



Fixed-point digital IIR filter Design using two-stage ensemble evolutionary algorithm

Er. Daljit Singh Bajwa
ECE&BBSBEC,
Fatehgarh Sahib, India

Er. Karamjeet Singh
ECE&BBSBEC,
Fatehgarh Sahib, India

Navpreet Kaur Chahal
ECE &BBSBEC,
Fatehgarh Sahib, India

Abstract: The research on optimal design of Infinite-impulse response (IIR) filter design based on various optimization techniques including evolutionary algorithms EA's, has gained much attention in recent years. Previously, digital IIR filter's parameters are encoded using floating point representation. Fixed point representation are used for encoding of digital IIR filter's parameters because it effectively save computational resources and more convenient for direct realization on hardware. On comparing fixed point representation with floating point representation, fixed point representation would make the search space miss much useful gradient information and therefore raises new challenges for continuous EA's. In this paper, first we will design digital low pass IIR filter. Then, analyze the fitness landscape properties of optimal digital low pass IIR filter. Based upon the fitness landscape investigation, apply two-state ensemble evolutionary algorithm to the above design optimal digital low pass IIR filter with fixed-point representation. In order to evaluate the performance of TEEA, we experimentally compare it with SaDE, jaDE, CLPSO, MUEDA on Low-pass, High-pass, Band-pass, Band-stop digital IIR filters with 10 different settings. Comparison is based on the standard deviation, mean and the number of successful runs. Based on the experimentally compared result, we conclude that TEEA has higher convergence speed, better exploration, and higher success rate. In order to benchmark TEEA, we apply TEEA further to some more difficult problems with shorter word length and higher order. TEEA perform satisfactory on task of shorter word length and higher order as well.

Keywords- Digital infinite-impulse response (IIR) filter Order, Continue evolutionary algorithm, Fixed-point representation.

I. INTRODUCTION

Digital IIR filter is one of the most commonly used computation tools in the digital signal processing systems. Digital IIR filters are used many application such as high-speed and low-power communication transceivers, they are also used in routinely employed as accustom designed digital block [1]. IIR Filters are infinite response filter, they have impulse response of infinite duration. IIR filter are recursive filter in which feedback is present from output side to the input side. IIR filter response is depending upon the previous output samples, present and past input samples. Digital IIR filters are one of the most frequently used computational tool used in digital signal processing. It is used in much application such as high-speed and low-power communication transceivers systems. Difference equation of IIR filter is [2]:

$$Y(z) = \sum_{k=1}^N a_k y(m-k) + \sum_{k=0}^M b_k x(m-k)$$

In previous work, the bilinear transformation approach is used as one of the early technique and it is widely adopted. Using bilinear transformation approach, a digital filter is transformed to a corresponding analog low-pass (LP) filter. Then, the analog low-pass filter is designed using different method such as Butterworth, cheybhev type 1, cheybhev type 2 and elliptic are used. This procedure needs much pre-knowledge and shows poor performance in many cases. The filters designed by these methods are always encoded by using floating-point representation. Using floating-point representation is impractical among the hardware design and requires more computational power. The research is stimulated on the more effective optimization approaches with the less prior knowledge and higher accuracy to obtain the digital IIR filter with fixed-point representation.

Fixed-point representation: Fixed-point representation is a decimal representation of the numbers as a string of digits with decimal point. The number right side to the decimal point is known as fractional number and number left side to the decimal point is called integer part of the number. General representation of fixed-point representation is [2,4]:

$$X = (b_{-A}, \dots, b_{-1}, b_0, b_1, b_2, \dots, b_B) r$$

$$X = \sum_{i=-A}^B b_i r^{-i} \quad 0 \leq b_i \leq r^{-1}$$

where b_i = digits, r = radix or base, A = integer digits, B = fractional digits

A fixed-point representation allow to cover a range of numbers with the resolution

$$\Delta = \frac{x_{max} - x_{min}}{m - 1}$$

Where $m = 2^b$ is a number of levels, b = number of bits or digits.

A basic characteristic of the fixed-point resolution is to provide the uniform resolution throughout the given number range. Therefore Δ increases directly with increase in dynamic range.

Comparison between Floating-Point and Fixed-Point Representation: 1) Fixed-point representation consume less power and less costly for portable application as compare to floating point representation. 2) Fixed-point representation is efficiently save computational resources and is more convenient for direct realization on hardware whereas floating-point is impractical in hardware design and requires higher computational power. 3) Multiplication of two floating-point representation leads to overflow whereas no such overflow occur in fixed-point representation. 4) Fixed-point representation provide uniform resolution throughout the given number range whereas floating-point resolution provide the finer resolution for small number range and provide the coarse resolution for large number range. 5) Fixed-point representation would make the search space miss much useful gradient information and rises new challenges for continuous EAs.

II. RELATED WORK

Two-stage based ensemble optimization framework for large-scale global optimization: This paper is based on the design of two-stage based ensemble optimization evolutionary algorithm framework for large scale global optimization. This algorithm is implemented by two sub-optimizers. According to different implementation conditions, EOE framework can easily generated, modified and altered flexibly. To analyze the effect of EOE's components, compare its performance on diverse kinds of problem with its two sub-optimizers and three variants. In order to show its superiorities over the LSGO algorithms, compare its performance with six classical LSGO algorithms on the CSGO test functions of IEEE Congress of Evolutionary computation (CEC 2008). To evaluate the performance of EOE by experimental comparison with four state-of-the-arts LSGO algorithms on the test function of CEC 2010 LSGO competition [6].

Evaluation of Two-Stage Ensemble Evolutionary Algorithm for Numerical Optimization: This paper is tried to extend the application area of TSEA from specific engineering problems to numerical optimization problems by altering its sub-optimizers. The experimental studies contain three aspects in this paper ,firstly , the TSEA framework are experimentally compared by comparing TSEA with its sub-optimizers on 26 test functions. Secondly, TSEA is compared with diverse state-of-the-art evolutionary algorithms to show its advantage. Thirdly, the performance of TSEA is further compared with 4 classical memetic algorithm (MA's) on CECO's test functions. The experimental results show the excellent effectiveness, efficiency and reliability of TSEA [7].

Self-adapting control parameters in differential evolution: a comparative study on numerical benchmark problems: This paper presents an efficient technique for adapting control parameter settings associated with differential evolution (DE). The DE algorithm has only the few control parameters, which are kept fixed throughout the evolutionary process. Self-adapting control parameter in differential algorithm is a new version of the DE algorithm which is used for obtaining self-adaptive control parameter settings that show good performance on numerical benchmark problems. The results show that self-adapting control parameter in differential algorithm with self-adaptive control parameter settings is better than, or comparable to, the standard DE algorithm and evolutionary algorithms when considering the quality of the solutions obtained [17]. Future work is to make the population size adaptive and to do experiment with different population sizes.

Two-stage ensemble memetic algorithm: Function optimization and digital IIR filter design: This paper proposed a design of Two-Stage ensemble Memetic Algorithm (TSMA) framework to more appropriately synthesize the strengths of the evolutionary global search and local search techniques. The first optimization stage is local search techniques in which a competition is held among the candidate. The major idea of the first optimization stage is to choose the best local search technique and to obtain good initial state. The second optimization stage is to be implementing effective adaptive MA to pursue high-quality solution. A suit of the higher-order digital IIR filters are designed by surpassing the threshold. To evaluate the performance of TSMA, TSMA is compared with six state-of-the-art EAs, including MDE, SaDE, jDE, CLPSO, PSO-cf and IPOPOP-CMA-ES. This experiment study provided that TSMA has better effectiveness and efficiency over the other algorithms. Future work can be done on the impact of the choice of global search method, how to apply MAs to very large-scale global optimization tasks and applying TSMA to other real-world engineering problems [8].

Comprehensive Learning Particle Swarm Optimizer for Constrained mixed-variable Optimization problem: This paper is based on an improved particle swarm optimizer (PSO) for solving multimodal optimization problems with problem-specific constraints and mixed variables. The standard PSO is extended by using a comprehensive learning strategy, different particle updating approaches and a feasibility-based rule method. The advantages of CLPSO are illustrated by solving four mechanical design optimization problems. CLPSO improves performance in terms of search quality, efficiency and robustness. The numerical results of CLPSO are better than or equal to existing method. In the future work, feasibility –based rules are needed to improve because it is not completely reasonable that feasible solutions are considered better than infeasible solution in the rule. This could lead to the overpressure from the selecting feasible solution and lead to premature convergence[9].

Self-adaptive differential evolutionary algorithm using population size reduction and three strategies: this paper presents a self-adaptive differential evolutionary algorithm, called jDEL-scop for solving large-scale optimization problem with continuous variables. The self-adaptive differential evolutionary algorithm employs three strategies and a population size reduction mechanism. The performance of self-adaptive differential evolutionary algorithm is evaluated on a set of benchmark problems provided for the special issue on the stability of Evolutionary algorithm and the other metaheuristics for large scale continuous optimization problem. The non-parametric statistical procedure were performed for multiple comparison between the self-adaptive differential algorithm and three well know algorithms (jDEL-scop, DE, CHC). The result of this comparison shows that the self-adaptive differential algorithm can deal with large scale continuous optimization effectively. Self-adaptive differential algorithm also behaves significantly better than other three algorithms[10].

A Self-adaptive Mixed distribution based uni-variate estimation of distribution algorithm for large scale global optimization: Large scale global optimization is highly needed for many scientific and engineering applications. Large scale global optimization is a very important and difficult task in the optimization domain. Different algorithms have been proposed to tackle this challenging problem. But the use of estimation of distribution algorithms (EDAs) to large scale optimization problem is rare. This paper aims to investigating the behavior and performance of univariate EDAs mixed with different kernel probability densities via fitness landscape analysis. Based upon this analysis, a self adaptive uni-variate EDA with mixed kernels (MUEDA) is proposed. To assess the effectiveness and efficiency of MUEDA. The tasks of function optimization with dimension scale from 30 to 1500 are adopted. On comparing the recently published LSGO algorithms, MUEDA shows the excellent convergence speed, final solution quality and dimension scalability[11].

A modified differential evolutionary algorithm for the unconstrained optimization problems: In this paper, a modified differential evolutionary algorithm is proposed to solve unconstrained optimization problem. MDE improves the differential evolutionary algorithm in four aspects: firstly, each candidate solution in the MDE model has its own scale factor and crossover rate. Each scale factor is adjusted by using gauss distribution whereas each crossover rate is adjusted by using uniform distribution. Both of this distribution has the characteristics of randomness; therefore they are used to diversify the population. Secondly, in order to ensure the quality of the population, an external structure is the used and some good solutions in this external archive can be chosen for the candidate solutions. Thirdly, the MDE employs two mutation strategies to update the candidate solution, which is beneficial to improve the convergence of the proposed algorithm (MDE). Fourth, a central solution is produced by averaging all the candidate solutions and thus it can provide a promising and efficient alternative in evolutionary process that is potential searching directions. Experimental results show that MDE can yield better objective function values than the other six DE algorithms that are DE, SADE, ODE, JADE, NDE, MDE-pBX for some unconstrained optimization problems. Thus MDE is a precise and reliable method for solving an unconstrained optimization problem. Future work will be done on the elimination or relaxation of computational burden brought by the external structure[12].

III. MOTIVATION AND CONTRIBUTION

Most number of successful EA based digital IIR filter design; consider the parameters that take on value from a continuous-domain. Only few little research takes fixed-point representation into consideration.

In Embedded system's application and development, fixed-point implementation of digital IIR filter design are more and more common in real application. Therefore, optimal digital IIR filter design based on fixed-point digital IIR filter design based on fixed-point representation quickly gains significance. In the previous works on fixed-point digital IIR filter tackled some relatively simple problems and are lack of necessary comparison analysis. In this paper, this gap is trying to fill by applying a specific evolutionary algorithm to higher order and more difficult digital IIR filter design problems.

Digital IIR design is a multimodal optimization problem with complex landscape that inevitably cause premature convergence of most conventional EA's. The fitness landscape properties in this depicted in a 3-D space and discussed from the view of heuristic method. Based on the fitness landscape investigation, the difficulties of optimal digital IIR filter design are clearly revealed.

To handle the conflict between exploitation and exploration abilities of EA's must be balanced in a more effective and efficient way. To cope with the conflict between exploitation and exploration abilities we resort the Two-state ensemble evolutionary algorithm (TEEA). TEEA is different from the existing methods that use multiple population generation strategies in each iteration. The main idea behind the TEEA is to divide the optimization procedure into two stages:

1. global shrinking stage 2. Local exploration stage. The objective of the global shrinking stage ,is to shrink the searching scope to the promising area as quick as possible. The objective of local exploration stage is to explore the limited area extensively to find as good as possible solution. The exploration which is stage 2 in TEEA ,may consume too much computational cost and cannot effectively detect the promising area for the complex multimodal problems. The techniques with good exploitation capabilities can quickly converge to an optimal solution. However, by using some seed provided by global shrinking stage and randomly generating the other individuals, the local exploration stage can make up this demerit by its strong exploration capabilities is needed. To show advantages of TEEA, compared TEEA with several classical continuous EAs, are experimentally verified on four types of digital IIR filters with 10 different settings. In order to evaluate TEEA, we apply it to some more difficult problems with shorter word length and higher order. TEEA can provide a success rate of 100% on these hard tasks.

This paper is structured as follows: in section 2 digital IIR filter design problem is introduced. Then, the fitness landscape properties of digital IIR filter design problem are investigated. Section 3: can present the TEEA. Section 4: provides an experiment evaluation of the TEEA on designing low-pass, high-pass, and band-pass and band-stop digital IIR filter with 10 different settings. Four state-of-the-art continuous EAs are utilized to provide the comparison. Scalability analysis is carried out to illustrate the scalable characteristics of TEEA. Section 5 : contains brief conclusion and future work is outlined.

IV. PROBLEM STATEMENT AND ANALYSIS:

Design fixed-point digital IIR filter using TEEA because of its higher convergence speed, better exploration and higher success rate. The cascade form of an infinite-impulse response (IIR) filter can be described as [13,14]:

$$H(z) = k \prod_{k=1}^n \frac{1+b_k z^{-1}}{1+a_k z^{-1}} \prod_{i=1}^m \frac{1+d_{i1} z^{-1}+d_{i2} z^{-2}}{1+c_{i1} z^{-1}+c_{i2} z^{-2}}$$

Where k is gain , a_k and b_k ; $k= 1,2,3,\dots,n$ are the first-order coefficient , c_{i1} , c_{i2} , d_{i1} and d_{i2} ; $i= 1,2,3,\dots,m$ are the second-order coefficient of digital IIR filter. All the coefficients are fixed-point numbers, whose word length are set beforehand.

A. Fitness function definition:

The important task of a designer is to find the value of first order coefficient and second order coefficient that produce the desired response. In EAs, the fitness function represents the designing objective of the problem, which can formulated as minimization of the magnitude response error. This situation can simulated as the difference o the boundary of design requirements. The stability of the candidates should be guaranteed to make the designed filter feasible and implementable. The stability requirement of digital IIR filter has been summarized in paper [15]. The constraints presents in this paper can be expressed as follows:

$$\begin{aligned} -1 < c_{j2} < 1 & \text{ this equation represents the stability requirement for first-order block} \\ -1 - c_{j2} < c_{j1} < 1 + c_{j2} & \text{ this equation denotes the stability requirement for second-order block.} \end{aligned}$$

B. Investigation of fitness landscape characteristics :

In the introduction, much attention is paid on developing the more effective algorithms for digital IIR filter design. A proper analysis of the problem i.e, investigation of their difficulties has not been carried out yet. In order to provide comprehensive analysis, the fitness landscape of common instances of low-pass, high-pass, band-pass and band-stop filter design problem is plotted in fig 1. These figures are illustrated in 3-D space, while in the multi-variable hyperspace, the fitness landscape are much complicated. Further, the different choices of the variables may result in remarkable differences in the figures. It is impossible to find out the global optima by exact mathematical approaches without the enough pre-knowledge. Therefore, the following analysis are for the heuristic method. Problems belong to low-pass class, observed that the fitness landscape of low-pass digital IIR filter contains number of large basins between which are high and sharp mountains. But there are more than one basin that reach the fitness value of 0. Therefore, the optimization task on low-pass IIR filter design problem is relatively easy. Problem belongs to high-pass class; the fitness landscape has only one global optimum. The basin that contains the global optimum is smaller than the other basin. This kind of fitness landscape always misleads the optimizer or user into wrong search direction. Problem belongs to band-pass class, has the most complex fitness function landscape, which is full of local optima. The distribution of local optima is not regular at all. The exploration ability is one of the major challenges for heuristic method, is especially important to handle such problem. Problem belongs to band-stop class, the valley is towards the global optimum is very narrow. Besides this valley, the fitness landscape is relatively flat. We can conclude from these fitness landscape investigations: 1) there is no uniform fitness landscape for different digital IIR filter design problem. 2) all problems are belong to multi-modal optimization problem. 3) Different problems require different search abilities, some requires good exploitation ability and other requires strong exploration ability. Therefore, the optimal approach to design digital IIR filter is definitely a challenging problem

V. TWO-STAGE ENSEMBLE EVOLUTIONARY ALGORITHM

To solve the conflict between exploration and exploitation in digital IIR filter design, a serial two-stage optimization frame work is used in our new two-stage ensemble evolutionary algorithm. In the previous research, many hybrid algorithms have been developed to achieve better balance between exploration and exploitation [16-18]. TEEA divides

the search procedure into two stages [29]: 1) in the first stage, TEEA provide the search technique with the high-convergence speed is used to shrink the search region to more promising area. 2) In second stage, search technique with good exploration ability is used to explore the limited area extensively to get better solutions. In this paper, a self-adaptive mixed distribution based univariate estimation of distribution algorithm and modified differential evolutionary algorithm (MDE) are implemented to meet the above requirement. The detailed explanation of TEEA is as follow:

- 1) Estimation of distribution algorithms (EDs) uses the probabilistic models to describe the promising area in the solution space and use the probabilistic models to guide the generation of the candidate solutions for the next generation. To reduce the complexity of learning the probabilistic model, an univariate EDA, named as MUEDA [19] is adopted in TEEA. Univariate EDAs are suitable for relative simple problems where the decision variables are usually independent from each other [21]. For the enhancement of the exploration ability of the univariate EDA, a levy model is combined with a Gaussian model to guide the generation of the candidate solutions in MUEDA. MUEDA performs well in both convergence speed and final accuracy on multimodal problems. But MUEDA performance is unsatisfying in case of complex multimodal problems. Therefore, TEEA utilize the merits of the fast convergence of MUEDA to conduct the first stage of the search. Even in case of complex problem, MUEDA is able to shrink the search region towards the promising area within a few generations which could provide good initial status to the second stage.
- 2) Due to the limited performance of MUEDA in case of complex multimodal problems, MDE is implemented as a complement to conduct the second stage of the search in TEEA. Differential evolutionary (DE) makes use of differential information in the population and has experimentally shown very good performance on the multimodal good performance on the multimodal problem [19, 20]. The fitness landscape in a rugged region, MUEDA may fails to proceed to better evolution. MDE is an adoptively launched to re-boost the search. With the combination of both techniques , TEEA can effectively cope with complex multimodal problems.

TEEA utilizes MUEDA and MDE as its sub-optimizers. A serial structure is chosen to effectively combine the merits of two EAs in TEEA: MUEDA is implemented in the first stage to extract the global information and force the search population to approach the global optimum as close as possible. When the search space reached difficult area then MUEDA fails to generate better solutions after this MDE is triggered to continue the search. The initial population of MDE is a mixture of selected good solutions inherited from MUEDA and new random generated solutions. The MDE will continue the search until the termination condition is met. A trigger is defined based on the criteria that best solution found in current generation is not better than those in the previous generation [22].

A. Algorithm Framework Of TEEA:

TEEA utilizes MUEDA and MDE as its sub-optimizers. TEEA adopted a serial cooperative framework to take a serial cooperative framework to take advantages of the merits of these two algorithms. One key problem of serial cooperative framework is the decision when to stop using MUEDA and to start using MDE search . based on the observation the performance of MUEDA is decline may result in an extremely low convergence speed [19], trigger is defined based on the criteria that the best solution found in the criteria that the best solution found in the current generation is not remarkably better than those in the previous generations. The definition of this criterion is:

$$\frac{f_{og}(t-D) - f_{og}(t)}{f_{og}(t-D)} < \varphi$$

Where $f_{og}(t)$ is the fitness value of the best candidate solution found until generation t and φ is threshold that can be tuned to suit different problems.

The framework of TEEA is shown in table.1. to make TEEA algorithm robust for different problems, we make all the parameters of TEEA are self-adaptive to the dimension of the problem (except the population size).TEEA is self-adaptive to the dimension of the problem.

It can be observed from the table.1 that the genes of each individuals are transformed into the fixed-point numbers after being generated, both being randomly generated individuals in the beginning and generated by the evolutionary operators. During the search procedure, the optimizer TEEA works with floating-point representation.

B. Sub-optimizer 1: Self-adaptive mixed distribution based uni-variate estimation of distribution algorithm:

Probabilistic models are generally employed in estimation of distribution algorithms to describe the most promising area in the solution space and these models are used to guide the generation of the candidate solutions for the next generation. Uni-variate EDA named as MUEDA is adopted in TEEA in order to reduce the complexity of learning of probabilistic model. The exploration ability of the uni-variate EDA is enhanced by combining a levy model with Gaussian model to guide the generation of the candidates in MUEDA. In case of unimodal and many simple multimodal problems, MUEDA performs well in both convergence speed and accuracy but in case of complex multimodal problems its performance is unsatisfying. TEEA utilize the merit of fast convergence of MUEDA to conduct the first stage of the search [21, 22].

Table 1: Procedure of TEEA

TEEA

Input :

- Optimization task (including the criteria of determining the fitness values and the dimensionally);
- A transformation criteria (trigger)
- A termination conditions;

Output: the best solution is found.

Step (1) Optimization by MUEDA :

- **Step(1.0) Reproduction :** Generate the floating-point candidates by sampling the probabilistic model $P(t)$. Encode the floating-point candidates with fixed-point candidates.
- **Step(1.1)** Set $t=t+1$
- **Step (1.2) Update:** apply selection strategy to X_t and update $P(t)$ by the selected population X_t .
- **Step (1.3)** if the termination criterion is met, go to step 4; else go to step 2.

Step (2) Trigger: determine whether to trigger MDE or continue MUEDA. In case of the former, go to step (3), otherwise, go back to step (1).

Step (3) Optimization by MDE:

- **Step (3.1) MDE initialization:** inherent some solutions from the stage 1 and randomly generate the other individuals.
- **Step (3.2) Reproduction:** Generate the new floating-point population by De operators, which include mutation and crossover.
- **Step (3.3) Update:** Apply selection strategy to update X_t and X_{t+1} .
- **Step (3.4)** Set $t=t+1$
- **Step (3.5)** if the termination criterion is met, go to step 4; else go to step 3.

Step (4) Termination and output.

C. Sub-optimizer 2: Modified differential evolutionary algorithm:

Any evolutionary algorithm which is effectively coping with multimodal problems can be used as the second sub-optimizer. DE algorithm is used for this purpose because of its well-known robustness [23,24, and 25]. Limitation of MUEDA is overcome by using MDE as a second sub-optimizer to conduct the second stage of search. In case of rugged region of fitness landscape, MUEDA may be failed to provide the better solution than MDE is used to re-boost the search. In MDE, two most effective differential evolution strategies (DE/rand/1 and DE/current to best/1) are probabilistically adopted based on adaptive chaotic control mechanism. The strategy DE/rand/1 is used with a larger probability due to its universality. The parameters of CR and F are self-adaptive and dependent. The general rule is to provide relatively large CR when F is small and relatively small CR when F is large.

VI. EXPERIMENT

to provide comprehensive impression of the utility of the two-state ensemble algorithm, we adopt ten test problems belongs to the four different types of digital IIR filter with different settings which includes low-pass (LP), high-pass(HP), band-pass (BP) and band-stop (BS) filter. The detailed settings of 10 different problems are summarized in Table 2.

The parameters of digital IIR filter which are represented in equation 1, all restrict in range of $[-2 \ 2]$ but most the parameters are located in range of $[-1 \ 1]$. For word length n_L , when the parameter is in the range of $[-1 \ 1]$, then we use the $(n_L - 1)$ bits to represent the decimal part and 1 sign bit. For the parameter values in the range of the $[-2 \ -1) \cup [1 \ 2)$ then we use 1 bit to represent the integer part of the number $(n_L - 2)$ bits to represent the decimal part and 1 sign bit. In first experiment, all filter's coefficients with the word length of 16 bits are quantized. In order to verify the advantages of the TEEA, then we compare it with four-state-of-the-art EAs. The algorithms which are compare it with following algorithms shown in list below:

1. SaDE: self-adaptive differential evolution.
2. jaDE: self-adaptive control parameters in differential evolution
3. CLPSO: comprehensive learning particle swarm optimizer
4. MUEDA: self-adaptive mixed distribution based uni-variate estimation of distribution algorithm.

These four-state-of-the-art EAs have different strength, the CLPSO has top capability of handling the multi-modal problems amongst the various PSO variants; the convergence speed of the algorithm MUEDA are very high and they perform well on the uni-modal problems; whereas SaDE and jade have very good capability and the generality of handling the diverse problems.

Table 2: Settings of the test problems

Problem	Type	Pass-band	Stop-band	ω_1 (dB)	ω_2 (dB)	Order
1	LP	$[0 \ 0.5\pi]$	$[0.6\pi \ \pi]$	1	80	9
2	LP	$[0 \ 0.5\pi]$	$[0.6\pi \ \pi]$	1	110	11
3	LP	$[0 \ 0.5\pi]$	$[0.6\pi \ \pi]$	1	140	13
4	HP	$[0.6\pi \ \pi]$	$[0 \ 0.5\pi]$	1	80	8
5	HP	$[0.6\pi \ \pi]$	$[0 \ 0.5\pi]$	1	110	11
6	HP	$[0.6\pi \ \pi]$	$[0 \ 0.5\pi]$	1	140	13
7	BP	$[0.4\pi \ 0.6\pi]$	$[0 \ 0.3\pi] \cup [0.7\pi \ \pi]$	1	55	12
8	BP	$[0.4\pi \ 0.6\pi]$	$[0 \ 0.3\pi] \cup [0.7\pi \ \pi]$	1	70	14
9	BP	$[0.4\pi \ 0.6\pi]$	$[0 \ 0.3\pi] \cup [0.7\pi \ \pi]$	1	90	16
10	BS	$[0 \ 0.25\pi] \cup [0.75\pi \ \pi]$	$[0.4\pi \ 0.6\pi]$	1	55	9

A. Experimental results

In this experiment, for all the test problems, we carried out 30 independent runs for each of the five algorithm used in the paper. Statics are gathered from the results which are summarized in Table 3. The mean and the standard deviation and number of the success runs are recorded. If atleast one solution was discovered during its course whose fitness value is 0 then the run is considered to be successful. When all the 30 runs are successful the the average number of the function evaluations are required to find the global optima re recorded the depicts the convergence characteristics in terms of the best fitness value of the median run of the four compared algorithms.

B. Discussion

The problem 1-3 are belongs to the digital low-pass IIR filter design. These three low-pass digital IIR filter design problems are relatively easy as compared to the other problems. On low-pass IIR filter problems, only TEEA can provide 30 successful runs which are followed by the SaDE, which can obtain 100% succession two problems, but for the other algorithms successful runs are limited. But it is observed that the convergence speed of the TEEA is faster than the SaDE. The problem 4-6 belongs to the high-pass filter is similar to the first three ones, on which three provided the top performance MUEDA is in second place than SaDE. The high-pass filter problems requires more exploitation ability that is one of the most important strength of MUEDA. The other algorithms show poor performance in case of high-pass IIR filter design problems. The problem 7-9 belongs to the band-pass digital IIR filter problems are the most difficult problem in the test suite. Only TEEA will perform the satisfactory performance although it requires more NFS as compare than in the above problems. The problem 10 belongs to band-stop digital IIR filter design. The difficult level of the band-stop digital IIR filter problem is also very high.

In summary of the experimental results shows that the TEEA achieves the best performance on all the test problems according to an effectiveness and efficiency. It is also observed that at the beginning of an optimization procedure, the optimization speed of the TEEA is extremely high that is caused by its sub-optimizer MUEDA. As the optimization procedure is continues, the TEEA will quickly obtains the required filter while the other algorithms such as SaDE, jade, CLPSO, MUEDA are always trapped by the local optima. So, it can be concluded that the TEEA can effectively cope with the fixed-point digital IIR filter design problems.

C. Scalability Study

We design the two experimental for testing the scalability of the TEEA in order to fully benchmark TEEA as follow:

1. In above experiments the word length for all the test problems are set to be 16. It is interesting to see that the TEEA can obtain the fitness value of zero that is to find the required fixed-point digital IIR filter on the problem of all word length set. Shorter the word length more difficult is the problem. Thus the optimization procedure last longer as the word length decreases. Therefore the scalability of the TEEA in terms of the word length can be definitely considered as excellent.
2. In order show to robustness of the TEEA on the high-quality fixed-point digital IIR filter design, we apply TEEA to handle the low-pass problems with δ_2 scaling from the range of 80 to 300. The parameter settings and there experimental results are shown in Table 3. Based on the scalability test, we can conclude that the TEEA has the excellent and steady scalable capability of the handling complex fixed-point digital IIR filter design problem.

Table 3:Filter performance comparison on four types of fixed-point digital IIR filters.

Problem 1	Mean	Std	Sruns	NFE	Problem 2	Mean	Std	Sruns	NFE
TEEA	0.00E+00	0.00E+00	30	16,419	TEEA	0.00E+00	0.00E+00	30	33,520
SaDE	0.00E+00	0.00E+00	30	50,142	SaDE	0.00E+00	0.00E+00	30	103,441
jade	3.34E-01	9.06E-01	24	-	Jade	6.91E+00	7.48E+00	5	-
CLPSO	8.14E+00	4.85E+00	1	-	CLPSO	3.61E+01	9.84E+00	0	-
MUEDA	4.42E-02	2.42E-01	29	-	MUEDA	4.38E-01	2.32E+00	27	-

Problem 3	Mean	Std	Sruns	NFE	Problem 4	Mean	Std	Sruns	NFE
TEEA	6.82E-01	3.74E+00	30	55,717	TEEA	0.00E+00	0.00E+00	30	28,223
SaDE	9.38E+00	2.00E+01	24	-	SaDE	1.86E+00	2.99E+00	18	-
jade	3.66E+01	9.63E+00	0	-	Jade	4.30E+00	3.11E+00	4	-
CLPSO	7.81E+01	2.02E+01	0	-	CLPSO	1.02E+01	4.23E+00	0	-
MUEDA	3.88E+00	6.46E+00	14	-	MUEDA	2.15E+00	3.86E+00	11	-

Problem 5	Mean	Std	Sruns	NFE	Problem 6	Mean	Std	Sruns	NFE
TEEA	0.00E+00	0.00E+00	30	24,619	TEEA	0.00E+00	0.00E+00	30	36,020
SaDE	1.57E-01	5.19E-01	23	-	SaDE	7.93E+00	9.05E+00	13	-
jade	1.66E+00	1.82E+00	5	-	Jade	1.14E+01	6.55E+00	1	-
CLPSO	2.08E+00	7.77E+00	3	-	CLPSO	14.35697	3.819005	0	-
MUEDA	0.00E+00	0.00E+00	30	18,213	MUEDA	0.00E+00	0.00E+00	30	37,414

Problem 7	Mean	Std	Sruns	NFE	Problem 8	Mean	Std	Sruns	NFE
TEEA	0.00E+00	0.00E+00	30	35,590	TEEA	0.00E+00	0.00E+00	30	41,057
SaDE	9.17E-01	1.27E+00	12	-	SaDE	2.29E+00	3.05E+00	11	-
jade	2.59E+00	1.56E+00	1	-	Jade	8.33E+00	2.45E+00	0	-
CLPSO	4.57E-01	2.93E+00	6	-	CLPSO	8.229372	1.888515	0	-
MUEDA	1.21E+01	1.70E+00	0	-	MUEDA	2.88E+01	3.46E+00	0	-

Problem 9	Mean	Std	Sruns	NFE	Problem 10	Mean	Std	Sruns	NFE
TEEA	0.00E+00	0.00E+00	30	71,690	TEEA	0.00E+00	0.00E+00	30	46,117
SaDE	1.47E+01	7.92E+00	5	-	SaDE	5.05E+00	6.18E+00	13	-
jade	2.23E+01	4.40E+00	0	-	Jade	3.50E+01	5.51E+00	4	-

CLPSO	9.5742 61	7.41134 6	0	-	CLPSO	8.53E+00	9.09E+0 0	0	-
MUEDA	7.10E +01	5.11E+0 0	0	-	MUEDA	2.19E+02	1.03E+0 2	2	-

Table 4 the t-test results of comparing TEEA with the other four algorithms (the final results of 25 runs)

	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6
TEEA vs SaDE	=	=	s+	s+	s+	s+
TEEA vs jaDE	s+	s+	s+	s+	s+	s+
TEEA vs CLPSO	s+	s+	s+	s+	s+	s+
TEEA vs MUEDA	+	+	s+	s+	=	=
	Problem 7	Problem 8	Problem 9	Problem 10		
TEEA vs SaDE	s	s+	s+	s+		
TEEA vs jDE	s	s+	s+	s+		
TEEA vs CLPSO	+	s+	s+	s+		
TEEA vs MUEDA	+	s+	s+	s+		

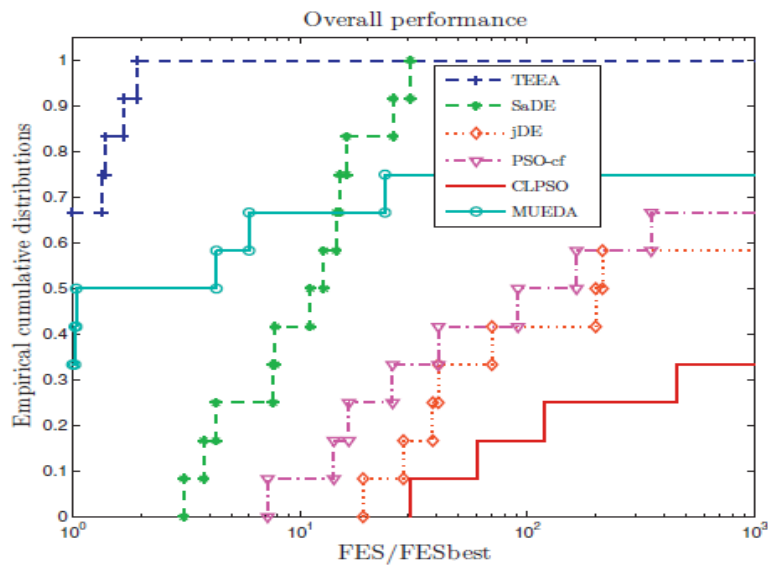


Fig 1. Empirical distribution of normalized success performance on digital IIR filter design problems.

Table 3. Scalability test of LP fixed-point digital IIR filter.

Problem	Type	δ_1 (dB)	δ_2 (dB)	Order	Sruns	NFE
S1	LP	1	80	9	30	16,419
S2	LP	1	110	11	30	33,220
S3	LP	1	140	13	30	55,717
S4	LP	1	220	21	30	100,400
S5	LP	1	300	27	30	158,900

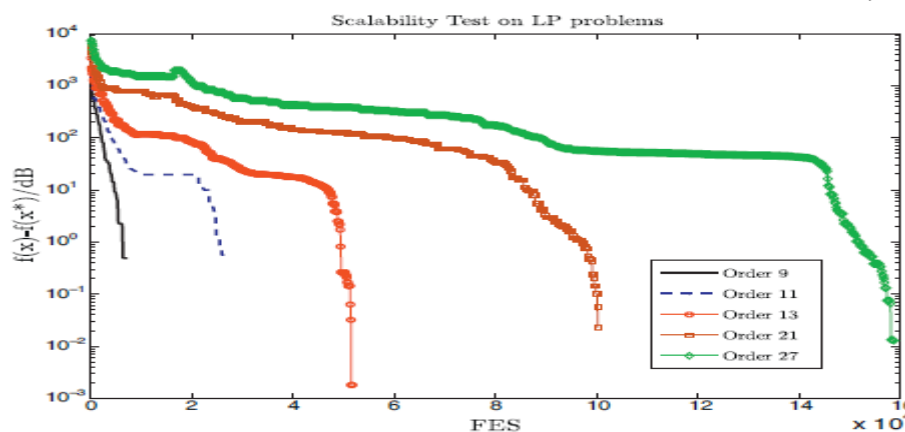


Fig. 2 Scalability test of TEEA on LP problems

VII. CONCLUSION AND FUTURE SCOPE

The fixed-point representation can effectively save the computational resources and is more convenient for the direct realization on the hardware application, we design the Two-stage ensemble evolutionary algorithm for the fixed-point representation digital IIR filter design. To evaluate the performance of the TEEA, we designed two experiments: first is to show the advantages of the TEEA over the other state-of-the-art EAs, we can experimentally compare TEEA with four EAs, including saDE, jade, MUDEA, CLPSO. This experimental result verifies that the TEEA has better effectiveness, efficiency and the higher successful rate. Secondly, to evaluate the scalable characteristics of the TEEA, we apply it to some more difficult problems with the shorter word length, higher order and lower δ_2 . The scalability study confirms that for the fixed-point digital IIR filter, the shorter will be the word length more difficult is the problem. The performance of the TEEA on the scalability test problems is still satisfactory. It can be concluded that the TEEA is definitely suitable for the fixed-point digital IIR filter design. But there are still many unsolved problems of this field, such as: how to reduce the computational cost on the high-quality filter design, how to obtain the digital IIR filter with shorter word length, and so on is the future work.

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