4 people for teamwork. Students got several problems to solve in different situations. Perelman [9] proposed interesting geometric problems in an enjoyable way in all (primary, secondary and advanced) levels of education.On the basis of his book authors of the present paper gained ideashow to reformulate certain problems in a more challenging way.

1. The first task to start to find the treasure was to build a good road. In order to do it triangulation was carried out. One student of each group was asked to leave the seminar room and to act like headmen. 3 students in each group were asked to act like workers measuring distances between three marker poles with laser. One group of 3 was asked to give incorrect measurements and the rest of the other groups to give correct measurements to students who had left the room and asked to enter the room and check workers' measurements. Students found out which groups of students gave the correct measurements and which group's measurements were incorrect with the help of their knowledge on triangle inequality: the sum of the lengths of any two sides must be greater than or equal to the length of the remaining side. If a representative of a group responded well then the group got maximum points and moved on to the next situational game. As public knowledge, the principle for triangle inequality goes back to antiquity (Book VI of Euclid's elements).

The target knowledge can be presented in a succinct, official form, which may not be clear and certainly not very convincing to students.

We could have asked students to solve the same problem like: Given three numbers representing the lengths of three line segments. Can a triangle be constructed out of them?

When you already understand mathematics concepts and you want to practice your skills you can find lots of exercises and problems related to this or other topics in a workbook such as Gyöngyösi [4].

2. The next situation was when groups arrived at a castle. There was a key in the tower which students had to get to open the castle where the treasure was. Ladders of different lengths could have been borrowed to climb the tower but the price increased as the length of the ladder increased. Students had to find the ladder which was just long enough to reach the key (based on the average body height, a student approximately could reach up to 2 m high from the top of the ladder and in addition to the length of the ladder the span between the legs of the ladder were given. The group which selected the only good ladder out of four which was the most cost-effective got maximum points. As public knowledge, students had to use Pythagorean theory to calculate the correct length.

We could have asked students to solve the same problem like: Calculate the height of an isosceles triangle if the length of the legs and the base are given.

3. Entering the courtyard of the castle three vicious dogs (students were asked to act their roles) at the points of a triangle were guarding the way. Students could pass the three dogs safe only if they found the only point which the dogs could not reach. According to the instructions this point was of equal distance from the where the dogs were standing. Students had to find this point. As public knowledge, properties of circle were described in Euclid's elements (Book III).

We could have asked students to solve the same problem like: Find the centre of the circumscribed circle in a given triangle.

4. Then there were three different ways for the competing groups to go on. An elevator, stairs and a lifting crane leading up to different heights. To choose the right way the contestants had to calculate how high the attic of the castle was. They had access to the information on the image of a webcam: the length of the shadow of the building with the attic and the shadow of a straight stick of one meter long. After the proportion of reduction was given students could calculate the real height of the attic using the knowledge of: Two triangles are similar if and only if corresponding sides are proportional, i.e. there is a fixed number n such that the length of a side in one of the two triangles is n times the length of the corresponding side in the other triangle.

As public knowledge, the principle was described in Euclid's elements (Book VI).

We could have asked students to solve the same problem like: Two similar triangles are given. Calculate the similarity ratio.

5. Before entering the attic students had to tile the floor of the attic. It was a square-based floor with a diagonal of one unit and students had 8 two-dimensional figures: a square, a parallelogram, two triangles, one medium triangle and two small triangles and a circle. Tiles had to be put down on the floor tight and without overlap(see Tangram game). One of the planar figures was the odd one out.

Students had to find this shape by calculating areas of all given planar figures where all necessary data were given. The group(s) that managed to solve all problems correctly got the treasure.

This is a very useful game to develop students' creativity. It can help students to understand the concept of similar and congruent shapes. Calculation skills also can be developed.

We could have asked students to solve the same problem like: Calculate the area of certain two-dimensional figures.

## V. SOME FIRST RESULTS

A teaching method of this kind explained above was first used in the autumn 2014. The course had previously not involved situational games (at least not as part of the official curriculum). For this and other reasons, a "modest" approach was adopted with situational games being involved in just a few seminars per week, and results of computer generated tests before the oral exam were not considered in the final grades. The main purpose of this invention was to get basic information on the effects of problems and the feasibility of more revised designs in this and similar courses.

The goal of these elements of situational games and computer generated tests was to support students' active learning that require more teamwork applying their knowledge of theoretical blocks, but we did not expect that the limited experiment would result in major document impact of this kind. However, we analysed more locally the performance of students both on computer generated tests and during their oral exam. We present the results of two groups. Also, secondary and more informal evidence was collected from the students, in particular from students' responses to questionnaire items about their experience with situational games in the course; some of the evidence is also presented below.

16 students were involved in the research and before the exam they were asked to fill in a computer generated test on exam tasks at the end of the term. After they completed these test papers they had to pass an oral exam. Their results are shown in Figure 1. The first columns of each pair represent results of students gained from automatic question generated written tests and the second columns represent their results in the oral exam.

Students were asked to fill in a computer generated test on the topic they learnt during the term. There were 30 multiple-choice questions.

See for example:

Complete the following sentences!

1. The distance between two different planes is the length of a segment <..........> both planes with its <..........> on the two given planes, respectively.
2. The angle of two lines is the angle of <..........> parallel to the given lines intersecting at an arbitrary <..........> in the space.
3. Perpendicular <..........> of a triangle intersect in one <..........> which is called the centre of the <..........>.

In order to fill the test in a printed format the user has to underline the word judged to be appropriate or write it where the dots are. Figure 1 shows an example of a test sheet in the form students received to fill in before the exam.

Figure 1 shows the test results of students who filled the same test on definitions and theorems learnt at the basics of geometry course during the term. Students had to fill in a test of 30 questions in which 2 or even 3 words were missing from each sentence (a definition or a theorem) which had to be filled in and no response options were given. The oral exam contained the same topics in geometry. Students had 20 minutes to fill in the written test and then they had 20 minutes to prepare for a randomly chosen topic in geometry out of 12 exam topics and they had 10 minutes to answer the examiner's questions in the oral exam. Result given by the teacher during the oral exam was their final result.

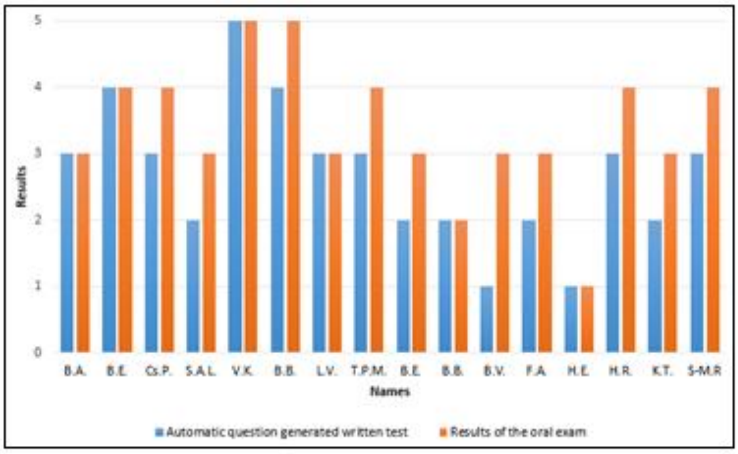


Fig.1 Performance of students

The students generally got a little bit better grades in the oral exam than in the written test. In six cases they got the same grades in both exams.Interestingly there was one student who would have failed in the written test, however, finally she got a satisfactory (3 out of 5) result in the oral exam.

We notice the similarity of results probably because both written and oral questions require students to know similar combination of geometric knowledge.

We finally conclude that virtually all students have passed the exam. Only one of them admitted that she had not got any time to prepare for the exam and she did not attend the lessons either.

Performance of students during the oral exam shows that they managed to activate stronger theoretical perspectives during the oral exam than during the written tests or that the teacher managed to remain benevolently objective. The difference between students' results in the written test and in the oral exam is not considerable which may mean that automatic question generated tests are quite reliable although they need to be improved to decrease this difference.

 

Computer generated tests also can be used to help students in learning concepts and prepare for their exams. Further research is needed in this context.

Results show that we may get more reliable results if we use both written and oral methods of examining students.

## VI. STUDENT QUESTIONNAIRE

It is not surprising that students with an overall lower performance also got weaker grades in taking part in situational games. Our purpose was not to make easy tasks but to include practical elements in the tasks and situational games that would ease the transition to more theoretical parts, at least for low achieving students. This would require a rather intensive study of these students' work during the course, which we did not manage to do.

Nevertheless, interesting evidence is found in the course evaluation, done by questionnaires towards the end of the course and with student replies being anonymous. Three questions concerned the use of situational games in the course. In the two first, students were asked to indicate their level of agreement with the propositions "I think that the use of situational games in the course contributed to my understanding of problems", and "I think that the use of situational games in the course improved my solutions". Here, the 15 responding students are divided rather equally on agreeing and disagreeing. The third question was "Comments and suggestions regarding the use of situational games in this course" and is certainly with open field response. As many as 10 students have responded to this field; they cannot be considered "representative" but they do represent strong opinions on the matter. These are also divided roughly in two halves, students who think the use of situational games is superfluous or tedious, and students who found it useful and interesting. Here are some examples of these comments (translated from Hungarian):

"I think it was good that situational games were relatively simple, interesting and without it certain exercises would have been more difficult to solve."

"Situational games are not necessary in a course like geometry. The pure mathematical methods are sufficient enough to understand and solve all the exercises(…) without artificially posed problems. In all my preparation for the exam I did everything without thinking of situational games and I did the situational game part without being surprised of any result."

"I liked situational games and I enjoyed teamwork. I never liked mathematics but now I started to find it interesting and I will apply this method in my teaching practice. I did get more understanding from the situational games."

The extent now (authors: of situational games use in the course) is fine, but it shouldn't be more. But it clearly helps your understanding.

## VII. CONCLUSIONS

New communication methods such as situational games were used and observed in a geometry course. The purpose was to provide students with help in their problem solving difficulties and enhance their learning activities. We think that two minorities of students can be identified according to the results in Figure 1:

1. Relatively successful students who got the same results on computer tests and in the exam who do not think that situational games have their place in a "real" mathematics course or these were not necessary to understand concepts better.
2. Students who performed better in the oral exam than on computer tests, who have troubles with several exercises, particularly the more abstract and theoretically demanding ones, but who appreciate and succeed with the parts where situational games can be used. This is partially because these are relatively easy but also, at least for (and according to) some students, because it helps their "understanding"

Making the use of situational games optional could probably satisfy the first group.

The needs and work of students in the latter group has to be studied more intensively than we were able to in this study. More students belong to this group than to the first one according to Figure 1.

On the other hand if teachers measure knowledge in two different ways (orally and written), it can ensure the objectivity of the measurement. Moreover, it is fast and objective to measure knowledge by automatic computer generated tests. Results of students from different colleges and universities can become more comparable. Through a standard credit system institutions may become more permeable which is especially important when students study abroad a few semesters through a scholarship and they want some results to be adopted in their home institution. Another advantage of computer generated testing is that they can be applied for any curriculum and at any age.

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