



## An Optimization Model for A Supply Chain Decision Problem of Fish Processed Products Considering Inventory and Routing

Open Darnius\*

Department of Mathematics,

University of Sumatera Utara, Indonesia

Herman Mawengkang

Department of Mathematics,

University of Sumatera Utara, Indonesia

---

**Abstract**— *Perishable products, such as fish processed, provide extra challenges to the supply chain due to their limited shelf-life. In this paper we propose an integer programming model for the inventory-location problem with routing costs. A supplier distributes multiple fish processed products to multiple retailers with deterministic demand through a set of distribution centers. The model can determine how many warehouses to open, where to locate them, and which customers to allocate to them. We develop a direct feasible search approach for solving the model.*

**Keywords**— *Supply Chain, Vehicle Routing, Inventory, Integer Programming, Direct Search.*

---

### I. INTRODUCTION

Supply chain management in the process industries has long been used as a tool to define production and distribution policies, as well as product allocation. This is the case of [9] who described the modelling of a supply chain composed of raw material vendors, primary and secondary plants (each one with inventories of raw materials and finished products), distribution centres, warehouses and customer areas. Later, [10] used supply chain optimisation to analyse the impact of scale, complexity (the operating costs are a function of the utilisation rates and number of products being processed in each facility) and weight of each cost factor (e.g. production, transportation and allocation costs) on the optimal design and utilisation patterns of the supply chain systems.

[13] described an MILP model, which combined production, distribution and marketing and involved plants and sales points, to cover the relevant features required for the complete supply chain management of a multi-site production network. [11] developed a Capacitated Plant Location Problem (CPLP) type model for planning and coordination of production and distribution facilities for multiple commodities, comprising raw materials suppliers, production sites, warehouses and customer areas. The authors followed a holistic approach to the supply chain, resulting in a deterministic, steady-state, multiechelon problem.

[12] considered both integrated and decoupled production and distribution planning problem, consisting of multiple plants, retailers, items over multiple periods. The author proposed mixed integer optimisation models and a two-phase heuristic solution to maximise the total net profit.

To design an efficient supply chain, we need to consider problems at the strategic, tactical, and operational levels. For example, we need to choose suppliers, decide on the location of production, and choose the transportation modality from the suppliers and to the customers. We also need to consider the production, storage, and transportation requirements for different products. In general, these problems are large scale problems that are complex in nature. Historically, these decisions were treated separately. This led to sub-optimality and excessive costs. Recently, supply chain managers and researchers have realized the importance of integration of supply chain decisions. Many researchers showed significant savings when considering a combination of the aforementioned decisions into a single model. Many models presented in the literature combine two of the supply chain decisions into one single model. However, few models integrate all three decisions and solve them simultaneously [16].

In this paper we consider supply chain decision problem which arises in marine fisheries industry in Indonesia. Marine fisheries play an important role in the economic development of Indonesia. This industry could also provide employment to people who live at coastal areas, to increase the financial gain of local government, and to conserve sustainability. Fisheries industrial sector can be classified into three different parts, i.e., open sea fishing, fish cultivation and processed fish. This paper is focusing on the latter sector.

Generally the processed fish industry in Indonesia can be found at the coastal area. There are a lot of varieties of fish processed can be produced, such as smoked fish, salted fish, crunchy bashed of fish, fish bowl, terrain (fish preserved), etc. The management of fish processed industry is still dominated by the local small traditional business, using conventional management strategy. They need supply chain management to distribute the fish productions to cities in Indonesia which are located in several islands. The expired date of fish processed products is short. This type of product, called perishable product, can be consumed whether they are top fresh or a few days old, but after their expiry date, they are usually deemed unfit for sale by the retailers.

Perishable products, such as fish processed, would impose additional difficulties to the supply chain management due to their limited shelf-life. These difficulties include the duration of storing the products due to the expired date. Hence, quantities delivered to retailers are limited by the shelf-life of goods as well as the retailer's holding capacity.

The objective of our model is to construct an integrating production, inventory and routing decisions that minimizes total costs while meeting the following requirements: decide how many warehouses are needed, where to locate the opened warehouses, and how to allocate customers to them; find out how much to keep in inventory at any time period; and understand how to build the vehicles' routes starting from opened warehouse to customers and back to that warehouse. In general distribution problems can be considered as a combinatorial optimization model. Therefore the model is formulated as a Mixed Integer Program (MIP). A direct feasible search approach is developed for solving the model.

Supply chains with perishable products have been studied in different lines of research. Some researchers extended the economic order quantity (EOQ) policy for inventory models which include perishable products. For example, [2] proposed an inventory model for a perishable product where the demand rate is a function of the on-hand inventory, and the holding cost is nonlinear. Moreover, [6] proposed stock-dependent selling rate model where the backlogging function was assumed to be dependent on the amount of demand backlogged. [1] extended their model by introducing a time-proportional backlogging rate. [14] a model of joint replenishment and delivery for perishable products. In a different line of research, [3] extended the vehicle routing problem with time-windows discussed in a number of papers (e.g. [4] and [7]), by considering the randomness of the perishable food delivery process. However, both EOQ and VRP extensions lack the integration of inventory and transportation decisions. Hence, the problem of sub-optimality might arise.

[5] combined inventory and routing components into one model. They proposed a column generation-based heuristic to solve the model. They showed significant savings when using their model. On the other hand, [8] presented a joint location-inventory model for blood distribution system. Recently, [15] proposed an integer programming model which integrates production, inventory and routing plan of perishable products.

## II. MODEL FORMULATION

This model deals with the distribution of several fish processed products  $N$  from a single manufacturer to a set of retailers,  $I$ , through a set of warehouses that can be located at various pre-determined sites,  $W$ .

To model the problem there are several assumptions.

- (i) The retailers deal with deterministic demand.
- (ii) The fleet of vehicles are homogen with the same capacity.
- (iii) Out-of-stock situations never occur.
- (iv) The fish products have a fixed shelf-life.
- (v) The inventory holding cost for warehouses is the same for all candidate warehouses. (vi) Inventory level at retailers is limited by two constraints, namely: physical capacity at retailers site and shelf-life of products.
- (vi) Upper bound inventory level at customer sites is only defined by the perishability restrictions [5]

Let  $G = (V, E)$  be an undirected network where  $V$  is a set of nodes comprised of a subset  $I$  of  $m$  potential warehouse sites and a subset  $J = V/I$  of  $n$  customers.  $E$  is a set of route connecting each pair of nodes in  $V$ . We define a feasible route as a route in which a vehicle starts from a candidate warehouse, visits a number of retailers, and comes back to the same warehouse. As such, a feasible route does not pass by more than one warehouse. As a result, the number of possible feasible routes will be  $W \times 2^I$ , where  $W$  and  $I$  are the number of warehouses and retailers, respectively. Feasible routes and the associated parameters ( $\alpha_{ir}$  and  $\beta_{rw}$ , as explained later) are required to define the model and, therefore, generated before solving the model. We assume vehicle capacity to be larger than the maximum customer demand at any time period. Moreover, in any time period, each vehicle travels at most on one route, and customers are visited at most once.

The objective is to minimize total cost incurred in the inventory-location problem. The total cost consists of three components. They are as follows:

- (i) warehouse fixed-location cost: the cost to establish and operate a warehouse;
- (ii) retailer unit-inventory holding cost: the cost to store products at retailer; and
- (iii) routing cost: the cost associated with delivering the goods from warehouse to retailers.

To formulate the problem, we use a similar notation as that used in [5], as follows.

Sets

- $N$  = Set of products,  $N = 1, \dots, n$   
 $W$  = Set of candidate warehouses  $W = 0, \dots, |W|$   
 $I$  = Set of retailers,  $I = 0, \dots, |I|$   
 $V$  = Set of nodes,  $V = W \cup I$   
 $T$  = Set of time periods  $T = 1, \dots, |T|$   
 $R$  = Set of all feasible routes  
 $K$  = Set of homogeneous vehicles  $K = 1, \dots, |K|$

Parameters

- $F_w$  = Fixed cost of opening and operating warehouse  $w \in W$   
 $C$  = Vehicle capacity  
 $\tau_{max}$  = Maximum shelf-life

- $d_{ijt}$  = Demand of product  $j \in N$  at customer  $i \in I$  in time period  $t = 1, \dots, T, \dots, T + \tau_{max} - 1$   
 $u_{ijt}$  = Upper bound inventory level of product  $j \in N$  at customer  $i \in I$  in time period  $t \in T$ ,  $u_{it} = \left( \sum_{t' > t}^{t' + \tau_{max}} d_{it'} \right)$   
 $h_{ijt}$  = Inventory holding cost of product  $j \in N$  at customer  $i \in I$  in time period  $t \in T$   
 $I_{ij0}$  = Inventory level of product  $j \in N$  at customer  $i \in I$  at the beginning of time period  $t = 1$   
 $\beta_{rw}$  =  $\begin{cases} 1 & \text{if route } r \in R \text{ visits warehouse } w \in W; \\ 0 & \text{otherwise} \end{cases}$   
 $c_{vv'}$  = Transportation cost from node  $v \in V$  to node  $v' \in V$   
 $c_r$  = Transportation cost of route  $r \in R$

#### Decision Variables

- $I_{it}$  = Inventory level of product  $j \in N$  at customer  $i \in I$  at the end of time period  $t \in T$   
 $\theta_{rt}$  =  $\begin{cases} 1 & \text{if route } r \in R \text{ is selected in time period } t \in T; \\ 0 & \text{otherwise} \end{cases}$   
 $m_w$  =  $\begin{cases} 1 & \text{if warehouse is opened at location } w \in W; \\ 0 & \text{otherwise} \end{cases}$   
 $a_{jirt}$  = Quantity of product  $j \in N$  delivered to customer  $i \in I$  by route  $r \in R$  in time period  $t \in T$

#### 4. The Model

The problem can be formulated as a MIP problem which has a mathematical form as follows.

$$\min \sum_{w \in W} f_w m_w + \sum_{t \in T} \left( \sum_{r \in R} c_r \theta_{rt} + \sum_{j \in N} \sum_{i \in I} h_{jit} I_{jit} \right) \quad (1)$$

$$s.t. \quad \sum_{r \in R} \theta_{rt} \leq 1 \quad \forall t \in T \quad (2)$$

$$\sum_{i \in I} a_{jirt} \leq C \theta_{rt} \quad \forall j \in N, r \in R, t \in T \quad (3)$$

$$I_{ijt-1} + \sum_{r \in R} \alpha_{jir} a_{jirt} = d_{jit} + I_{jit} \quad \forall j \in N, i \in I, t \in T \quad (4)$$

$$I_{ijt} \leq u_{ijt} \quad \forall i \in I, t \in T, j \in N \quad (5)$$

$$\theta_{rt} \leq \sum_{w \in W} \beta_{rw} m_w \quad \forall r \in R, t \in T \quad (6)$$

$$\sum_{r \in R} \theta_{rt} \leq |K| \quad \forall t \in T \quad (7)$$

$$\theta_{rt} \in \{0, 1\} \quad \forall r \in R, t \in T \quad (8)$$

$$m_w \in \{0, 1\} \quad \forall w \in W \quad (9)$$

$$\alpha_{jirt}, I_{jit} \geq 0 \quad \forall j \in N, i \in I, r \in R, t \in T \quad (10)$$

The objective function(1) shows the sum of costs included in this model. The first term represents the cost of opening and operating the selected warehouses, whereas the second term represents the transportation cost. The last term is the inventory holding cost. Constraints (2) guarantee that a customer is visited once at most in any time period. Constraints (3) account for the vehicle capacities. Inventory balance equations are represented in constraints (4). Constraints (5) ensure that the inventory level at a customer never exceeds the total demand in the next  $(\tau_{max} - 1)$  consecutive time periods. Constraints (6) guarantee that routes start and end with open warehouses only. Constraint (7) limit the maximum number of routes at any time period to be no greater than the number of vehicles. Finally, constraints (8) and (9) restrict  $\theta_{rt}$  and  $m_w$  to be binary, and constraints (10) ensure that quantities to be shipped to customers and inventory levels are non-negative.

### III. THE BASIC APPROACH

Consider a mixed integer linear programming (MILP) problem with the following form.

$$\text{Minimize} \quad P = c^T x \quad (11)$$

$$\text{Subject to} \quad Ax \leq b \quad (12)$$

$$x \geq 0 \quad (13)$$

$$x_j \text{ integer for some } j \in J \quad (14)$$

A component of the optimal basic feasible vector  $(x_B)_k$ , to MILP solved as continuous can be written as

$$(x_B)_k = \beta_k - \alpha_{k1}(x_N)_1 - \dots - \alpha_{kj}(x_N)_j - \dots - \alpha_{kn} - m(x_N)_N n - m \quad (15)$$

Note that, this expression can be found in the final tableau of Simplex procedure. If  $(x_B)_k$  is an integer variable and we assume that  $\beta_k$  is not an integer, the partitioning of  $\beta_k$  into the integer and fractional components is that given

$$\beta_k = [\beta_k] + f_k, 0 \leq f_k \leq 1 \quad (16)$$

suppose we wish to increase  $(x_B)_k$  to its nearest integer,  $([\beta_k] + 1)$ . Based on the idea of suboptimal solutions we may elevate a particular nonbasic variable, say  $(x_N)_{j^*}$ , above its bound of zero, provided  $\alpha_{kj^*}$ , as one of the element of the vector  $\alpha_{j^*}$ , is negative. Let  $\Delta_{j^*}$  be amount of movement of the non variable  $(x_N)_{j^*}$ , such that the numerical value of scalar  $(x_B)_k$  is integer. Referring to Eqn. (15),  $\Delta_{j^*}$  can then be expressed as

$$\Delta_{j^*} = \frac{1-f_k}{-\alpha_{kj^*}} \quad (17)$$

while the remaining nonbasic stay at zero. It can be seen that after substituting (16) into (17) for  $(x_N)_{j^*}$  and taking into account the partitioning of  $\beta_k$  given in (16), we obtain

$$(x_B)_k = [\beta_k] + 1$$

Thus,  $(x_B)_k$  is now an integer.

It is now clear that a nonbasic variable plays an important role to integerize the corresponding basic variable. Therefore, the following result is necessary in order to confirm that must be a non-integer variable to work with in integerizing process.

**Theorem 1.** Suppose the MILP problem (11)-(14) has an optimal solution, then some of the nonbasic variables.  $(x_N)_j, j = 1, \dots, n$ , must be non-integer variables.

**Proof:**

Solving problem as a continuous of slack variables (which are non-integer, except in the case of equality constraint). If we assume that the vector of basic variables consists of all the slack variables then all integer variables would be in the nonbasic vector  $x_N$  and therefore integer valued.

#### IV. DERIVATION OF THE METHOD

It is clear that the other components,  $(x_B)_{i \neq k}$ , of vector  $x_B$  will also be affected as the numerical value of the scalar  $(x_N)_{j^*}$  increases to  $\Delta_{j^*}$ . Consequently, if some element of vector  $\alpha_{j^*}$ , i.e.,  $\alpha_{ij^*}$  for  $i \neq k$ , are positive, then the corresponding element of  $x_B$  will decrease, and eventually may pass through zero. However, any component of vector  $x$  must not go below zero due to the non-negativity restriction. Therefore, a formula, called the minimum ratio test is needed in order to see what is the maximum movement of the nonbasic  $(x_N)_{j^*}$  such that all components of  $x$  remain feasible. This ratio test would include two cases.

1. A basic variable  $(x_B)_{j \neq k}$  decreases to zero (lower bound) first.
2. The basic variable,  $(x_B)_k$  increases to an integer.

Specifically, corresponding to each of these two cases above, one would compute

$$\theta_1 = \min_{i \neq k | \alpha_{ij^*} > 0} \left\{ \frac{\beta_i}{\alpha_{ij^*}} \right\} \quad (18)$$

$$\theta_2 = \Delta_{j^*} \quad (19)$$

How far one can release the nonbasic  $(x_N)_{j^*}$  from its bound of zero, such that vector  $x$  remains feasible, will depend on the ratio test  $\theta^k$  given below

$$\theta^k = \min(\theta_1, \theta_2) \quad (20)$$

Obviously, if  $\theta^k = \theta_1$ , one of the basic variable  $(x_B)_{j \neq k}$  will hit the lower bound before  $(x_B)_k$  becomes integer. If  $\theta^k = \theta_2$ , the numerical value of the basic variable  $(x_B)_k$  will be integer and feasibility is still maintained. Analogously, we would be able to reduce the numerical value of the basic variable  $(x_B)_k$  to its closest integer  $[\beta_k]$ . In this case the amount of movement of a particular nonbasic variable,  $(x_N)_{j^*}$ , corresponding to any positive element of vector  $\alpha_{j^*}$ , is given by

$$\Delta_{j^*} = \frac{f_k}{\alpha_{kj^*}} \quad (21)$$

In order to maintain the feasibility, the ratio test  $\theta^*$  is still needed.

Consider the movement of a particular nonbasic variable,  $\Delta$ , as expressed in Eqns.(18) and (21).The only factor that one needs to calculate is the corresponding element of vector  $\alpha$ . A vector  $\alpha_j$  can be expressed as

$$\alpha_j = B^{-1} \alpha_j, j = 1, \dots, n - m \quad (22)$$

Therefore, in order to get a particular element of vector  $\alpha_j$  we should be able to distinguish the corresponding column of matrix  $[B]^{-1}$ . Suppose we need the value of element  $\alpha_{kj^*}$ , letting  $v_k^T$  be the  $k$ -th column vector of  $[B]^{-1}$ , we then have

$$v_k^T = e_k^T B^{-1} \quad (23)$$

Subsequently, the numerical value of  $\alpha_{kj}^*$  can be obtained from

$$\alpha_{kj}^* = v_k^T \alpha_j^* \quad (24)$$

in Linear Programming (LP) terminology the operation conducted in Eqns. (23) and (24) is called the pricing operation. The vector of reduced costs  $d_j$  is used to measure the deterioration of the objective function value caused by releasing a nonbasic variable from its bound. Consequently, in deciding which nonbasic should be released in the integerizing process, the vector  $d_j$  must be taken into account, such that deterioration is minimized. Recall that the minimum continuous solution provides a lower bound to any integer-feasible solution. Nevertheless, the amount of movement of particular nonbasic variable as given in Eqns. (18) or (21), depends in some way on the corresponding element of vector  $\alpha_j$ . Therefore it can be observed that the deterioration of the objective function value due to releasing a nonbasic variable  $(x_N)_{j^*}$  so as to integerize a basic variable  $(x_B)_k$  may be measured by the ratio

$$\left| \frac{d_k}{\alpha_{kj}^*} \right| \quad (25)$$

where  $|\alpha|$  means the absolute value of scalar  $a$ .

In order to minimize the deterioration of the optimal continuous solution we then use the following strategy for deciding which nonbasic variable may be increased from its bound of zero, that is,

$$\min_j \left\{ \left| \frac{d_k}{\alpha_{kj}^*} \right| \right\}, j = 1, \dots, n - m \quad (26)$$

From the “active constraint” strategy and the partitioning of the constraints corresponding to basic ( $B$ ), superbasic ( $S$ ) and nonbasic ( $N$ ) variables, we can write [5].

$$\begin{bmatrix} B & S & N \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_b \\ x_N \\ x_S \end{bmatrix} = \begin{bmatrix} b \\ b_N \end{bmatrix} \quad (27)$$

or

$$Bx_b + Sx_N + Nx_S = b \quad (28)$$

$$x_N = b_N \quad (29)$$

The basis matrix  $B$  is assumed to be square and nonsingular, we get

$$x_B = \beta - Wx_S - \alpha x_N \quad (30)$$

Where

$$\beta = B^{-1}b \quad (31)$$

$$W = B^{-1}S \quad (32)$$

$$\alpha = B^{-1}N \quad (33)$$

Expression (30) indicates that the nonbasic variables are being held equal to their bound. It is evident through the “nearly” basic expression of Eqn. (30), the integerizing strategy discussed in the previous section, designed for MILP problem can be implemented. Particularly, we would be able to release a nonbasic variable from its bound, Eqn.(29) and exchange it with a corresponding basic variable in the integerizing process, although the solution would be degenerate.

**Theorem 2.** Suppose the MILP problem has a bounded optimal continuous solution, then we can always get a non-integer  $y_j$  in the optimum basic variable vector.

**Proof.**

1. If these variables are nonbasic, they will be at their bound. Therefore they have integer value.
2. If a  $y_j$  is superbasic, it is possible to make  $y_j$  basic and bring in a nonbasic at its bound to replace it in the superbasic.

However, the ratio test expressed in (20) cannot be used as a tool to guarantee that the integer solution found still remains in the feasible region.

## V. PIVOTING

Currently, we are in a position where particular basic variable,  $(x_B)_k$  is being integerized, thereby a corresponding nonbasic variable,  $(x_N)_{j^*}$ , is being released from its bound of zero. Suppose the maximum movement of  $(x_N)_{j^*}$  satisfies

$$\theta^* = \Delta_{j^*}$$

such that  $(x_B)_k$  is integer valued. To exploit the manner of changing the basis in linear programming, we would be able to move  $(x_N)_{j^*}$  into  $B$  (to replace  $(x_B)_k$ ) and integer-valued  $(x_B)_k$  into  $S$  in order to maintain the integer solution. We now have a degenerate solution since a basic variable is at its bound. The integerizing process continues with a new set of  $[B, S]$ . In this case, eventually we may end up with all of the integer variables being superbasic.

**Theorem 3.** A suboptimal solution exists to the MILP problem in which all of the integer variables are superbasic.

**Proof.**

1. If all of the integer variables are in  $N$ , then they will be at bound.
2. If an integer variable is basic it is possible to either
  - interchange it with a superbasic continuous variable, or
  - make this integer variable superbasic and bring in a nonbasic at its bound to replace it in the basis which gives a degenerate solution.

The other case which can happen is that a different basic variables  $(x_B)_{i \neq k}$  may hit its bound before  $(x_B)_k$  becomes integer. Or in other words, we are in a situation where

$$\theta^* = \Delta_1$$

In this case we move the basic variable  $(x_B)_j$  into  $N$  and its position in the basic variable vector would be replaced by nonbasic  $(x_B)_{j^*}$ . Note that  $(x_B)_k$  is still a non-integer basic variable with a new value.

## VI. THE ALGORITHM

First we solve the mixed integer programming model Eq. (1) – (10) by relaxing the integer restriction. After solving the relaxed problem, the procedure for searching a suboptimal but integer-feasible solution from an optimal continuous solution can be described as follows.

Let

$$x = [x] + f, \quad 0 \leq f \leq 1$$

be the (continuous) solution of the relaxed problem,  $[x]$  is the integer component of non-integer variable  $x$  and  $f$  is the fractional component.

Stage 1.

Step 1. Get row  $i^*$  the smallest integer infeasibility, such that  $\delta_{i^*} = \min\{f_i, 1 - f_i\}$

(This step is taken in order to get a minimum deterioration of the objective function).

Step 2. Do a pricing operation

$$v_{i^*}^T = e_{i^*}^T B^{-1}$$

Step 3. Calculate  $\sigma_{ij} = v_{i^*}^T \alpha_j$

With  $j$  corresponds to

$$\min_j \left\{ \left| \frac{d_j}{\alpha_{ij}} \right| \right\}$$

Calculate the maximum movement of nonbasic  $j$  at lower bound and upper bound.

Otherwise go to next non-integer nonbasic or superbasic  $j$  (if available). Eventually the column  $j^*$  is to be increased from LB or decreased from UB. If none go to next  $i^*$ .

Step 4.

Solve  $B\alpha_{j^*} = \alpha_{j^*}$  for  $\alpha_{j^*}$

Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic  $j^*$  from its bounds.

Step 6. Exchange basis

Step 7. If row  $i^* = \{\emptyset\}$  go to Stage 2, otherwise

Repeat from step 1.

Stage 2. Pass1 : adjust integer infeasible superbasics by fractional steps to reach complete integer feasibility.

Pass2 : adjust integer feasible superbasics. The objective of this phase is to conduct a highly localized neighbourhood search to verify local optimality.

## VII. CONCLUSION

This paper presents a joint location-inventory-routing model for supply chains of fish processed products which can be regarded as perishable products. The model is formulated as Mixed Integer Program (MIP). We solve the model using the feasible neighbourhood search approach.

In the model we assume that the maximum shelf-life time for each fish product is known products. In reality each fish processed product shelf-life time depends on temperature and humidity. Moreover, in this model it is assumed that goods maintain their values up to the expiry date, when they will be of no value. A more realistic scenario might be that products lose their value gradually throughout their lifetime.

## REFERENCES

- [1] C. Dye and L. Ouyang. An eq model for perishable items under stock-dependent selling rate and time-dependent partial backlogging. European Journal of Operational Research, 163(3):776-783, 2005.

- [2] B. Giri and K. Chaudhuri. Deterministic models of perishable inventory with stock-dependent demand rate and nonlinear holding cost. *European Journal of Operational Research*, 105(3):467-474, 1998.
- [3] C. Hsu, S. Hung, and H. Li. Vehicle routing problem with time-windows for perishable food delivery. *Journal of food engineering*, 80(2):465-475, 2007.
- [4] Y. Koskosidis, W. Powell, and M. Solomon. An optimization-based heuristic for vehicle routing and scheduling with soft time window constraints. *Transportation Science*, 26(2):69, 1992.
- [5] T. Le, A. Diabat, J. Richard, and Y. Yuehwern. A column generation-based heuristic algorithm for an inventory routing problem with perishable product. In Press, 2011.
- [6] G. Padmanabhan and P. Vrat. Eoq models for perishable items under stock dependent selling rate. *European Journal of Operational Research*, 86(2):281-292, 1995.
- [7] T. Sexton and Y. Choi. Pickup and delivery of partial loads with " soft" time windows. *American Journal of Mathematical and Management Sciences*, 6(3):369-398, 1986.
- [8] Z. Shen, C. Coullard, and M. Daskin. A joint location-inventory model. *Transportation Science*, 37(1): 40-55, 2003.
- [9] Cohen, M.A., Lee, H.L., Strategic analysis of integrated production-distribution systems – models and methods. *Oper. Res.* 36, 216–228, 1988.
- [10] Cohen, M.A., Moon, S., An integrated plant loading model with economies of scale and scope. *Eur. J. Oper. Res.* 50, 266–279, 1991.
- [11] Jayaraman, V., Pirkul, H., Planning and coordination of production and distribution facilities for multiple commodities. *Eur. J. Oper. Res.* 133, 394–408, 2001.
- [12] Park, Y.B., An integrated approach for production and distribution planning in supply chain management. *Int. J. Prod. Res.* 43, 1205–1224, 2005.
- [13] Timpe, C.H., Kallrath, J., Optimal planning in large multi-site production networks. *Eur. J. Oper. Res.* 126, 422–435, 2000.
- [14] L. C. Coelho and G. Laporte. Optimal Joint Replenishment and Delivery of Perishable Products. *CIRRELT-2013-20*, 2013.
- [15] S. M. Seyedhosseini and S. M. Ghoreyshi. An Integrated Model for Production and Distribution Planning of Perishable Products with Inventory and Routing Considerations. *Mathematical Problems in Engineering*, Vol. 2014, Article ID 475606, Hindawi Publishing Corporation, 2014.
- [16] A. Hiassat and A. Diabat. A Location-Inventory-Routing-Problem with Perishable Products. *Proceedings of the 41st Int. Conf. On Computers & Industrial Engineering*, pp. 386-391, LA, USA, 2011.