



Design of Digital FIR Filters Using Differential Evolution Algorithm Based on Reserved Genes

Er. Karamjeet Singh

ECE Department & BBSBEC, Fatehgarh Sahib
India

Navdeep Kaur

ECE Department & BBSBEC, Fatehgarh Sahib
India

Abstract- *The research on optimal design of Finite impulse response (FIR) filter based on various optimization techniques, including evolutionary algorithm (EAs), has gained much attention in recent years. The Digital filter design problem involves the determination of coefficients to meet a set of design specification. Evolutionary algorithms are able to solve the optimization problems by imitating some aspects of natural evolution. Differential Evolution is an effective approach for solving these problems. The New Differential evolution algorithm based on reserve genes (Eclectic Differential Evolution) have been used here for the design of finite impulse response filters. This algorithm is applied in order to obtain the actual filter response as close as possible to the ideal response. In this method, the new vectors can be produced by the combination of genes of selected chromosomes. These new vectors as the new individuals are evolved with other individuals in the population and increase the diversity of population. This method enhances the ability to search the optimal solution of algorithm and avoid local optimum solution. The performance of this method has been compared to other algorithms like DE, eDE and new eDE.*

Keywords— *FIR filter, Genetic Algorithm, Differential Evolutionary Algorithm, Eclectic DE*

I. INTRODUCTION

Filtering is a process by which frequency spectrum of a signal can be modified, re-shaped, or manipulated according to the desired specifications. The digital filter is a digital system that can be used to filter the discrete-time signals. Due to the vast application of signal system, the topic of digital signal process attracts many notifications of researchers. The design of filter then becomes an import problem to be solved. There are two different types of digital filters: finite impulse response (FIR) filters and infinite impulse response (IIR) filters. An FIR filter is that whose impulse response is of finite duration. The output of such a filter is calculated from the current and previous input values. This type of filter is hence said to be non-recursive filter. On the other hand, an IIR filter is one whose impulse response continues for ever in time (infinite duration). The current output of IIR filter depends upon previous output values. This type of filter is hence said to be recursive filter. Compared to IIR filters, the main advantages of FIR filters are (1) finite impulse response,(2)easy to optimize,(3) linear phase,(4) they are always stable filters. FIR filters has been widely applicable for the image processing, data transmission, signal processing etc. There are also many traditional methods which design FIR filters, such as window function, Frequency Sampling, least mean square error etc. Windowing method is the most popular among them. These various types of windows limit the infinite length impulse response of ideal filter into a finite window to design an actual response. But various windowing methods do not allow proper control of the frequency response in the various frequency bands and other filter parameters such as transition width [1].

The nature of designing of FIR filters is an optimization problem. Therefore, some researchers have attempted to develop the design methods based on various modern global optimization algorithms. Currently, heuristic optimization algorithms such as genetic algorithms, tabu search, simulated annealing algorithms and particle swarm intelligence have been widely used in the optimal design of digital filters[1],[4]-[15].

Genetic algorithms (GA) [2]-[3] is used for the FIR filter design in several works, such as[11]-[12],[16].Although standard GA perform well for finding the promising regions of the search space, GA has two disadvantages : slow convergence and precocity [17]-[18]. In order to overcome these disadvantages of the GA in numeric optimization problems, a differential evolution (DE) algorithm has been introduced by Storn and Price [19]. DE is formed by Ken Price who attends to solve the Chebychev polynomial problem which is proposed by Rainer Storn. Ken Price proposed a method of disturbing the group vector by a difference vector and then the solution of the Chebychev polynomial problem is improved. Consequently, researchers adopted DE for solving their problems. There are mainly three types of DE. The basic idea of DE is that the weighted difference between the individuals is added to a third vector which is optimized using selection, crossover and mutation operators as in GA. The offspring is selected for the next generation only if it has a better fitness than its parent. There are only a few studies related to the application of the DE algorithm for designing digital filter [5]-[6],[20]-[23].In

[21]-[22], the method of designing an IIR filter using a DE algorithm is investigated. In [23], the article proposes a solution to the problem of designing of multi criterion filter. In [5]-[6], [20] explains; the design of digital FIR filters based on the DE algorithm. In [7], the article is proposed for designing real two-dimensional (2-D) finite duration impulse response (FIR) digital filters with complex valued frequency responses. In [11], the article is proposed for design of digital FIR filters with sum power-of-two (SPT) coefficients.

In this work, DE algorithm based on reserved genes is proposed. In this new DE algorithm, the new vectors are produced by the reserved genes. Hence, it can increase the diversity of population and better search the optimal solution. It is chosen to design real-coefficient FIR filters, due to their importance in engineering practice. Globally minimizing such a numerical function means to find the optimal filter parameters, this new DE algorithm is sufficient to the applications. The nature of designing digital filter is to find global optimization in a high dimensional search space. The new DE algorithm is more effective in search in a high dimensional search space compared with original differential evolution (DE) or GA. The comparison of performance of the design methods like DE based on reserved genes, DE, GA, and least square algorithm (LSQ) is presented for digital FIR filters.

The paper is organized as follows-Section 2 presents a basic DE algorithm, especially Eclectic Differential Evolution (eDE). Section 3 describes DE3 based on reserved genes. Section 4 describes the application of this new DE algorithm to the design of digital FIR filters. Section 5 compares the new and modified algorithm with other algorithms. It concludes paper in the section 6.

II. DIFFERENTIAL EVOLUTIONARY ALGORITHM

A DE algorithm for global optimization has been proposed by Storn and Price in year 1995. It is a new heuristic approach for minimizing the non-linear and non-differentiable continuous space function. It uses few control parameters, which has made the algorithm very much popular particularly for parallel computation purpose. This particular type of evolutionary approach comprises of four steps: namely Initialization, Mutation, Crossover or Recombination and Selection. It mainly involves the following parameters: (1) size of population N, (2) dimension D (also known as chromosome length) of a individual (also known as variables), (3) Scaling factor F, (4) Crossover probability CR. In DE, N solution vectors are randomly created at the begin. This population is successfully improved by applying operators like mutation, crossover, and selection. The types of DE, are called DE1, DE2, DE3. The basic algorithm of new DE algorithm is DE3.

A. DE3 (Eclectic Differential Evolution):

The evolution of DE3 is described by the equation:

$$V_{i,G+1}(t) = [x_{r,G}(t) + x_{i,G}(t)]/2 + F \cdot [x_{r,G}(t) - x_{t,G}(t) + x_{r1,G}(t) - x_{r2,G}(t)] \quad (1)$$

in which $r, r1, r2 \in [0, N-1]$ are randomly chosen and they must be different from each other. Fitness of $\vec{x}_{r,G}(t)$ is not less than the fitness of $\vec{x}_{i,G}(t)$. F is usually set between random numbers 0 and 2.

B. Crossover and Selection

The parent vector is mixed with the mutated vector to produce trial vector as:

$$U_{i,G+1}(t) = \begin{cases} v_{i,G+1}(t) & \text{if } (\text{rnd}_t \leq \text{CR}) \\ X_{i,G}(t) & \text{if } (\text{rnd}_t > \text{CR}) \end{cases} \quad (2)$$

Or $t = \text{rn}_i$
And $t \neq \text{rn}_i$

in which $\text{rnd}_t \in [0, 1]$ is also a random number, $\text{CR} \in [0, 1]$, $\text{rn}_i \in D$. The performance of the produced trial vector and its parent are compared and the better one is selected. Then the better one of the trial solution and its parent wins the competition; this provides the significant advantage of converging performance over the GA.

III. DE ALGORITHM BASED ON RESERVED GENES

In this paper, eclectic differential evolution is the basic form of DE algorithm. The new eDE algorithm employs reserved genes of selected individual to increase the diversity of population during the evolution process. This method enhances the ability of global search. This algorithm is effective to avoid the problem of local optimal solution. The average fitness of the previous generation population is compared with the average population fitness of the present generation to determine whether convergence. If the result of the comparison is same and the fitness of best individual is also equal, it will use reserved genes of selected individuals to be out of local optimum solution.

A. Array reserved genes

Initially, population is constructed from the solutions randomly and distributed within the search space uniformly. 2 arrays which are named as elite and loser are used. The elite array stores the best individual and the loser array stores the worst individual. The genes of the individual in these arrays can be replaced with other genes in each generation. One different combination of the genes can form a new individual and this new individual is implemented by using differential evolution with other individuals. The population diversity is improved by this method. 2 arrays which are named as best1 and best2

must be set. They stored the contemporary global best individual and the previous global best individual. These arrays must reserve the contemporary and the previous average fitness, too.

B. Differential evolution

Selected an individual from population is named x , then another individual named y whose fitness is not less than the fitness of individual x is randomly selected. The other two individuals are randomly selected. These selected individuals employ the equation (3), (4) to perform crossover and mutation step. This method will not only maintain the population diversity, but also improves the convergence speed. It is known as an eclectic method of selection. It can generate new individual named as z which is compared with x and comparative results are as follows:

- a) The fitness of individual z is respectively higher than the fitness of x and y . It will be replaced x with z . The selected part of genes of z are stored in elite array and the selected part genes of x are stored in loser array. In this algorithm, selected part genes are randomly selected from these individuals.
- b) The fitness of z is higher than the fitness of x but smaller than fitness of y . It will be replaced x with z . The selected part genes of individual y are stored in elite array and the selected part genes of x are stored in loser array.
- c) The fitness of z is smaller than the fitness of x and y . The z is eliminated. But the selected part genes of individual y are stored in elite array and the selected part genes of z are stored in loser array. Then the individual z and elite array employs the equation (3), (4) to perform crossover and mutation. It generates the new individual named $z1$. If the fitness of $z1$ is higher than the fitness of x , it will be replaced x with $z1$. It performs same crossover and mutation process. The purpose is to add new individual to the population.

C. Mutation

First, n individuals where $n < N$ is selected. It has been suggested $n = 10$ to be more suitable in this paper. Second, these selected individuals and elite array employs the equation (3), (4) to perform crossover and mutation respectively. Probability of mutation is very small.

D. To reduce the number of identical individuals

If each generation population contains many individuals with the same fitness, it will be easy to fall into local optimum and easy to convergence. Hence, it must eliminate the same individuals. But, it reserves a small number of the same individual for the convergence. The number of the same individuals is counted in each generation. It can only reserve m the same individuals in each generation. It has been suggested $m = 5$ to be more suitable in this paper. If the number is equal to m , m_{th} is selected to perform crossover and mutation process with loser array. The generated new individual is named as v .

- a) If fitness of v is higher than the fitness of m_{th} individual. It will be replaced m_{th} individual with v . The selected part genes of individual v are stored in elite array and the selected part genes of the m_{th} individual are stored in loser array.
- b) If fitness of v is small than the fitness of m_{th} individual. It will also be replaced m_{th} individual with the v . Then the selected part genes of v are stored in loser array and the selected part genes of m_{th} individual are stored in elite array. This can be also considered as degraded. But it increases the diversity of the population. Then the bad individual must replace one of the same individual. But this does not cause degradation to the entire population.

E. Examination whether convergence

While the best individual within both the previous generation and the present generation is equal, the average fitness of the previous generation and the present generation is compared. It is determined whether convergence by this method. If the average fitness of the previous generation and the present generation are same, it will perform differential evolution to use the elite array and the loser array. The evolved new individual is called $v1$.

- a) If the fitness of $v1$ is greater than the fitness of elite. It will be replaced elite with $v1$.
- b) If the fitness of $v1$ is smaller than the fitness of loser. It will be replaced loser with $v1$.

It will replace one individual with the $v1$. This individual is also randomly selected and differential evolution is run $n1$ times. A total of $m1$ individuals are replaced with the different $v1$. In this paper, $n1$ is taken as 100.

IV. DIGITAL FILTER DESIGN

The transfer function of an FIR filter is given by the equation 3

$$H(z) = \sum_{n=0}^N a_n z^{-n} \quad (3)$$

where a_n represents the filter coefficients to be determined in the design process and N represents the polynomial order of the function. There are various parameters in filter design. They are the stopband and passband normalized frequencies (ω_s, ω_p), the passband and stopband ripple δ_s and δ_p , the stopband attenuation and the transition width. In any filter design problem, some of the parameters are fixed while others are determined. In this paper, the new DE3 algorithms are used in order to obtain the actual filter response as close as possible to the ideal response. This article discusses the most widely used FIR filter whose length is odd and the order is even. The length of filter is $N + 1$ means the number of a_n is also $N + 1$. The

individuals is represented by an . In each iteration of algorithm, these individuals generate new offspring, which is the new set of coefficients. Fitness of each individuals is calculated using the new coefficients. This fitness is used to improve the search in each iteration and result obtained after a certain number of iterations or after the error is below a certain limit is taken to be the final result. Because its coefficients are matched, the problem dimension reduces by a factor of 2. The $N/2 + 1$ coefficients are then flipped and concatenated to calculate the required $N + 1$ coefficients. The least mean squared error is used to evaluate the individuals. It calculates the mean squared error between the frequency response of the ideal and the actual filter. An ideal filter has a magnitude response of 1 on the passband and a magnitude of 0 on the stopband. Hence, the error for this fitness function is the squared difference between the magnitudes of this filter and the filter designed using the evolutionary algorithms. The individuals having smaller evaluation values represent the better filters with better frequency response. The expression of the LMS function is shown below:

$$E = \min \sum_{k=0}^K [H_i(w_k) - H_d(e^{jwk})]^2 \tag{4}$$

where $H_i(w_k)$ is the magnitude response of the ideal filter and $H_d(e^{jwk})$ is the magnitude response of the designed filter. And K is the number of samples which is used to calculate the error. The fitness function of the algorithm is the equation (4).

V. RESULTS

The simulations have been realized for digital filters with the order of 20 and 30. That means the length of coefficients are 21 and 31, respectively. The sampling number was taken as 100. In each case, passband and stopband cut off frequencies are taken as 0.25 and 0.3 respectively. The algorithm is run by 10 times. The best coefficients of obtained from the filter with the order of 20 design have been found by the two DE methods are given in Table I

TABLE I: Coefficients of Designed Filter with order of 20

$a(n)$	The new eDE	eDE
$a(1) = a(21)$	0.554768	0.553634
$a(2) = a(20)$	0.623148	0.659745
$a(3) = a(19)$	-0.104930	0.095943
$a(4) = a(18)$	-0.175062	-0.173815
$a(5) = a(17)$	0.092302	0.131932
$a(6) = a(16)$	0.072044	0.050340
$a(7) = a(15)$	-0.073634	-0.079656
$a(8) = a(14)$	-0.023390	-0.001796
$a(9) = a(13)$	0.053707	-0.048625
$a(10) = a(12)$	0.001293	-0.005897
$a(11)$	-0.033445	-0.028730

TABLE II: Coefficients of Designed Filter with order of 30

$a(n)$	The new eDE	eDE
$a(1) = a(31)$	0.537893	0.573111
$a(2) = a(30)$	0.629212	0.618676
$a(3) = a(29)$	-0.073100	-0.123265
$a(4) = a(28)$	-0.204775	-0.158371
$a(5) = a(27)$	0.066889	0.107711
$a(6) = a(26)$	0.096059	0.069130
$a(7) = a(25)$	-0.045437	-0.086152
$a(8) = a(24)$	-0.053169	-0.024950
$a(9) = a(23)$	0.065302	0.023417
$a(10) = a(22)$	0.037115	-0.037643
$a(11) = a(21)$	-0.039848	-0.004266
$a(12) = a(20)$	-0.007813	0.012842
$a(13) = a(19)$	0.034844	0.000519
$a(14) = a(18)$	-0.016373	-0.014295
$a(15) = a(17)$	-0.023184	-0.033998
$a(16)$	0.012797	-0.01714

TABLE III: LMS ERROR VALUES FOR ALGORITHM

Algorithms	The filter with the order of 20	The filter with the order of 30
The new eDE (best)	0.0425	0.0808
The eDE(best)	0.3565	0.2407
The new eDE (worst)	0.2097	0.6098
The eDE(worst)	2.2602	4.4554
The new eDE (Average)	0.1077	0.2188
The eDE(Average)	0.9991	2.2631

The magnitude responses of the digital FIR filters designed using the new eDE, eDE, DE and LSQ algorithms for the filter of 20th order are given in Figure 1. As seen from Figure 1, for the passband region, the new eDE produces a better response than the others. The filters designed by the new eDE algorithm based on reserved genes have sharper transition band responses than that produced when the LSQ algorithm and the eDE algorithm are used. For the stopband and passband region, the filter designed by the new eDE method produce better responses than the others. The best coefficients that obtained from the filter having order of 30 have been found by the two methods are given in Table II. Notice that the population size of the eDE is 400 rather than 100 because it can produce better coefficients for comparison.

The magnitude responses of the digital FIR filters designed using the new eDE, eDE algorithms for the filter of 30th order are given in Figure 2. As seen from Figure 2, for the passband region, the new eDE produces a better response than the others. The filters designed by the new eDE algorithm have sharper transition band responses than that produced when the basic eDE algorithm are used. For the stopband and passband region, the filters designed by the new eDE methods produce better responses than the others. In order to compare the algorithms in terms of their convergence speed, Figure 3 shows which the evolution of best solutions obtained when the new eDE and the eDE algorithm are employed. From the figures drawn for this filter, it is concluded that the new eDE algorithm is significantly faster than the eDE algorithm for finding the optimum filter. The new eDE algorithm converges to a much lower fitness in lesser number of iterations.

In Table III, LMS error values obtained for the two algorithms are given. From the table it is clear that the performances of the new eDE algorithm are much better to each other in terms of LMS error. When the new eDE algorithm is employed for the filter with the order of 30, the population size of the eDE is taken as 400.

For the higher order filter, the eDE algorithm find the better solution is more difficult .When the order of the filter is 30, the eDE algorithm must increase the size of population for better solution. Otherwise, a large number of iterations are required. Experimental results show that the new eDE algorithm is more fit to design the filters having higher order. As the genes are reserved, the global search capability of the new eDE algorithm is improved. So, it is better than the basic eDE algorithm especially treatment of high-dimensional problems.

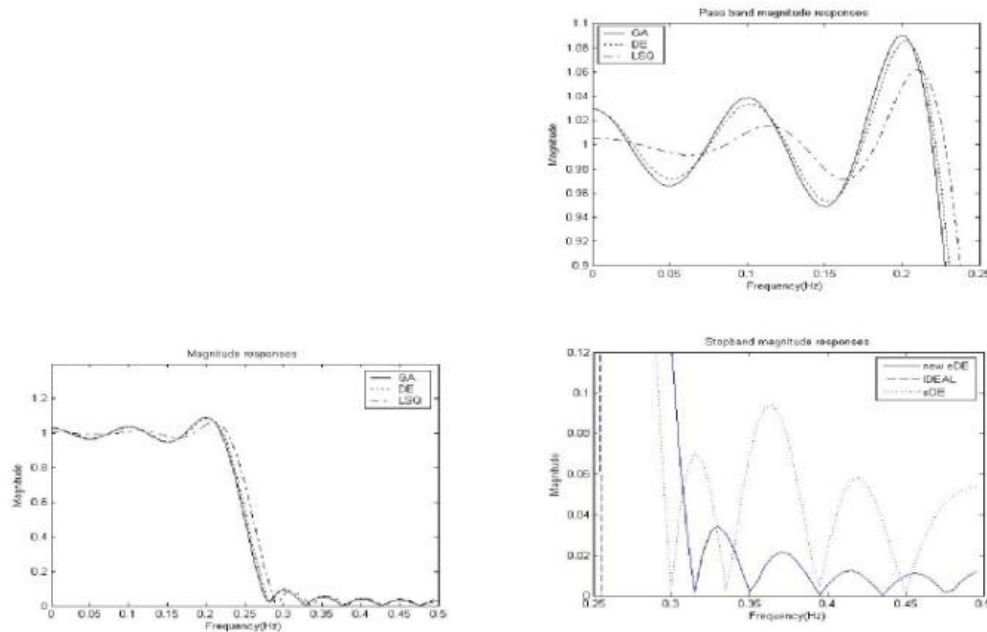
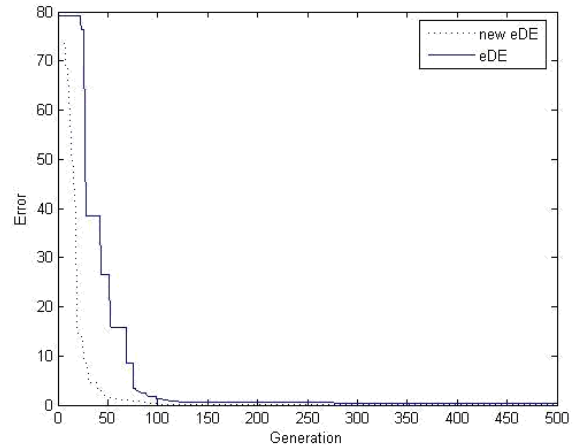
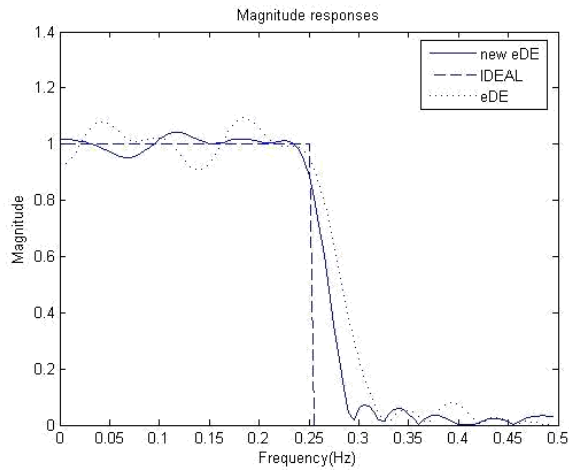


Figure 1. Magnitude Response of the Filter with order 20



(a) Error graph for design filters with the order of 20.

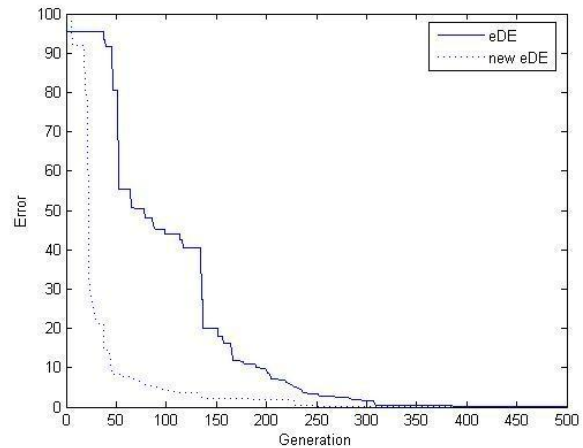
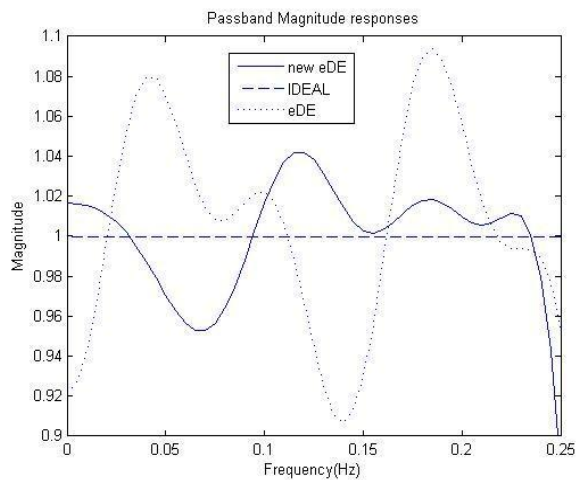


Figure 2 Magnitude Response of the filter with order 30

(b) Error graph for design filters with the order of 30

VI. CONCLUSION

The new eDE algorithm has been applied to the design of digital FIR filters with different orders that is 20 and 30. As a result of individual's reserved genes, the diversity of population is increased. It can save individual from being trapped in local minima, thus guiding them towards the global solution and avoids local solution. In many different experiments, the new eDE algorithm is more suitable to design the filters with higher order and the eDE or GA is fit to design the filters with lower order. It would perform much better and faster to obtain approximation of filter coefficients than others. This method gains the ability to search the optimal solution of algorithm. This algorithm is effective to avoid the local optimal solution. Results show that it is better than the eDE algorithm especially treatment of high-dimensional problems. But with this fitness function, lower ripples were also achieved but at the cost of wider transition width. The fitness function can be in combination with other algorithms to reduce the cost of wider transition width. Further research is also required to improve this new eDE algorithm and to be integrated with evolvable hardware.

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