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# Weak Non–Linear Waves in Fluid with Internal State–Variables

**Nahid Fatima**
Amity University,
Gurgaon, India

***Abstract – In present paper, an attempt has been made to discuss the behaviour of weak–non–linear waves in fluid with several internal state variables. Behaviour of finite amplitude gas dynamic disturbance headed by a planer, cylindrical or spherical wave front in characteristic plane is investigated and result is applied for a binary dissociating gas.***

***Key words – Weak, Non-linear waves, Internal state variables.***

## I. INTRODUCTION

Certain papers [2], [3], [4], [5], [7 ]deal with the analysis of formation of plane shock waves in one dimensional unsteady flow with discontinuities resulting from the motion of a piston. Clark [5] has discussed growth and decay behaviour of plane waves propagating through a spatially uniform but time dependent chemically reacting gas mixture in a general state of disequilibrium. Sharma and Shyam[12] have discussed behaviour at the wave head of a finite amplitude gas dynamic disturbance in a chemically reacting fluid. Pandey and Chaturvedi [11] have discussed weak waves in reacting gases. Ojha and Tiwari [10] have considered propagation of spherical shock–waves in non–ideal atmosphere.

Studies of non–linear wave by using the progressive–wave theory have been carried out by several authors. Germain [9] reviewed the theory of progressive waves for wide applications in several fields. Fusco and Engelbreckt [8] presented the asymptotic analysis of non–linear waves in rate dependent media to study the high and low frequency wave processes and obtained an evolution equation for visco–elastic media. Shukla et.al. [13] have applied progressive wave approach to study decay behaviopur of a saw–tooth profile in chemically–reacting gases.Clark and Rodgers [6] have investigated the structure of plane steady shock–wave in a gas with several internal energy modes. Becker and Böhme [1] have discussed the structure of compression wave for n–parallel relaxation modes. Colemann and Gurtin [7] have discussed the growth and decay of discontinuities in fluids with internal state variables.

Equations governing one dimensional motion of a fluid with several internal state variables, neglecting various transport effects, are given by

$$\rho,_t + u\rho,_r + \rho u,_r + \frac{\in \rho u}{r} = 0 \qquad (1.1)$$

$$\rho u,_t + \rho u u,_r + p,_r = 0 \qquad (1.2)$$

$$p,_t + up,_r + \rho a_f^2 \left( u,_r + \frac{\in u}{r} \right) + \sum_{i=1}^{N} \left( \frac{h,_{Ci}\, w_i}{-1+\rho h,_p} \right) = 0 \qquad (1.3)$$

$$c_{i,t} + uc_{i,r} = \frac{w_i}{\rho}, (i = 1,..., N). \qquad (1.4)$$

where t is time, r is the distance of the axis or the centre of symmetry from a plane , ‘ρ’ is the density, p is the pressure, ‘u’ is the gas velocity, ‘$c_i$’ concentration of the $i^{th}$ species and $w_i$ rate of production of $i^{th}$ species respectively. $a_f$ is the frozen sound speed given by

$$a_f^2 = (p,_\rho)_{S,c_i} = \frac{\rho h, \rho}{1-\rho h,_p} \qquad (1.5)$$

where subscripts S,ci denote that the derivative is taken with these quantities held constant, while S is entropy and h being enthalpy given by

$$h = h(p, S, c_1, c_2, ...c_n)$$

Thus,

$$h,_\rho = \left( \frac{\partial h}{\partial \rho} \right)_{p,c_i}$$

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$$h,_{p} = \left(\frac{\partial h}{\partial p}\right)_{\rho, c_i}$$

and $\in$ = 0, 1, 2 refers to the case of a planer cylindrical and spherical motion respectively and comma followed by an index denote partial differentiation with respect to that index.

## II. EQUATIONS IN CHARACTERISTIC FORM

Following Wegener [14], we choose axis of symmetry in the direction of propagation of wave and introduce two characteristic variables $\alpha$ and $\beta$ defined as follows:

$$\frac{dr}{dt} = \frac{r,_\alpha}{t,_\alpha} = u + a_f, \quad \text{where } \beta(r,t) = \text{constant} \qquad (2.0a)$$

and

$$\frac{dr}{dt} = \frac{r,_\beta}{t,_\beta} = u - a_f, \quad \text{where } \alpha(r,t) = \text{constant} \qquad (2.0b)$$

Applying transformation

$$f,_t = \frac{r,_\alpha (f),_\beta - r,_\beta (f),_\alpha}{J}$$

and

$$f,_r = \frac{t,_\beta (f),_\alpha - t,_\alpha (f),_\beta}{J} \qquad (2.1)$$

where $$J = \begin{vmatrix} r,_\alpha & r,_\beta \\ t,_\alpha & t,_\beta \end{vmatrix} \neq 0,$$

equations (1.1) to (1.4) reduces to

$$a_f(\rho,_\alpha t,_\beta + \rho,_\beta t,_\alpha) + \rho(u,_\alpha t,_\beta - u,_\beta t,_\alpha) + \frac{\in \rho u}{r} J = 0 \qquad (2.2)$$

$$\rho a_f (u,_\alpha t,_\beta + u,_\beta t,_\alpha) + (p,_\alpha t,_\beta - p,_\beta t,_\alpha) = 0, \qquad (2.3)$$

$$(p,_\alpha t,_\beta + p,_\beta t,_\alpha) + \rho a_f (u,_\alpha t,_\beta - u,_\beta t,_\alpha) \qquad (2.4)$$

and $$+\left(\frac{\in \rho u}{x}\right) a_f J + \sum_{i=1}^{N} \frac{(h,_{ci} w_i) J}{a_f(-1 + \rho h,_\rho)} = 0$$

$$a_f [(c_i),_\beta t,_\alpha + (c_i),_\alpha t,_\beta] = \frac{w_i J}{\rho} \qquad (2.5)$$

Adding and subtracting equation (2.3) and (2.4), we have

$$p,_\alpha + \rho a_f u_\alpha = -\left[\frac{\left(\sum_{i=1}^{N} h,_{c_i} w_i\right)}{(-1 + \rho h,_p)} t,_\alpha + \frac{\rho \in u}{r} t,_\alpha a_f^2\right]$$

(2.6)

and

$$p,_\beta - \rho a_f u_\beta = -\left[\frac{\left(\sum_{i=1}^{N} h,_{c_i} w_i\right)}{(-1 + \rho h,_p)} + \frac{\rho \in u}{r} a_f^2\right] t,_\beta$$

(2.7)

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If unperturbed field ahead of the wave whose behaviour is to be investigated is assumed to be spatially uniform, all r derivatives of equations (1.1) to (1.4) vanishes and thus, we have

$\rho_{0,t} = 0$ i.e. $\rho_0 = \text{constant},$ (2.8)

$\rho_0 u_{0,t} = 0$ i.e. $u_0 = 0,$ (2.9)

$$p_{0,t} = \sum_{i=1}^{n} -\left( \frac{h,_{c_i} w_i}{(-1+\rho h,_p)} \right)_0 \quad (2.10)$$

$$c_{i0,t} = \left( \frac{w_i}{\rho} \right)_0 \quad (2.11)$$

$i = (1, 2, ..., n).$

where subscript 0 indicates a value in the background field. From equation (2.8) to (2.11) the background state can be visualized in terms of a fixed vessel uniformly filled with the gas mixture which is at rest. Perturbations of the background state will be assumed to propagate through the mixture behind the wave–front $\beta = 0$. Continuity of the variable $p, \rho, u$ and $c_i$ ($i = 1,...N$) at $\beta = 0$ is essential but discontinuities in their derivatives are permitted. Any derivatives with respect to $\alpha$ must be continuous, discontinuities can appear only in the $\beta$ derivatives.

## III. BEHAVIOUR AT THE WAVE–FRONT

Differentiating equation (2.6) with respect to '$\beta$' and equation (2.7) with respect to '$\alpha$' and subtracting one from the other and evaluating the resulting equation at $\beta = 0^+$, we get

$$\begin{aligned} 2\rho_0 a_{f_0} u,_{\alpha\beta^+} + a_{f_0}[u,_{\beta^+} \rho_{0,\alpha} + \rho,_{\beta^+} u_{0,\alpha}] &= \frac{\in \rho_0 a_{f_0}^2}{r}[-u,_{\beta^+} t_{0,\alpha}] \\ -\sum_{i=1}^{n} \frac{h,_{ci_0} w_{i_0}}{(-1+\rho_0 h,_{p_0})^2}&[-\rho,_{\beta^+} h,_{p_0} t_{0,\alpha} - \rho_0 h,_{p\beta^+} t_{0,\alpha}] \\ + \frac{1}{(1-\rho_0 h,_{p_0})}&\left[\sum (h,_{ci_{\beta^+}} w_{0i}) t_{0,\alpha}\right] \\ + \frac{1}{(1-\rho_0 h,_{p_0})}&\left[\sum (h,_{ci_0} w_{i,\beta^+}) t_{0,\alpha}\right] \end{aligned} \quad (3.1)$$

Applying initial conditions given by (2.8) and (2.9) and $\beta = 0^+$ equations (2.2) to (2.5) reduces to

$$a_{f_0} \rho,_{\beta^+} = \rho_0 u,_{\beta^+} \quad (3.2)$$

$$t_{0,\alpha} p,_{\beta^+} - p_{0,\alpha} t,_{\beta^+} = \rho_0 a_{f_0} t_{0,\alpha} u,_{\beta^+} \quad (3.3)$$

$$t_{0,\alpha} c_{i,\beta^+} - t,_{\beta^+} c_{i0,\alpha} = 0, \quad (3.4)$$

Combining equation (3.1) with (3.2) to (3.4), we get

$$\frac{\partial}{\partial \alpha} \log[(\rho_0 a_{f_0})^{1/2} u,_{\beta^+}] = \frac{1}{2}\left[\Lambda_1 - \frac{\in a_{f_0}}{r}\right] t_{0,\alpha} \quad (3.5)$$

where

$$\Lambda_1 = \frac{-1}{(-1+\rho_0 h,_{p_0})}\left[\sum_{i=1}^{N} \frac{(h,_{ci\beta^+} w_{0i})}{\rho_0 a_{f_0} u,_{\beta^+}}\right] - \frac{1}{(-1+\rho_0 h,_{p_0})}\left[\sum_{i=1}^{N} \frac{(h,_{ci_0} w_{i,\beta^+})}{(\rho_0 a_{f_0}) u,_{\beta^+}}\right] + \sum_{i=1}^{N} \frac{(h,_{ci_0} w_{i_0})}{(1-\rho_0 h,_{p_0})^2 (\rho_0 a_{f_0}) u,_{\beta^+}}[\rho,_{\beta^+} h,_{p_0} + \rho_0 h,_{p\beta^+}].$$

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Integrating equation (3.5), we have $u_{,\beta^+} = u_{,\beta i^+}\left(\frac{\rho_{,0i}\, a_{f_{0i}}}{\rho_0 a_{f_0}}\right)^{1/2} \exp\left\{\int_{t_i}^{t} \frac{1}{2}\left(\Lambda_1 - \frac{\in a_{f_0}}{r}\right)dt\right\},$ (3.6)

where $u_{,\beta^+ i}$ is the value of $u_{,\beta^+}$ at the initial time t = $t_i$. From equation (2.6) and equation (2.7), after some manipulation, we have

$$p_{,\alpha}\, t_{,\beta} + p_{,\beta}\, t_{,\alpha} = \rho a_f (u_{,\beta}\, t_{,\alpha} - u_{,\alpha}\, t_{,\beta}) - \frac{1}{(-1+\rho h_p)}\left[2\sum_{i=1}^{N}(h_{,ci}\, w_i) - \frac{2\rho \in u a_f^2}{r}\right] t_{,\alpha}\, t_{,\beta} \quad (3.7)$$

From equation (2.0a) &(2.0b) when evaluated at β = $0_+$ yields

$$2a_{f_0}(t_{,\beta^+})_\alpha + u_{,\beta^+}\, t_{0,\alpha} + (a_{f,\beta^+} t_{0,\alpha} + a_{f_0,\alpha} t_{,\beta^+}) = 0 \quad (3.8)$$

From the above equation, we get

$$(t_{,\beta^+})_{,\alpha} + \left(\frac{\gamma+1}{4}\right)\frac{u_{,\beta^+}\, t_{0,\alpha}}{a_{f_0}} + \frac{\gamma}{2a_{f_0}^2 \rho_0 (1-\rho_0 h_{,p_0})}\cdot\left[\sum_{i=1}^{N}(h_{,ci}\, w_{i0}) t_{0,\alpha} t_{,\beta^+}\right] = 0 \quad (3.9)$$

Equation (3.9) can be written as

$$(t_{,\beta^+})_{,\alpha} + \left(\frac{\gamma+1}{4}\right)\frac{u_{,\beta^+}\, t_{0,\alpha}}{a_{f_0}} + \Lambda_2 t_{0,\alpha} t_{,\beta^+} = 0 \quad (3.10)$$

where $\Lambda_2 = \frac{1}{2p_0(1-\rho_0 h_{,p_0})}\sum_{i=1}^{N}(h_{,ci_0}\, w_{i_0}).$

Integrating equation (3.10), we have

$$t_{,\beta^+} = t_{,\beta i^+} \exp\left\{-\int_{t_i}^{t} \Lambda_2(t)dt\right\}$$
$$- u_{,\beta_i}^{+}\, e^{-\int_{t_i}^{t}\Lambda_2(t)dt} \int_{t_i}^{t}\left(\frac{\gamma+1}{4a_{f_0}}\right)\left(\frac{\rho_0 a_{f_{0i}}}{\rho_0 a_{f_0}}\right)^{1/2} e^{\int_{t_i}^{t}\frac{1}{2}\left(\Lambda - \frac{\in a_{f_0}}{r}\right)dt} .dt$$

(3.11)

Where Λ = $\Lambda_1 + \Lambda_2$ and $u_{\beta^+ i}$ is value of $u_{\beta^+}$ at t = $t_i$.

$$\frac{u_{,r^+}}{u_{,ri^+}} = \frac{\left(\frac{\rho_{0i} a_{f_{0i}}^3}{\rho_0 a_{f_0}^3}\right)^{1/2} u_{,\beta i}^{+} \exp\left\{\int_{t_i}^{t}\frac{1}{2}\left(\Lambda(t) - \frac{\in a_{f_0}}{r}\right)dt\right\}}{1 + \frac{1}{2}(\gamma+1) u_{,\beta i}^{+} \int_{t_i}^{t}\left\{\left(\frac{\rho_{0i} a_{f_{0i}}^3}{\rho_0 a_{f_0}^3}\right)^{1/2} .e^{\int_{t_i}^{t}\left\{\Lambda(t) - \frac{\in a_{f_0}}{r}\right\}dt} .dt\right\}}$$

(3.12)

## IV. DISCUSSION

For a plane wave ∈ = 0, equation (4.14) reduces to the form as obtained by Clarke [25], and therefore, all his conclusions follow immediately.

Here, we shall consider the following situations in which the wave–front is of cylindrical or spherical geometry.

**Case I:**

If the medium ahead is one of uniform equilibrium, in that case $w_i = 0$ as a result of which $p_0$, $a_{f_0}$ etc. are constants and Λ < 0.

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$R(t) = R_0 + a_{f_0} t$, where $R_0$ is the position of the wave front at time t = 0. Here $t_i$ has been set equal to zero for convenience. Thus, equation (3.12) reduces to

$$u,_{r^+} = \frac{u,_{r_i^+} (R_0/R)^{\in/2} \exp(-|\Lambda| t)}{1 + \left(\frac{\gamma+1}{2}\right) u,_{r_i^+} \int_0^t (R_0/R)^{\gamma/2} \exp(-|\Lambda| t) dt} \tag{4.1}$$

$$(u,_{r^+})_C = \begin{cases} 2|\Lambda|/(\gamma+1) & \text{for } \in = 0 \\ 2\left(\frac{|\Lambda| a_{f_0}}{\pi R_0}\right)^{1/2} \frac{\exp(-|\Lambda| R_0/a_{f_0})}{(\gamma+1)\operatorname{erfc}(|\Lambda| R_0/a_{f_0})^{1/2}} & \in = 1 \\ \frac{2a_{f_0} \exp(-|\Lambda| R_0/a_{f_0})}{(\gamma+1) R_0 E_i(|\Lambda| R_0/a_{f_0})} & \text{for } \in = 2 \end{cases}$$

then $u,_{r^+} \to 0$ as $t \to \infty$, the wave damps out ultimately. But if $u,_{r_i^+} < 0$ and $|u,_{r_i^+}| > (u,_{r^+})_C$ then there exists finite time $t_s$ given by

$$t_s = \frac{1}{|\Lambda|} \log\left\{1 - \frac{2|\Lambda|}{|u,_{r_i^+}|(\gamma+1)}\right\}^{-1} \qquad \text{for } \in = 0$$

and

$$\int_0^{t_s} (R_0/R)^{\in/2} \exp(-|\Lambda| t) dt = 2|u,_{r_i^+}|(\gamma+1) \qquad \text{for } \in = 1,2$$

such that $|u,_{r_i^+}| \to \infty$ as $t \to t_s$, i.e. the wave terminates into a shock at an instant $t_s$. Thus, we find that a compression wave steepens up into a shock after a finite time only if the initial discontinuity associated with the wave is sufficiently strong. From the above expressions of $(u,_{r^+})_C$, we can see that $\frac{\partial (u,_{r^+})_C}{\partial |\Lambda|} > 0$ which means that the chemical reactions in the flow have a effect on the tendency of the wave surface to grow into a shock in the sense that an increase in $|\Lambda|$ will cause $(u,_{r^+})_C$ to increase and thus delays the shock formation. Also, $\frac{\partial (u,_{r^+})_C}{\partial R_0} < 0$ implies that the curvature has a stabilizing effect and that an increase in the initial curvature causes an increase in $(u,_{r^+})_C$. For $|u,_{r_i^+}| = (u,_{r^+})_C$, it follows from (4.1) that at a plane compression wave head the discontinuity propagates with the constant initial strength and at a cylindrical and spherical wave head they propagate according to

$$u,_{r^+} = -\frac{2}{(\gamma+1)} \left(\frac{|\Lambda| a_{f_0}}{\pi R}\right)^{1/2} \frac{\exp(-|\Lambda| R/a_{f_0})}{\operatorname{erfc}(|\Lambda| R/a_{f_0})^{1/2}} \tag{4.2}$$

and

$$u,_{r^+} = -\frac{-2a_{f_0} \exp(-|\Lambda| R)}{(\gamma+1) R E_i(|\Lambda| R)} \tag{4.3}$$

respectively.

**Case II:**

If the medium ahead is in a state of disequilibrium, i.e. $w_0 \neq 0$

(3.12) can be written in the following form

$$u,_{r^+} = \frac{u,_{r_i^+}(R_0/R_0 + \bar{a}_{f_0} t)^{\in/2} \exp(\bar{\Lambda} t)}{1+\left(\frac{\gamma+1}{2}\right) u,_{r_i^+} \int_0^t R_0/(R_0+\bar{a}_{f_0}\hat{t})^{\in/2} \exp(|\Lambda|\hat{t}) d\hat{t}} \tag{4.4}$$

and $\bar{\Lambda}$ indicate suitable mean value over the interval $t_i$ to t and $t_i$ has been equal to zero for convenience.

From equation (4.4), if $u,_{r_i^+} < 0$ and $\bar{\Lambda} > 0$, a finite time $t^*_s$ is given by

$$t_s^* = \frac{1}{\Lambda} \log\left\{1+\frac{2\bar{\Lambda}}{|u,_{r_i^+}| \in +1|}\right\} \quad \text{for} \in = 0 \tag{4.5}$$

and

$$\int_0^{t^*_s} \frac{R_0}{(R_0+\bar{a}_{f_0} t)^{\in/2}} \exp(\bar{\Lambda}(t)) dt = \frac{2}{|u,_{r_i^+}|(\gamma+1)} \quad \text{for} \in = 1,2 \tag{4.6}$$

Thus, we find that in a state of disequilibrium sufficiently far from equilibrium the discontinuity associated with a compression wave, no matter how small always breakup into a shock after a finite time. It follows from (4.5) and (4.6) that

$$\frac{\partial t^*_s}{\partial \bar{\Lambda}} < 0 \quad \text{and} \quad \frac{\partial t^*_s}{\partial R_0} < 0,$$

mean that the chemical rate process in the flow accelerates the steepening of a compression wave to grow into shock.

## V. BINARY DISSOCIATING GAS

Equations governing the one dimensional unsteady motion of a binary dissociating gas, neglecting the various transport effects are given by

$$\rho,_t + u\rho,_x + \rho u,_x + \frac{\in u\rho}{x} = 0, \tag{5.1}$$

$$\rho u,_t + \rho u u,_x + p,_x = 0, \tag{5.2}$$

$$p,_t + up,_x + a_f^2\left(\rho u,_x + \frac{\in \rho u}{x}\right) + f(p,\rho,c)(\gamma-1)\rho d = 0, \tag{5.3}$$

$$c,_t + uc,_x = + f(p, \rho, c) \tag{5.4}$$

the coefficient $\in$ = 0, 1, 2 refers to the case of a planar, cylindrical and a spherical motion respectively. ρ is the density, 'p' is the pressure, u is the gas velocity, 'x' the distance of the axis of symmetry respectively. The frozen sound speed is given by

$$a_f^2 = (p,_\rho)_{s,c_i} - \frac{\rho h,_\rho}{(\rho h,_p - 1)} \quad , \tag{5.5}$$

and

$$h = \left(\frac{\gamma}{\gamma-1}\right)\frac{p}{\rho} + cd$$

$$h,_\rho = \left(\frac{\partial h}{\partial \rho}\right)_{p,c},$$

$$h,_p = \left(\frac{\partial h}{\partial p}\right)_{\rho,c}.$$

Now applying the transformation (2.1) in equations (5.2 & 5.3) we have

$$(p,_\alpha + \rho a_f u,_\alpha)t,_\beta - (p,_\beta + \rho a_f u,_\beta)t,_\alpha = 0, \tag{5.6}$$

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$$(p,_\beta - \rho a_f u,_\beta)t,_\alpha + (p,_\alpha + \rho a_f u,_\alpha)t,_\beta + \frac{\in \rho u}{x} J a_f + \frac{f(\gamma-1)d\rho.J}{a_f} = 0 \qquad (5.7)$$

Adding equation (5.6) and (5.7), we have

$$(p,_\alpha + \rho a_f u,_\alpha) =, -\left[\left(\frac{\in \rho u}{x}\right)a_f^2 + f(\gamma-1)d\rho\right]t,_\alpha \qquad (5.8)$$

and on subtracting, we have

$$(p,_\beta - \rho a_f u,_\beta) = -\left[\left(\frac{\in \rho u}{x}\right)a_f^2 + f(\gamma-1)d\rho\right]t,_\beta, \qquad (5.9)$$

Unperturbed condition in this case become

$\rho_{0,t} = 0$, i.e. $\rho_0 = \text{constt}$; (5.10)

$\rho_0 u_{0,t} = 0$, i.e. $u_0 = 0$, (5.11)

$p_{0,t} = -f(p_0\rho_0,c_0)d(\gamma-1)\rho_0$, (5.12)

$c_{0,t} = +f(p_0\rho_0,c_0)$, (5.13)

where subscript 0 indicates a value in the background field. From equation (5.10) to (5.13), the background state can be visualized in terms of a fixed vessel uniformly filled with the gas mixture which is at rest. The pressure $p_0$ changes as reactants–species $c_0$ varies.

The resulting equation at $\beta = 0_+$, we get

$$2\rho_0 a_{f_0} u,_{\alpha,\beta^+} + (a_{f_0}\rho_0)_\alpha u,_{\beta^+} = \frac{\in \rho_0 a_{f_0}^2}{x}[-u,_{\beta^+} t_{0,\alpha}]$$
$$+ f_0(\gamma-1)[d,_{\beta^+} t_{0,\alpha}] + (\gamma-1)[f,_{\beta^+} t_{0,\alpha}] + f(\gamma-1)d\{\rho,_{\beta^+} t_{0,\alpha}] \qquad (5.14)$$

$$a_f(\rho,_\alpha t,_\beta + \rho,_\beta t,_\alpha) + \rho(u,_\alpha t,_\beta - u,_\beta t,_\alpha) + \frac{\in \rho u}{x}J = 0, \qquad (5.15)$$

$$\rho a_f(u,_\alpha t,_\beta + u,_\beta t,_\alpha) + (p,_\alpha t,_\beta - p,_\beta t,_\alpha) = 0, \qquad (5.16)$$

$$(p,_\alpha t,_\beta + p,_\beta t,_\alpha) + a_f(u,_\alpha t,_\beta - u,_\beta t,_\alpha) = 0. \qquad (5.17)$$

$$a_f(t,_\alpha c,_\beta + c,_\alpha t,_\beta) = f(p,\rho,c), \qquad (5.18)$$

Applying initial conditions given by (5.10) and (5.11) and $\beta = 0^+$, equation (5.15) to (5.17) reduces to

$$a_{f_0}\rho,_{\beta^+} = \rho_0 u,_{\beta^+} \qquad (5.19)$$

$$t_{0,\alpha}p,_{\beta^+} - p_{0,\alpha}t,_{\beta^+} = \rho_0 a_{f_0} t_{0,\alpha} u,_{\beta^+} \qquad (5.20)$$

$$t_{0,\alpha}c,_{\beta^+} - c_{0,\alpha}t,_{\beta^+} = 0, \qquad (5.21)$$

Combining equation (5.14) with (5.18) to (5.20), we get

$$\frac{\partial}{\partial\alpha}\log\left[(\rho_0 a_{f_0})^{1/2} u,_{\beta^+}\right] = \frac{1}{2}\left[\Lambda_1 - \frac{\in a_{f_0}}{x}\right]t_{0,\alpha} \qquad (5.22)$$

where

$$\Lambda_1 = -\frac{f_0(\gamma-1)d,_{\beta^+}}{(\rho_0 a_{f_0})u,_{\beta^+}} - \frac{f(\gamma-1)\rho,_{\beta^+}}{\rho_0 a_{f_0} u,_{\beta^+}} - \frac{(\gamma-1)d_0 f,_{\beta^+}}{(\rho_0 a_{f_0} u,_{\beta^+})}$$

Integrating equation (5.21), we have

$$u,_{\beta^+} = u,_{\beta_i^+}\left(\frac{\rho,_{0i} a_{f_{0i}}}{\rho_0 a_{f_0}}\right)^{1/2} \exp\left\{\int_{t_i}^{t}\frac{1}{2}\left(\Lambda_1 - \frac{\in a_{f_0}}{x}\right)dt\right\} \qquad (5.23)$$

where $u,_{\beta_i^+}$ is the value of $u,_{\beta^+}$ at the initial time t = t*. For equation (5.8) and equation (5.9) after some manipulation, we have

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$p,_\alpha t,_\beta + p,_\beta t,_\alpha = \rho a_f[u,_\beta t,_\alpha - u,_\alpha t,_\beta] + 2f(\gamma - 1)\rho d\, t,_\alpha t,_\beta$

$$-\left(\frac{2\rho \in u}{x}\right) a_f^2 t,_\alpha\, t,_\beta \quad (5.24)$$

From equation (5.8),

$$2a_{f_0} t,_{\beta^+\alpha} + u,_{\beta^+} t_{0,\alpha} + (a_{f,\beta^+} t_{0,\alpha} + a_{f_0} t,_{\beta^+}) = 0 \quad (5.25)$$

From the above equation, we get

$$t,_{\beta^+\alpha} + \left(\frac{\gamma+1}{4}\right)\frac{u,_{\beta^+}\, t_{0,\alpha}}{a_{f_0}} + \Lambda_2 t_{0,\alpha} t,_{\beta^+} = 0 \quad (5.26)$$

where $\Lambda_2 = \dfrac{\rho_0}{2p_0}(\gamma - 1)f_0 d_0.$

Integrating equation (5.26), we have

$$t,_\beta^+ = t,_{\beta_i}^+ \exp\left\{-\int_{t_i}^{t} \Lambda_2(t)dt\right\}$$

$$-u,_{\beta_i^+} e^{-\int_{t_i}^{t}\Lambda_2(t)dt} \int_{t_i}^{t}\left(\frac{\gamma+1}{4a_{f_0}}\right)\left(\frac{\rho_{0i} a_{f_{0i}}}{\rho_0 a_{f_0}}\right)^{1/2} e^{\int_{t_i}^{t}\frac{1}{2}\left(\Lambda - \frac{\in a_{f0}}{x}\right)} .dt \quad (5.26)$$

where $\Lambda = \Lambda_1 + \Lambda_2$.

which shows that results of article four is also satisfied in this case.

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