



## Stochastic Analysis of a Two Priority Unit Standby System with Imperfect Switch and Correlated Failure and Repair Times

Praween Kumar\*, Anu Sirohi

Department of Mathematics, Bharat Institute of Technology,  
Meerut, India

**Abstract:** The paper presents the Markov model a two-priority unit standby system with imperfect switch and correlated failure and repair times. It is assumed that the system stops functioning only when both the units fail. The joint distribution of failure and repair times of each unit failed either partially or totally is taken to be bivariate exponential distribution having the joint density function. The repair time distribution of transfer switch is taken to be negative exponential. Formulas for various reliability indices of the system including availability, busy period analysis of the repairman, and profit function analysis by using the regenerative point technique are derived.

**Keywords:** Redundancy; Standby system; Stochastic; Failure and Repair times; Correlation.

### I. INTRODUCTION

One way to improve the system reliability is to add redundant components. Standby redundancy is a technique widely used to improve system reliability and availability. In general there are three types in redundancy i.e. cold, hot, and warm standby. Cold standby implies that the inactive components have a zero failure rate and cannot fail while in standby state. Hot standby implies that an inactive component has the same failure rate as when it is in operation. Warm standby is an intermediate case and it implies that an inactive component has a failure rate between that for the cold and hot standby. It is also called dormant failure in some papers.

In this paper a two priority unit standby systems with imperfect switch and correlated failure and repair times is studied. For such a system, when priority unit fails, the ordinary unit becomes active and the system is working. This type of systems found applications in various fields including air conditioner and air cooler, medical diagnosis, redundant system testing, power plant, network design and so on.

In the past decades, many articles concerning the reliability, availability, cost function analysis and some other characteristics of two priority unit standby systems have been published. Among them, a two-unit system with imperfect switch [1], two priority unit with imperfect switch and by considering arbitrary distributions [2], switch failure without priority [3], cold standby with correlated failures and repairs [4], warm standby redundant system with two types of repair facilities [5], repair machine failures [6], multi component systems with correlated failure and repair [7], correlated life times [8]. Various related models have been developed and many formulas have been derived, resulting in a large body of literature, for those general results [9-10]. Other papers investigate concept of two units parallel system with waiting time, repair machine failure and repair times [11-13]. Some recent papers on this and related topics can be found in [14].

Though the reliability and availability of cold standby systems have been reported in the above research, these results are limited to some essential conditions. The purpose of this paper is to present cost function analysis of the general two priority unit standby systems with imperfect switch and correlated failure and repair times. Several authors have analyzed two identical/non-identical unit standby systems with different concepts and obtained various characteristics of interest by using the theories of semi-Markov process, regenerative process, Markov-renewal process and supplementary variable techniques. Followed by that, in this model we used regenerative point technique to obtain various characteristics.

### II. MODEL DESCRIPTION

#### 2.1. System description

There are some real systems such as power generator and transmission systems where some new equipment groups need to be added because of the requirement for more output of the system. In this model, initially the system comprises two non-identical units (unit-1 and unit-2). Unit-1 is named as priority (p) unit and the unit-2 is named as ordinary (o) unit. The satisfactory operation of any of the unit is sufficient to do the job. The p-unit of the system has three modes-normal, partial failure and total failure (N, P and F) whereas o-unit has only two modes-normal and total failure (N and F). The p-unit either is in N or P mode gets priority in operation over the o-unit. Initially o-unit is kept into cold standby and p-unit is in operation. When both the units are operative then their failure times are assumed to be correlated random variables having their joint distribution as B.V.E with density function as given below-

$$f(x_1, x_2) = \alpha_1 \alpha_2 (1-r) \exp[-\alpha_1 x_1 - \alpha_2 x_2] I_0(2\sqrt{\alpha_1 \alpha_2 r x_1 x_2})$$

where  $x_1, x_2, \alpha_1, \alpha_2 > 0; 0 \leq r \leq 1$  and  $I_0 \left( 2\sqrt{\alpha_i \beta_i r_i xy} \right) = \sum_{j=0}^{\infty} \frac{(\alpha_i \beta_i r_i xy)^j}{(j!)^2}$ .

## 2.2. Assumptions

1. The system is a general two-priority unit standby system with imperfect switch and correlated failure and repair. The satisfactory operation of any of the unit is sufficient to do the job.
2. The priority unit, either it is in N or P mode gets priority in operation over ordinary unit.
3. When p-unit fails, it is instantaneously replaced by o-unit with the help of switching device, which may be found perfect or imperfect at the time of need with fixed probabilities  $\theta$  and  $(1 - \theta)$  respectively.
4. The priority unit cannot enter into F-mode without passing through P-mode.
5. The ordinary unit operates only when the priority unit enters in F-mode.
6. The operation and repair of a partially failed unit are possible simultaneously.
7. Each repaired unit works as good as new.
8. The repair time distribution of transfer switch is taken to be negative exponential with parameter  $\eta$ .
9. When the repair of a totally failed priority unit is restarted one or more times, its repair time (residual repair time) is taken to be independent having the negative exponential distribution with parameter  $\beta_2$ .

## III. NOTATIONS AND STATES OF THE SYSTEM

To write the states of the system we define the following symbols for both the units: -

- \* / ~ : Symbol for Laplace/ Laplace Stieltjes transform of a function.
- $N_{1o}, N_{1or}$  : p-unit is in N-mode and operative, in P-mode and operative as well as under repair.
- $N_{2s}, N_{2o}$  : o-unit is in N-mode and standby, operative.
- $F_{1r}, F_{1wr}$  : p-unit is in F-mode and under repair, waiting for repair.
- $F_{1r}, F_{1wr}$  : p-unit is in F-mode and under re-started repair, waiting for restarted repair.
- $F_{2r}$  : o-unit is in F-mode and under repair.
- $T_r$  : Transfer switch is under repair.
- $X_i (i=1, 2, 3)$  : Random variables denoting the failure times of p-unit from N-mode to P-mode and P-mode to F-mode respectively for  $i=1, 2$  and of o-unit from N-mode to F-mode for  $i=3$ .
- $Y_i (i=1, 2, 3)$  : Random variables denoting the repair times of p-unit from P-mode to N-mode and F-mode to N-mode respectively for  $i=1, 2$  and of o-unit from F-mode to N-mode for  $i=3$ .
- $g_i(.)$  : Marginal p. d. f of  $X_i$ .

Considering the above symbols in view of the assumptions we have the following states of the system-

<b>Up States</b>		<b>Down States</b>
$S_0 = (N_{1o}, N_{2s}),$	$S_1 = (P_{1or}, N_{2s})$	$S_3 = (F_{1w}, N_{2s}, T_r)$
$S_2 = (F_{1r}, N_{2o}),$	$S_5 = (F_{1r}, N_{2o})$	
<b>Failed States</b>		
$S_4 = (F_{1wr}, F_{2r})$		

The transition diagram of the system model along with failure time variables, and repair rates is shown in fig. 1. In figure we observe that the epochs of transition into all the states  $S_0$  to  $S_8$  are regenerative points.

## IV. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Let  $Q_{ij}(t)$  is the one step unconditional transition probability, which can be obtained by using the simple probabilistic arguments, as follows-

$$\begin{aligned}
 Q_{01}(t) &= \int_0^t P[\text{system transits from } S_0 \text{ to } S_1 \text{ during } u, u+du] \\
 &= \int_0^t \alpha_1 (1-r_1) e^{-\alpha_1(1-r_1)u} du \\
 &= 1 - e^{-\alpha_1(1-r_1)t}
 \end{aligned}$$

Similarly other probabilistic arguments can be obtained, then the non-zero elements of the transition probability matrix  $P = (p_{ij})$  for the model are as follows-

$$p_{01} = p_{32} = 1, \quad p_{54} = \frac{\alpha_3(1-r_3)}{\beta_2 + \alpha_3(1-r_3)}, \quad p_{50} = \frac{\beta_2}{\beta_2 + \alpha_3(1-r_3)}$$

The steady-state unconditional steady-state transition probabilities  $p_{ij|x}$  can be obtained by taking  $t \rightarrow \infty$  using the

result  $p_{ij} = \int p_{ij|x} \cdot g(x) dx$  as follows-  $p_{10|x} = \beta_1 e^{-\alpha_1 r_1 x} \sum_{j=0}^{\infty} \frac{(\alpha_1 \beta_1 r_1 x)^j}{(j!)^2} \frac{\Gamma(j+1)}{[\beta_1 + \alpha_2(1-r_2)]^{j+1}}$

$$= \beta_1' e^{-\alpha_1 r_1 x} \sum_{j=0}^{\infty} \frac{(\alpha_1 \beta_1' r_1 x)^j}{(j!)^2}, \text{ where } \beta_1' = \frac{\beta_1}{\beta_1 + \alpha_2(1-r_2)}$$

$$= \beta_1' e^{-\alpha_1 r_1 x(1-\beta_1')}$$

$$p_{12|x} = \theta \left[ 1 - \beta_1' e^{-\alpha_1 r_1 x(1-\beta_1')} \right], \quad p_{13|x} = \bar{\theta} \left[ 1 - \beta_1' e^{-\alpha_1 r_1 x(1-\beta_1')} \right]$$

$$p_{20|x} = \beta_2' e^{-\alpha_2 r_2 x(1-\beta_2')}, \text{ where } \beta_2' = \frac{\beta_2}{\beta_2 + \alpha_3(1-r_3)}$$

$$p_{24|x} = 1 - \beta_2' e^{-\alpha_2 r_2 x(1-\beta_2')}, \quad p_{45|x} = e^{-\alpha_3 r_3 x} \sum_{j=0}^{\infty} \frac{(\alpha_3 r_3 x)^j}{j!} = 1 \quad \forall x$$

we observe that  $p_{10|x} + p_{12|x} + p_{13|x} = 1$  and  $p_{20|x} + p_{24|x} = 1$ .

The above conditional transitional probabilities involve  $r_i$ . For  $r_i=0$  they give the corresponding unconditional probability with correlation zero. The unconditional transition probabilities with correlation coefficient are as follows-

$$p_{10} = \frac{\beta_1'(1-r_1)}{(1-r_1\beta_1')}, \quad p_{12} = \left[ 1 - \frac{\beta_1'(1-r_1)}{(1-r_1\beta_1')} \right], \quad p_{13} = \bar{\theta} \left[ 1 - \frac{\beta_1'(1-r_1)}{(1-r_1\beta_1')} \right], \quad p_{20} = \frac{\beta_2'(1-r_2)}{(1-r_2\beta_2')}$$

$$p_{24} = 1 - \frac{\beta_2'(1-r_2)}{(1-r_2\beta_2')}, \quad p_{45} = 1$$

so that  $p_{10} + p_{12} + p_{13} = 1$ , and  $p_{20} + p_{24} = 1$

Let  $X_i$  denotes the sojourn time in state  $S_i$ , then the mean sojourn time in state S is given by-  $\psi_i = \int P(X_i > t) dt$

The mean sojourn times in various states are as follows:

$$\psi_0 = \frac{1}{\alpha_1(1-r_1)}$$

$$\psi_{1|x} = \frac{1 - \beta_1' e^{-\alpha_1 r_1 x(1-\beta_1')}}{\alpha_2(1-r_2)}, \text{ so that } \psi_1 = \{\alpha_2(1-r_2)\}^{-1} \left[ 1 - \frac{\beta_1'(1-r_1)}{(1-r_1\beta_1')} \right]$$

$$\psi_{2|x} = \frac{1}{\alpha_3(1-r_3)} \left[ 1 - \beta_2' e^{-\alpha_2 r_2 x(1-\beta_2')} \right], \text{ so that } \psi_2 = \{\alpha_3(1-r_3)\}^{-1} \left[ 1 - \frac{\beta_2'(1-r_2)}{(1-r_2\beta_2')} \right]$$

$$\psi_3 = \frac{1}{\eta}, \quad \psi_{4|x} = \frac{1 + \alpha_3 r_3 x}{\beta_3}, \text{ so that } \psi_4 = \frac{1}{\beta_3(1-r_3)}, \quad \psi_5 = \frac{1}{\beta_2 + \alpha_3(1-r_3)}$$

## V. ANALYSIS OF RESULTS

### 5.1 Reliability

Let the random variable  $T_i$  be the time to system failure when the system starts its operation from state  $S_i \in E$ , then the reliability of the system is given by-

$$R_i(t) = P(T_i > t)$$

To given  $R_i(t)$ , we assume that failed state  $S_4$  as absorbing states of the system. Using the simple probabilistic arguments, one can easily develop the recurrence relations among  $R_i(t)$ ;  $i=0, 1, 2, 3$ . Taking the Laplace transformation of the relations and simplifying the resulting set of algebraic equations for  $R_0^*(s)$ , we get -

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} = \frac{Z_0^* + q_{01}^* (Z_1^* + q_{12}^* Z_2^* + q_{13}^* Z_3^*)}{1 - q_{01}^* (q_{10}^* + q_{12}^* q_{20}^*)}$$

### 5.2 Mean Time to System Failure

The mean time to system failure is given by

$$E(T_0) = \lim_{s \rightarrow 0} R_0^*(s)$$

Using the results:  $-z_i^*(0) = \psi_i$  and  $q_{ij}^*(0) = p_{ij}$ , we get

$$E(T_0) = \int R_0(t) dt = R_0^*(s)|_{s=0} = \frac{N_1(0)}{D_1(0)} = \frac{\psi_0 + \psi_1 + p_{12}\psi_2 + p_{13}\psi_3}{1 - p_{10} - p_{12}p_{20}}$$

### 5.3 Availability

Let  $A_i^p(t)$  and  $A_i^o(t)$  be the probability that the system is up at epoch t due to p-unit and o-unit when initially it starts functioning from state  $S_i \in E$ . Here, in this a dichotomous variable  $\delta$  is introduced which takes two values 1 and 0 respectively when the p-unit operates in N-mode and P-mode. By using probabilistic arguments we get-

$$A_0^{p*}(s) = \frac{N_2(s)}{D_2(s)}$$

where  $N_2(s) = (\delta Z_0^* + q_{01}^* \bar{\delta} Z_1^*) (1 - q_{45}^* q_{54}^*)$

and  $D_2(s) = [1 - q_{01}^* q_{10}^* - q_{01}^* (q_{12}^* + q_{13}^* q_{32}^*) q_{20}^*] (1 - q_{45}^* q_{54}^*) - q_{01}^* (q_{12}^* + q_{13}^* q_{32}^*) q_{24}^* q_{45}^* q_{50}^*$

Now Steady state availabilities of the system with p-unit is given by-

$$A_0^p = \lim_{t \rightarrow \infty} A_0^p(t) = \lim_{s \rightarrow 0} s A_0^{p*}(s) = \lim_{s \rightarrow 0} \frac{s N_2(s)}{D_2(s)}$$

By putting s=0, we get an indeterminate form, so by applying L-hospital rule, the above result becomes-

$$A_0^p = \lim_{s \rightarrow 0} \frac{N_2(s)}{D_2'(s)} = \frac{N_2}{D_2} \text{ (say)}$$

where  $N_2 = (\delta \psi_0 + \bar{\delta} \psi_1) p_{50}$

and  $D_2 = p_{50} (\psi_0 + \psi_1) + (p_{12} + p_{13}) p_{50} \psi_2 + p_{13} p_{50} \psi_3 + p_{24} (p_{12} + p_{13}) (\psi_4 + \psi_5)$

Similarly the availability of the system due to the o-unit is given by-

$$A_0^{o*}(s) = \frac{N_3(s)}{D_2(s)}$$

where  $N_3(s) = q_{01}^* (q_{12}^* + q_{13}^* q_{32}^*) [(1 - q_{45}^* q_{54}^*) Z_2^* + q_{24}^* q_{45}^* Z_5^*]$

Again, steady state availabilities of the system with o-unit is given by-

$$A_0^o = \lim_{t \rightarrow \infty} A_0^o(t) = \lim_{s \rightarrow 0} s A_0^{o*}(s) = \lim_{s \rightarrow 0} \frac{s N_3(s)}{D_2(s)}$$

By putting s=0, we get an indeterminate form, so by applying L-hospital rule, the above result becomes-

$$A_0^o = \lim_{s \rightarrow 0} \frac{N_3(s)}{D_2'(s)} = \frac{N_3}{D_2} \text{ (say)}$$

where  $N_3 = (p_{12} + p_{13}) (p_{50} \psi_2 + p_{24} \psi_5)$  and  $D_2$  is given as above.

### 5.4 Busy Period Analysis

Let  $B_i^p(t)$ ,  $B_i^o(t)$  and  $B_i^T(t)$  be the respective probabilities that the repairman is busy in the repair of p-unit, o-unit and transfer switch respectively at time t when system initially starts from state  $S_i \in E$ . Here, in this a dichotomous variable  $\phi$  is introduced which takes two values 1 and 0 respectively when the repairman is busy in partially failed unit and totally failed unit. By using probabilistic arguments we get-

$$B_0^{p*}(s) = \frac{N_4(s)}{D_2(s)}$$

where  $N_4(s) = q_{01}^* (1 - q_{45}^* q_{54}^*) \phi Z_1^* + q_{01}^* (q_{12}^* + q_{13}^* q_{32}^*) \bar{\phi} [(1 - q_{45}^* q_{54}^*) Z_2^* + q_{24}^* q_{45}^* Z_5^*]$  and  $D_2(s)$  is given as in the section 5.3

Now, the steady state probability  $B_0^p$  is given by-

$$B_0^p = \lim_{t \rightarrow \infty} B_0^p(t) = \lim_{s \rightarrow 0} \frac{sN_4(s)}{D_2(s)} = \frac{N_4(0)}{D_2'(0)} = \frac{N_4}{D_2}$$

where  $N_3 = p_{50}\phi\psi_1 + (p_{12} + p_{13})\bar{\phi}(p_{50}\psi_2 + p_{24}\psi_5)$

By similar probabilistic arguments the recursive relations for  $B_i^o(t)$  and  $B_i^T(t)$  can be developed. By taking their L.T, we have-

$$B_0^{o*}(s) = \frac{N_5(s)}{D_2(s)} \quad \text{and} \quad B_0^{T*}(s) = \frac{N_6(s)}{D_2(s)}$$

where  $N_5(s) = q_{01}^*(q_{12}^* + q_{13}^*q_{32}^*)q_{24}^*Z_4^*$  and  $N_6(s) = q_{01}^*q_{13}^*(1 - q_{45}^*q_{54}^*)Z_3^*$

The steady-state probabilities that the repairman is busy in the repair of o-unit and transfer switch is given by-

$$B_0^o = \lim_{s \rightarrow 0} B_0^{o*}(s) = \frac{N_5}{D_2} \quad \text{and} \quad B_0^T = \lim_{s \rightarrow 0} B_0^{T*}(s) = \frac{N_6}{D_2}$$

where  $N_4 = N_4(0) = (p_{12} + p_{13})p_{24}\psi_4$  and  $N_5 = N_5(0) = p_{13}p_{50}\psi_3$

## VI. PROFIT FUNCTION ANALYSIS

Let  $K_0$  be the per unit up time revenue by the system when p-unit operates in N-mode,  $K_1$  is the revenue per unit of time by the system when p-unit operates in P mode and  $K_2$  is the revenue per unit of time by the system when o-unit operates. Similarly,  $K_3, K_4, K_5$  and  $K_6$  are the payments to the repair men per unit of time when he busy in repairing a partially failed p-unit, totally failed p-unit, failed o-unit and failed transfer switch respectively. Then net expected profit earned by the system during (0, t) is given by

$$P(t) = \text{Expected total revenue in } (0, t) - \text{Expected total expenditure in } (0, t) \\ = K_0\mu_{up}^{pN}(t) + K_1\mu_{up}^{pP}(t) + K_2\mu_{up}^o(t) - K_3\mu_b^{pP}(t) - K_4\mu_b^{pF}(t) - K_5\mu_b^o(t) - K_6\mu_b^T(t)$$

Where the mean up time of the system during (0, t) due to p-unit in N-mode is given by

$$\mu_{up}^{pN}(t) = \int_0^t A_o^{pN}(u) du \quad \text{So that} \quad \mu_{up}^{pN*}(s) = \frac{A_o^{pN*}(s)}{s}$$

Similarly other related terms can be evaluated. Now the expected profit per unit time in steady state is given by

$$P = \lim_{t \rightarrow \infty} \frac{P(t)}{t} = \lim_{s \rightarrow 0} s^2 P^*(s) \\ = K_0 \lim_{s \rightarrow 0} s A_0^{pN*}(s) + K_1 \lim_{s \rightarrow 0} s A_0^{pP*}(s) + K_2 \lim_{s \rightarrow 0} s A_0^{o*}(s) - K_3 \lim_{s \rightarrow 0} s B_0^{pP*}(s) - K_4 \lim_{s \rightarrow 0} s B_0^{pF*}(s) \\ - K_5 \lim_{s \rightarrow 0} s B_0^{o*}(s) - K_6 \lim_{s \rightarrow 0} s B_0^{T*}(s) \\ = K_0 A_0^{pN} + K_1 A_0^{pP} + K_2 A_0^o - K_3 B_0^{pP} - K_4 B_0^{pF} - K_5 B_0^o - K_6 B_0^T$$

## VII. GRAPHICAL STUDY OF SYSTEM BEHAVIOR

Here in this section we study the behavior of MTSF and net expected profit in steady state through graph w.r.t the various parameters. We have plotted these characteristics in fig 2 and fig3 respectively w.r.t the failure rate  $\alpha_1$  for three different values of repair rate  $\beta_1(0.30, 0.50, 0.70)$  and two values of correlation coefficient  $r_1(0.25, 0.75)$  whereas the other parameters are kept fixed as  $\alpha_2 = 0.30, \alpha_3 = 0.05, \beta_2 = 0.20, \beta_3 = 0.30, r_1 = r_2 = 0.50, \theta = 0.70, \eta = 0.10, K_0 = 500, K_1 = 200, K_2 = 300, K_4 = 150, K_5 = 100, K_6 = 25$

From fig 2, it is observed that the MTSF decreases uniformly with the increase in failure rate  $\alpha_1$  and it increases with the increase in repair rate  $\beta_1$  and correlation coefficient  $r_1$ . Also from fig 3 we note that the profit function P decreases almost linearly as  $\alpha_1$  increases. Further, it is obvious that the profit increases with the increase in repair parameter  $\beta_1$  and coefficient of correlation  $r_1$ . Thus we conclude that the higher correlation between the failure and repair times provides the better system performance.

### VIII. CONCLUSIONS

In this paper, a solution procedure was developed to find the reliability and mean time to system failure for a two-priority unit standby system with imperfect switch and correlated failure and repair times. In addition the availability, busy period of the repairman, expected number of repairs and the profit function were also considered. Some particular cases are also considered. This kind of study can be useful for investigating the working efficiency of various mechanical/ electrical systems.

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TRANSITION DIAGRAM

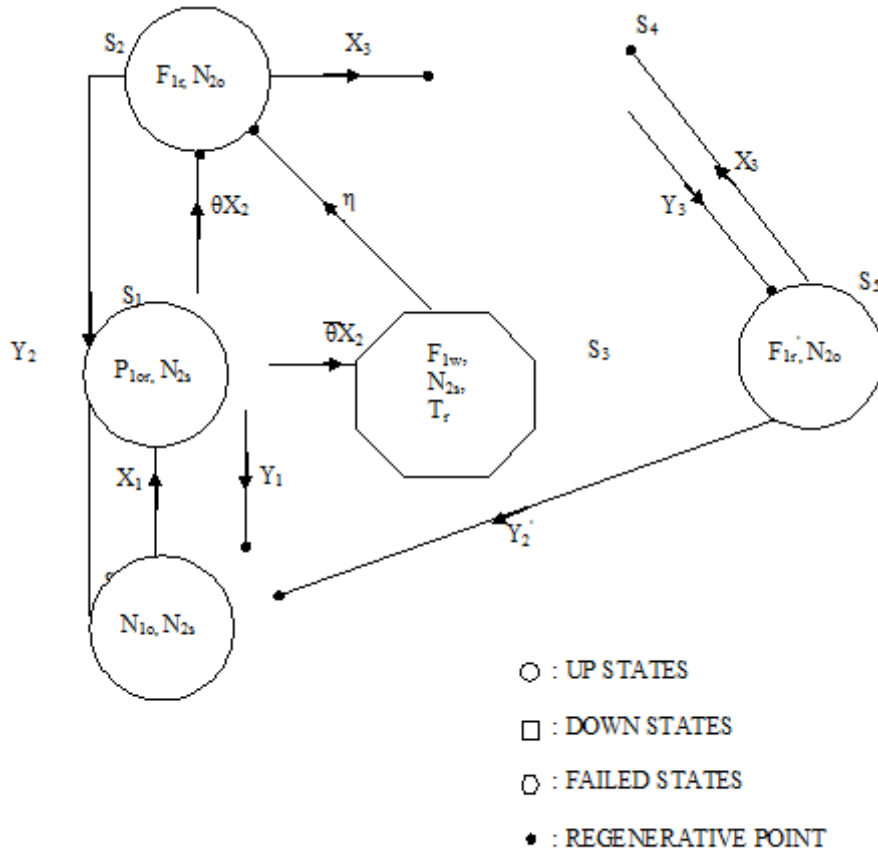


Fig 1

Behaviour of MTSF w.r.t.  $\alpha_1$

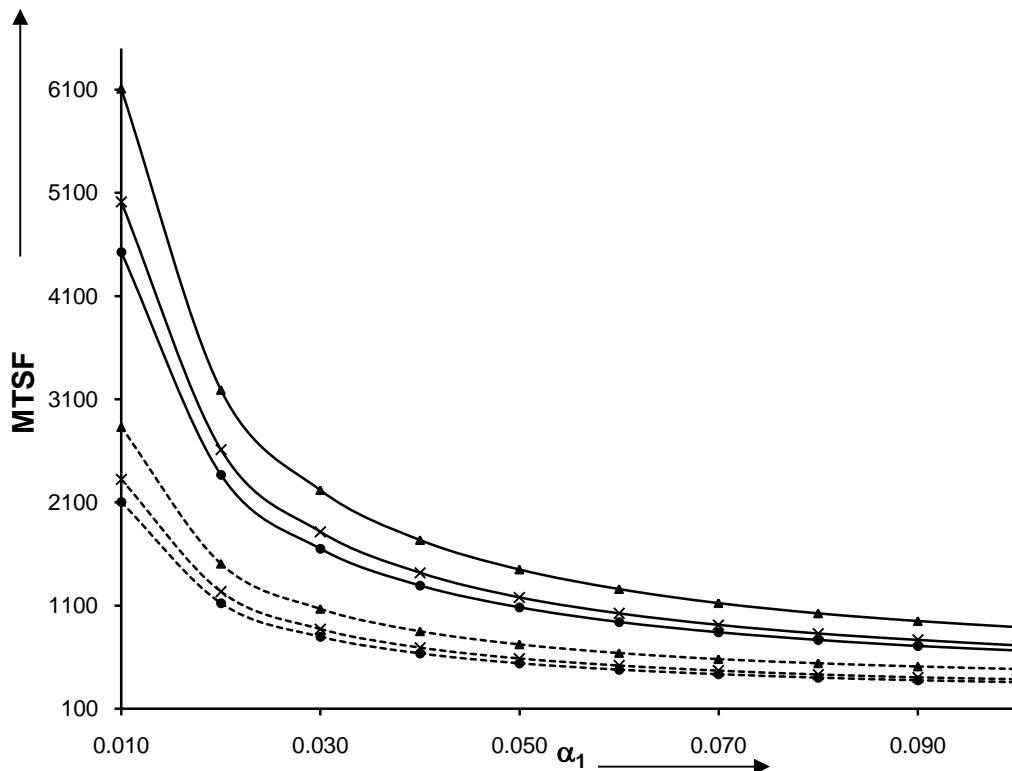


Fig 2

Behavior of Profit function P w.r.t  $\alpha_1$

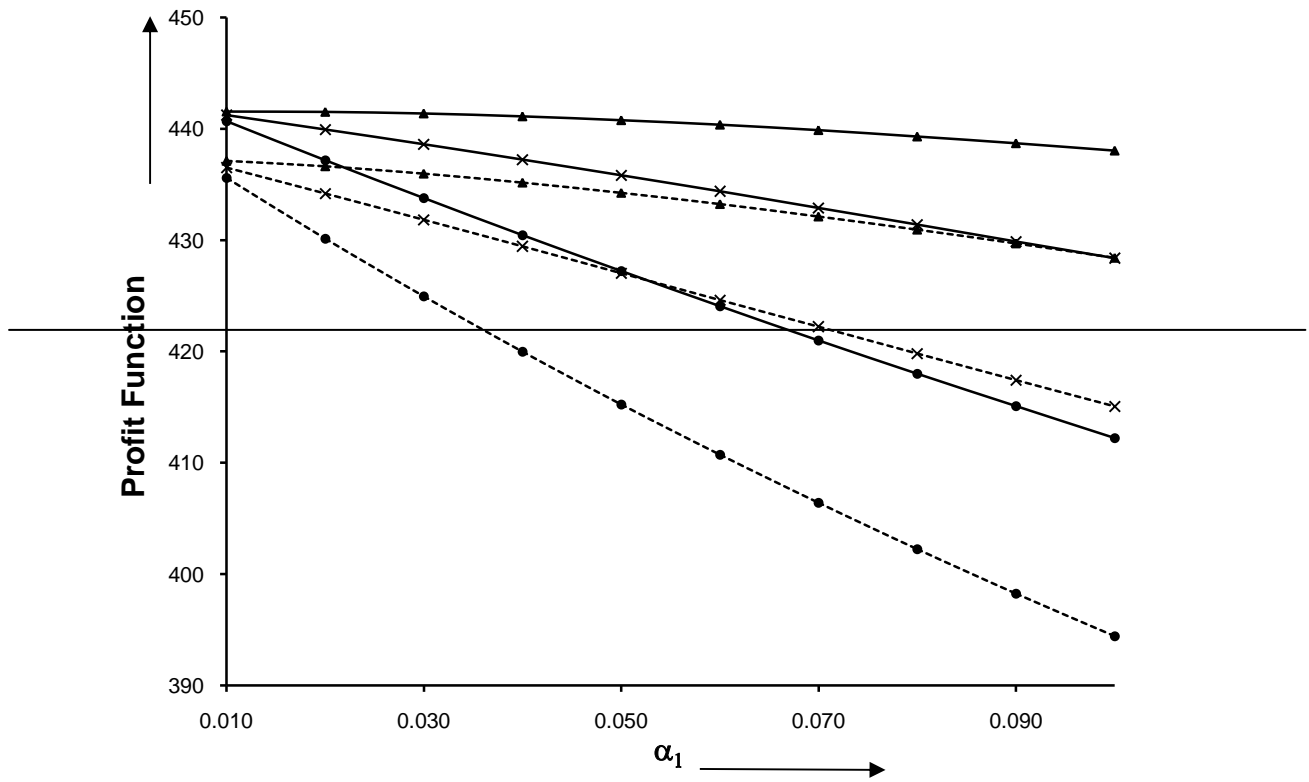


Fig 3